

## UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua  
Sidang Akademik 1995/96

Mac/April 1996

ZCC 542 - Teori Keadaan Pepejal II

Masa : [3 jam]

Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab mana-mana EMPAT soalan. Calon-calon boleh memilih menjawab kesemua soalan di dalam Bahasa Malaysia. Jika calon-calon memilih untuk menjawab di dalam Bahasa Inggeris, sekurang-kurangnya satu soalan wajib dijawab di dalam Bahasa Malaysia.

1. (a) Explain what is meant by screening of the electrostatic potential due to a point charge in a gas of mobile carriers. (10/100)

- (b) The Thomas-Fermi analysis leads to the following expression for the screened potential in k space:

$$\phi(k) = (e/\epsilon_0) (k^2 + k_s^2)^{-1}$$

where  $k_s$  is a constant. By taking spherical polar coordinates along  $r$  show that the Fourier transform is

$$\phi(r) = (e/4\pi\epsilon_0 r) \exp(-r/r_s)$$

and find the expression for  $r_s$  in terms of  $k_s$ . Is  $r_s$  an increasing or a decreasing function of the carrier density  $n$ ?

(30/100)

- (c) Draw sketches to compare  $\phi(r)$  with the unscreened Coulomb potential and to show how  $\phi(r)$  varies with  $n$ . (20/100)

- (d) Explain what is meant by the Mott metal-insulator transition. (20/100)

- (e) When the Schrödinger equation for a single particle is solved with  $\phi(r)$  as the potential, it is found that a bound state occurs for  $r_s > 0.84a_0$ , where  $a_0$  is the Bohr radius. Use this fact to explain why the Mott transition occurs. (20/100)

[The Fourier transform is defined by

$$\phi(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3k \phi(\vec{k}) \exp(i \vec{k} \cdot \vec{r})$$

Tables of integrals give

$$\int_0^{\infty} x \sin(mx) (a^2 + x^2)^{-1} dx = (\pi/2) \exp(-ma) \quad \text{for } m > 0 \text{ and } a > 0.]$$

2. (a) The semiclassical equation of motion for a wave packet in a semiconductor or a metal is

$$m^* (dv/dt + v/\tau) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

where  $m^*$  is the effective mass and  $q$  is the charge. Explain how this equation is used for  $\omega\tau \gg 1$  to describe plasma effects and for  $\omega\tau \ll 1$  to give an account of transport properties.

(20/100)

- (b) Sketch the normal-incidence reflectivity of light from a doped semiconductor for a range of frequencies including the plasma frequency. Identify the general spectral region (e.g. FIR or visible) and relate the reflectivity to the dielectric function  $\epsilon(\omega)$ .

(40/100)

- (c) Fig. 2.1 shows the temperature dependence of the resistivity of various sodium samples and Fig 2.2 shows data at higher temperatures for several metals. The temperature axis in Fig. 2.2 is scaled by the Debye temperature  $\theta_D$ . Discuss the data in terms of electron scattering mechanisms.

(40/100)

