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UNIVERSITI SAINS MALAYSIA

First Semester Examination  
Academic Session 2004/2005

October 2004

**ZCT 211E/2 - Vector Analysis**  
*[Analisis Vektor]*

Duration : 2 hours  
*[Masa : 2 jam]*

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Please check that this examination paper consists of **FOUR** pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi **EMPAT** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer all **FIVE** (5) questions. Students are allowed to answer all questions in Bahasa Malaysia or in English.

**Arahan:** *Jawab kesemua **LIMA** (5) soalan. Pelajar dibenarkan menjawab semua soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]*

1. (a) What vector must be added to the two vectors  $\hat{i} - 2\hat{j} + 2\hat{k}$  and  $2\hat{i} + \hat{j} - \hat{k}$ , so that the resultant may be a unit vector along the x-axis?  
 [(a) Apakah vektor yang perlu ditambah kepada dua vektor  $\hat{i} - 2\hat{j} + 2\hat{k}$  dan  $2\hat{i} + \hat{j} - \hat{k}$  supaya paduannya adalah satu vektor unit sepanjang paksi-x?  
 (20/100)
- (b) If  $\vec{A} = 4\hat{i} + 6\hat{j} + 2\hat{k}$  and  $\vec{B} = \hat{i} + 6\hat{j} + \hat{k}$ , find the magnitudes and direction cosines of  $(\vec{A} + \vec{B})$  and  $(\vec{A} - \vec{B})$ .  
 [(b) Jika  $\vec{A} = 4\hat{i} + 6\hat{j} + 2\hat{k}$  dan  $\vec{B} = \hat{i} + 6\hat{j} + \hat{k}$ , cari magnitud dan kosinus arah bagi  $(\vec{A} + \vec{B})$  dan  $(\vec{A} - \vec{B})$ .  
 (40/100)
- (c) A car is driven eastward for a distance of 5 kilometers, then northward for 3 kilometers and then in a direction  $30^\circ$  east of north for 10 kilometers. Draw the vector diagram and determine the total displacement of the car from its starting point.  
 [(c) Sebuah kereta dipandu ke timur untuk jarak 5 kilometer, kemudian ke utara untuk jarak 3 kilometer, dan seterusnya di dalam arah  $30^\circ$  timur daripada utara untuk jarak 10 kilometer. Lukiskan gambarajah vektor dan tentukan jumlah sesaran kereta tersebut daripada titik permulaannya.  
 (40/100)
2. (a) If  $\vec{a}$  and  $\vec{b}$  are unit vectors and  $\theta$  is the angle between them, show that  
 [(a) Jika  $\vec{a}$  dan  $\vec{b}$  adalah vektor unit dan  $\theta$  adalah sudut diantaranya, tunjukkan
- $$\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|.$$
- (20/100)
- (b) Show that  $\vec{a}$  is perpendicular to  $\vec{b}$  if  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ .  
 [(b) Tunjukkan bahawa  $\vec{a}$  tegak lurus kepada  $\vec{b}$  jika  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ .  
 (20/100)
- (c) If  $\vec{A} = 4\hat{i} - 5\hat{j} + 3\hat{k}$ ,  $\vec{B} = 2\hat{i} - 10\hat{j} - 7\hat{k}$  and  $\vec{C} = 5\hat{i} + 7\hat{j} - 4\hat{k}$  deduce the values of  
 [(c) Jika  $\vec{A} = 4\hat{i} - 5\hat{j} + 3\hat{k}$ ,  $\vec{B} = 2\hat{i} - 10\hat{j} - 7\hat{k}$  dan  $\vec{C} = 5\hat{i} + 7\hat{j} - 4\hat{k}$  dapatkan nilai bagi

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(i)  $(\bar{A} \times \bar{B}) \cdot \bar{C}$  and [dan]

(ii)  $\bar{A} \times (\bar{B} \times \bar{C})$ .

(60/100)

3. (a) Show that  $\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$ .

[(a) Tunjukkan bahawa  $\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$ .]

(15/100)

(b) Find the volume of a parallelepiped whose three coterminus edges are described by the vectors  $\hat{i} + 2\hat{j}$ ,  $4\hat{j}$  and  $\hat{j} + 3\hat{k}$ .

