TERJEMAHAN

UNIVERSITI SAINS MALAYSIA

First Semester Examination
1998/99 Academic Session

August/September1998

ZCT 211/2 - Vector Analysis

Time: [2 hours]

Please check that the examination paper consists of NINE printed pages before you commence this examination.

Answer EIGHT questions only. Candidates may choose to answer all questions in the Malay Language. If candidates choose to answer in the English Language, it is compulsory to answer at least one question in the Malay Language.

1. Prove from the definition $\vec{A}.\vec{B} = AB\cos\theta$ that $\vec{i}.\vec{i} = 1$, $\vec{i}.\vec{j} = 0$ and $\vec{i}.\vec{k} = 0$ where $\vec{i}, \vec{j}$ and $\vec{k}$ are unit vectors along x, y and z axes.

(20/100)

Hence prove that scalar product with $\vec{i}$ acts as a projection operator on the x axis, i.e. $\vec{i}.\vec{A} = A_x$.

(10/100)

Vector $\vec{A}$ is equal to $A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ in one set of axes and to $A_{x'}\vec{i}' + A_{y'}\vec{j}' + A_{z'}\vec{k}'$ in another. By using projection with $\vec{i}$ prove that $A_x = c_{xx'}A_{x'} + c_{xy'}A_{y'} + c_{xz'}A_{z'}$ where $c_{xx'}$ is the cosine of the angle between the x and x' axes and so on.

(30/100)

State the corresponding results for A_y and A_z. (20/100)

State the transformation between scalars S and S'. (20/100)

2. $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$ is a vector field and ϕ is a scalar field. State the definitions of $\nabla.\vec{A}, \nabla \times \vec{A}, \nabla\phi$ and $\nabla^2\phi$.

(40/100)

...2/-

Prove from the definitions that $\nabla.(\nabla \times \bar{A}) = 0$ and $\nabla \times (\nabla\phi) = 0$ for any fields $\bar{A}$ and ϕ.

(20/100)

Calculate $\nabla\phi$ and $\nabla^2\phi$ for the scalar field $\phi(\bar{r}) = r = (x^2 + y^2 + z^2)^{1/2}$.

(40/100)

3. Two vector fields are $\bar{A} = 2xz^2\bar{i} + (y - z)\bar{j} - 3xyz\bar{k}$ and $\bar{B} = y^3 i - 4z\bar{j} + x^2z^2\bar{k}$. Evaluate $\nabla \times \bar{A}, \nabla \times \bar{B}, \nabla \times (\bar{A} \times \bar{B})$ and $\nabla.(\bar{A} \times \bar{B})$.

(100/100)

4. The position of a particle at t is given by

$$\bar{r}(t) = a\cos(\omega t)\bar{i} + a\sin(\omega t)\bar{j} + ut\bar{k}$$

Find expressions for the velocity $\bar{v}(t)$ and the acceleration $\bar{f}(t)$.

(20/100)

Describe in words the motion that $\bar{r}(t)$ represents and suggest a physical arrangement in which a particle would move in this way.

(50/100)

How would you modify the experimental arrangment to produce the path

$$\bar{r}(t) = a\cos(\omega t)\bar{i} + a\sin(\omega t)\bar{j} + (ut + \beta t^2)\bar{k}$$

(30/100)

5. Calculate the double integral $\int f(x,y)dxdy$ over the square $0 \le x \le a$, $0 \le y \le a$ where

(a) $f(x,y) = (x - y)^3$ (30/100)
(b) $f(x,y) = x^2y^3$ (30/100)

Calculate the volume integral $\int_V f(\bar{r})dV$ where V is the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ and $f(\bar{r}) = \sin(\pi x)\sin(\pi y)\sin(\pi z)$.

(40/100)

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6. State Gauss's and Stokes's theorems, defining carefully the terms you use. (60/100)

By using Gauss's theorem or otherwise evaluate $\int_S \vec{F}.\vec{n}dS$ where $\vec{F} = x^4\vec{i} + y^3\vec{j} + z^2\vec{k}$ and S is the surface of the cube $0 \le x \le a$, $0 \le y \le a$, $0 \le z \le a$. (40/100)

7. Write down the differential form of the equation of continuity relating the density ρ of a conserved quantity to the conservation current $\vec{J}$. (20/100)

Transform the equation into integral form and explain why the equation does describe continuity. (40/100)

Two of Maxwell's equations are $\nabla.\vec{E} = \rho / \varepsilon_0$ and $\nabla \times \vec{H} = \vec{J} + \varepsilon_0 \partial\vec{E} / \partial t$. Prove that the equation for continuity of charge follows from these equations. (40/100)

8. Poisson's equation for the gravitational potential ϕ is

$$\nabla^2\phi = -4\pi G\rho$$

Assume that ρ is constant and that ϕ is spherically symmetric, i.e. depends only on the radial coordinate r, so that $\nabla^2\phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right)$.

Integrate Poisson's equation twice to show that the general solution is

$$\phi = -\frac{2\pi G\rho}{3}r^2 + \frac{K_1}{r} + K_2$$

where K_1 and K_2 are constants of integration. (60/100)

.../4-

Find the gravitational field $\vec{F} = \nabla\phi = (F_r, 0, 0)$ and explain the meaning of the two terms in F_r.

(40/100)

9. It is known that one solution of the diffusion equation

$$\frac{\partial n}{\partial t} = C\frac{\partial^2 n}{\partial x^2}$$

is the function

$$n(x,t) = \frac{1}{(4\pi Ct)^{1/2}}\exp(-x^2/4Ct)$$

Confirm this by substitution.

(50/100)

Use the standard integral $\int_{-\infty}^{\infty}\exp(-y^2)dy = \pi^{1/2}$ to prove that $\int_{-\infty}^{\infty} n(x,t)dx = 1$ for all values of t.

(30/100)

State the physical reason for this independence of t. (20/100)

10. A potential $V(\vec{r})$ satisfied Poisson's equation $\nabla^2 V = 0$ and depends only on r and cos θ, $V(\vec{r}) = f(r)\cos\theta$. By substitution into Poisson's equation derive the differential equation satisfied by f (r).

(60/100)

Find the two values of n for which $f(r) = r^n$ is a solution of this equation.

(40/100)

You may assume that

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)$$

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