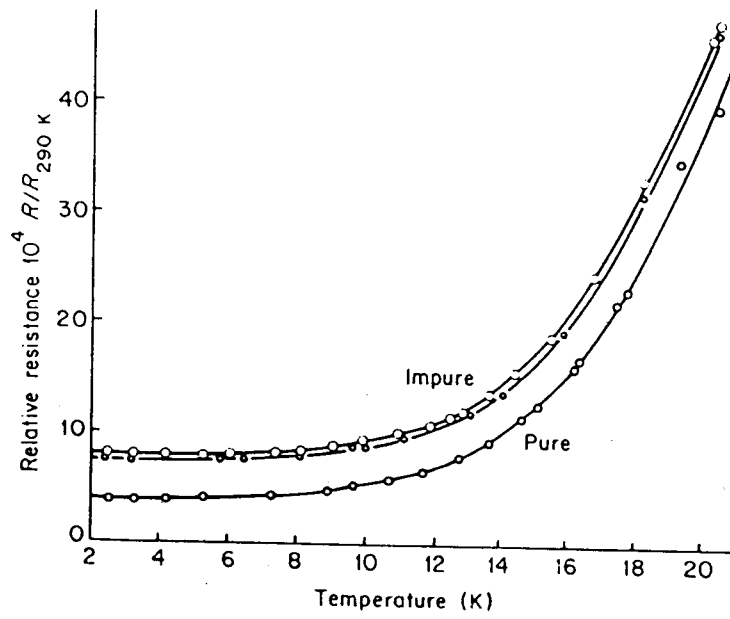


Fig. 2.1

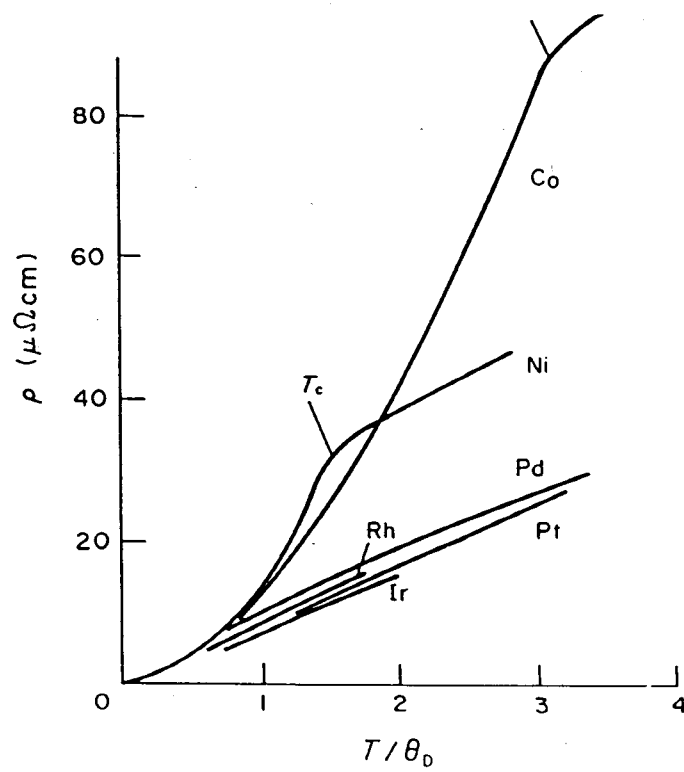


Fig. 2.2

3. (a) Draw a sketch to show a  $180^\circ$  Bloch domain wall in a ferromagnet. Explain the terms *exchange energy* and *anisotropy energy* and explain qualitatively how the balance between them determines the width of the Bloch wall. (20/100)
- (b) If  $S_1$  and  $S_2$  are the mean spin values on adjacent planes in your sketch the exchange energy may be written  $-J S_1 \cdot S_2$ . Assuming that the width is  $N$  layers and  $N \gg 1$  so that the angle  $\phi$  between adjacent layers is small show that on the assumption that  $\phi$  has the same value throughout the wall the exchange energy of a line of spins in the wall is  $W_{\text{ex}} = \pi^2 JS^2/2N$  and hence that the exchange energy per unit area is  $w_{\text{ex}} = \pi^2 JS^2/2Na^2$  where  $a$  is the lattice constant. Explain why a simple approximate expression for the anisotropy energy is  $w_{\text{anis}} = KNa$  where  $K$  is a constant. Minimise the total wall energy  $w = w_{\text{ex}} + w_{\text{anis}}$  and hence derive expressions for (a) the wall width  $N$  and (b) the minimum value of  $w$ . (40/100)
- (c) Sketch hysteresis loops to illustrate the difference between *soft* and *hard* magnetic materials and mention one application of each class. (20/100)
- (d) Explain the general form of the hysteresis loops in terms of pinning of domain walls and hence give a brief discussion of the principles of materials design for soft and hard behaviour. (20/100)
4. (a) Describe with suitable sketches *ferromagnetic*, *antiferromagnetic* and *ferrimagnetic* ordering. (20/100)
- (b) Explain what is meant by the *mean-field approximation* to the exchange energy in a ferromagnet. (10/100)
- (c) A ferromagnet is placed in a static external field  $B_0$  pointing along the  $z$  direction. Assume that the mean-field Hamiltonian is