[(b) Cari isipadu bagi paralelepiped di mana tiga sisi koterminus adalah diberi oleh vektor  $\hat{i} + 2\hat{j}$ ,  $4\hat{j}$  dan  $\hat{j} + 3\hat{k}$ .]

(15/100)

(c) Differentiate  $\frac{\bar{r} \times \bar{a}}{\bar{r} \cdot \bar{a}}$ ,  $\bar{a}$  being constant vector and  $\bar{r}$  is the position vector.

[(c) Bezakan  $\frac{\bar{r} \times \bar{a}}{\bar{r} \cdot \bar{a}}$ ,  $\bar{a}$  adalah vektor malar dan  $\bar{r}$  ialah vektor kedudukan.]

(20/100)

(d) Find  $\frac{\partial s}{\partial y}$ ,  $\frac{\partial^2 s}{\partial x^2}$ ,  $\frac{\partial^2 s}{\partial y^2}$ ,  $\frac{\partial^2 s}{\partial x \partial y}$  for the following functions:

[(d) Cari  $\frac{\partial s}{\partial y}$ ,  $\frac{\partial^2 s}{\partial x^2}$ ,  $\frac{\partial^2 s}{\partial y^2}$ ,  $\frac{\partial^2 s}{\partial x \partial y}$  bagi fungsi-fungsi berikut]

(i)  $\bar{s} = x \cos y \hat{i} + x \sin y \hat{j} + a e^{my} \hat{k}$ , where  $m$  is an integer [di sini  $m$  adalah integer]

(ii)  $\bar{s} = \frac{1}{2} \bar{a}(x+y)\hat{i} + \frac{1}{2} \bar{b}(x-y)\hat{j} + \frac{1}{2} xy \hat{k}$ .

(50/100)

4. (a) Prove  $\nabla r^n = nr^{n-2} \bar{r}$ , where  $n$  is an integer.

[(a) Buktikan  $\nabla r^n = nr^{n-2} \bar{r}$ , di sini  $n$  adalah integer]

(25/100)

(b) Show that  $\nabla \cdot \nabla \phi = \nabla^2 \phi$  where  $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ .

[(b) Tunjukkan bahawa  $\nabla \cdot \nabla \phi = \nabla^2 \phi$  di sini  $\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ .]

(25/100)

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- (c) Find curl of the following functions:  
 [(c) Cari keikalan fungsi-fungsi berikut:]

(i)  $\vec{f} = \frac{x\hat{i} + y\hat{j}}{x + y}$

(ii)  $\vec{g} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

(50/100)

5. (a) (i) Write the Maxwell's equations governing electromagnetic fields in a dielectric medium.  
 [(a) (i) Tuliskan persamaan Maxwell bagi medan elektromagnet dalam medium dielektrik.]

- (ii) What was the Maxwell's contribution toward Ampere's law?  
 [(ii) Apakah sumbangan Maxwell terhadap Hukum Ampere?]

(25/100)

- (b) (i) Write the Stoke's theorem in space.  
 [(i) Tuliskan teorem Stokes dalam ruang.]

(ii) If  $\oint \vec{E} \cdot d\vec{r} = -\frac{1}{c} \frac{\partial}{\partial t} \iint_s \vec{H} \cdot d\vec{s}$ ,

[(ii) Jika  $\oint \vec{E} \cdot d\vec{r} = -\frac{1}{c} \frac{\partial}{\partial t} \iint_s \vec{H} \cdot d\vec{s}$ ,

where  $s$  is any surface bounded by curve  $c$ ,

[di sini sebarang permukaan  $s$  disempadani oleh lengkung  $c$ .]

show that  $\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$

[tunjukkan bahawa  $\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}$ ]

(50/100)

- (c) Given that  $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$  when  $t = 2$  and  $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$  when  $t = 3$ .

- [(c) Di beri  $\vec{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$  bila  $t = 2$  dan  $\vec{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$  bila  $t = 3$ .

Show that  $\int_2^3 \vec{r} \cdot \frac{d\vec{r}}{dt} dt = 10$ .

Tunjukkan  $\int_2^3 \vec{r} \cdot \frac{d\vec{r}}{dt} dt = 10$ .

(25/100)