$$H_{\text{mf}} = -g\mu_B \sum_i (B_0 + \lambda M) S_i^z$$

where  $M$  is the magnetization and assume that  $S_i^z$  has eigenvalues  $\pm 1/2$ . Write down the expression for the mean value  $\langle S_i^z \rangle$  at temperature  $T$  and hence prove that  $M$  is given by

$$M = \frac{1}{2} N g \mu_B \tanh \left[ \frac{g \mu_B (B_0 + \lambda M)}{2 k_B T} \right]$$

(30/100)

- (d) Derive from this the expression for the Curie temperature  $T_C$ . Explain without detailed derivations how the equation is used to find (a) the temperature dependence of  $M$  and (b) the magnetic susceptibility  $\chi$  above  $T_C$ . Sketch the temperature dependences of  $M$  and  $\chi$ .  
(40/100)
5. (a) Describe the experiments on quantization of magnetic flux in conventional and high- $T_C$  superconductors and explain how they support the idea of a macroscopic wave function.  
(20/100)
- (b) Sketch the magnetization curves of (a) type I and (b) reversible type II superconductors. Include a careful definition and explanation of the axis variables.  
(20/100)
- (c) On your sketch (b) identify the mixed-state region and explain its physical nature.  
(20/100)
- (d) Describe the difference between reversible and irreversible type II behaviour and explain the difference in terms of flux-line pinning.  
(20/100)
- (e) Why is flux-line pinning essential in superconducting solenoid wires? Discuss the practical problems that arise due to the necessity for pinning.  
(20/100)
6. (a) Describe the ordering in a nematic liquid crystal and explain what is meant by a director profile.  
(20/100)
- (b) How can the director profile be controlled by (a) pinning at walls and (b) a static electric or magnetic field. State precisely what the ordering mechanisms are in fields.  
(20/100)
- (c) Fig. 6.1 shows a nematic cell of width  $d$  with pinning as indicated on the walls and an electric field  $E$  applied normal to the wall pinning direction. Describe the Frederiks transition that occurs at a critical value  $E_C$  of  $E$ .  
(20/100)

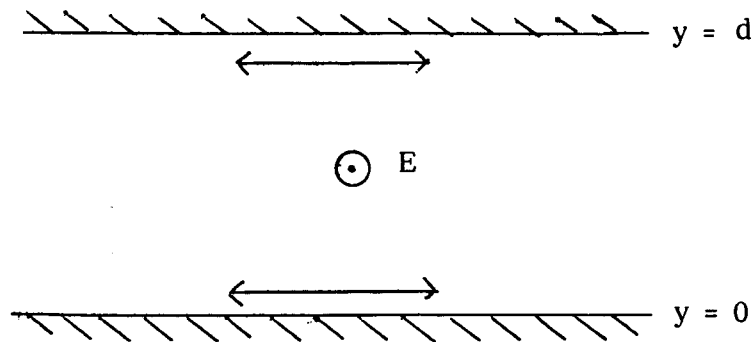


Fig. 6.1

- (d) The free energy may be written

$$F = \int_0^d \left\{ \frac{K_2}{2} \left( \frac{d\theta}{dy} \right)^2 - \frac{\chi}{2} E^2 \sin^2 \theta \right\} dy$$

where  $\theta$  is the angle between the director and the plane of the sketch. Explain the physical origin and significance of the two terms in the integrand. Show that for  $E < E_c$   $F = 0$ . Assume that for  $E$  just greater than  $E_c$   $\theta$  is non-zero but small varying linearly from zero at each wall to a maximum value  $\theta_M$  in the cell centre. Prove that

$$F = \left( \frac{2K_2}{d} - \chi E^2 \frac{d}{6} \right) \theta_M^2$$

and hence find an approximate expression for  $E_c$ .

(30/100)

- (e) Describe briefly without detailed derivations how the Euler-Lagrange equation is applied to find an exact expression for  $E_c$ .

(10/100)