UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama

Sidang Akademik 2004/2005

Oktober 2004

ZCA 110/4 - KALKULUS DAN ALJABAR LINEAR

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **LAPAN** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab kesemua <u>ENAM</u> soalan. Kesemuanya wajib dijawab dalam Bahasa Malaysia.

Diberi bersama kertas soalan ini ialah *A Brief Table of Integrals* (5 muka surat).

1.

(a) Tulis sistem persamaan linear berikut,

$$-x_1 + x_2 + 2x_3 = 5$$
$$2x_1 - x_2 + 5x_3 = 3$$
$$2x_1 + 5x_2 - 2x_4 = 0$$
$$3x_2 - x_3 + x_4 = 1.$$

dengan menggunakan nyataan matriks. Apakah matriks koefisien A sistem persamaan ini? Cari kofaktor α_{34} dan α_{34} bagi matriks A dan dengan menggunakan keputusan ini cari determinan bagi matriks koefisien A.

(b) Cari penyelesaian sistem ini dengan menggunakan kaedah Cramer.

(16 Markah)

2.

Cari titik-titik dan persamaan-persamaan bagi tangen pada lengkung $y = 2x^3 - 3x^2 - 12x + 20$ di mana tangennya adalah;

(a) bersudut tepat dengan garis $y = 1 - \dfrac{x}{24}$,

(b) selari dengan garis $y = \sqrt{2} - 12x$.

(16 Markah)

3.

(a) Nilaikan kamiran-kamiran berikut:

(i) $\displaystyle\int 2^{\tan x} \sec^2 x \, dx$

(ii) $\displaystyle\int_1^3 \frac{(\ln(v+1))^2}{v+1} \, dv$

(iii) $\displaystyle\int_{-2}^2 \frac{3 \, dt}{4 + 3t^2}$

(iv) $\displaystyle\int \frac{dt}{(t+1)\sqrt{t^2 + 2t - 8}}$.

(b) Cari luas kawasan di antara graf, $f(x) = 1 - \frac{x^2}{4}$, $-2 \le x \le 3$, dan paksi-x.

(16 Markah)

4.

Diberi terbitan pertama $y'(x) = x^4 - 2x^2$, suatu fungsi $y = f(x)$.

(a) Jika fungsi $y(x)$ ini melalui pusat koordinat $(0, 0)$, cari persamaan fungsi ini.

(b) Cari titik-titik minimum, maksimum, dan titik perubahan kecekungan fungsi ini.

(c) Cari selang-selang di mana fungsi ini bertambah dan selang-selang di mana ia berkurang.

(d) Cari selang-selang di mana fungsi ini bercekung ke atas dan selang-selang di mana ia bercekung ke bawah.

(e) Lukis fungsi $y(x)$ ini atas selang [-2, 2], di kertas graf yang dibekalkan. Tunjukkan titik-titik minimum, maksimum, dan titik perubahan kecekungannya atas

graf ini.

(16 Markah)

5.

Nilaikan kamiran-kamiran berikut:

(a) $\displaystyle\int \frac{\tan x \, dx}{\tan x + \sec x}$

(b) $\displaystyle\int \frac{x^3 + x^2}{x^2 + x - 2} \, dx$

(c) $\displaystyle\int \frac{9 \, dv}{81 - v^4}$

(d) $\displaystyle\int \frac{z + 1}{z^2(z^2 + 4)} \, dz$

(e) $\displaystyle\int x^5 \sin x \, dx$

(f) $\displaystyle\int_1^\infty \frac{\ln y}{y^3} \, dy.$

(18 Markah)

6.

(a) Bagi siri,

$$\sum_{n=1}^{\infty} \frac{(x + 4)^n}{n3^n},$$

cari (i) jejari siri dan (ii) selang menumpunya. Untuk nilai apa x siri itu menumpu

(iii) mutlak dan (iv) menumpu bersyarat?

(b) Cari perkembangan Siri Fourier bagi fungsi,

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2. & 0 < x < \pi. \end{cases}$$

(c) Cari perkembangan Siri Kosinus Fourier bagi fungsi,

$$f(x) = \begin{cases} 1. & 0 < x < 1/2 \\ 0. & 1/2 < x < 1. \end{cases}$$

(18 Markah)

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A Brief Table of Integrals

1. $\int u\,dv = uv - \int v\,du$

2. $\int a^u\,du = \dfrac{a^u}{\ln a} + C. \qquad a \neq 1. \qquad a > 0$

3. $\int \cos u\,du = \sin u + C$

4. $\int \sin u\,du = -\cos u + C$

5. $\int (ax + b)^n\,dx = \dfrac{(ax + b)^{n+1}}{a(n + 1)} + C. \qquad n \neq -1$

6. $\int (ax + b)^{-1}\,dx = \dfrac{1}{a}\ln|ax + b| + C$

7. $\int x(ax + b)^n\,dx = \dfrac{(ax + b)^{n+1}}{a^2}\left[\dfrac{ax + b}{n + 2} - \dfrac{b}{n + 1}\right] + C. \qquad n \neq -1, -2$

8. $\int x(ax + b)^{-1}\,dx = \dfrac{x}{a} - \dfrac{b}{a^2}\ln|ax + b| + C$

9. $\int x(ax + b)^{-2}\,dx = \dfrac{1}{a^2}\left[\ln|ax + b| + \dfrac{b}{ax + b}\right] + C$

10. $\int \dfrac{dx}{x(ax + b)} = \dfrac{1}{b}\ln\left|\dfrac{x}{ax + b}\right| + C$

11. $\int (\sqrt{ax + b})^n\,dx = \dfrac{2}{a}\dfrac{(\sqrt{ax + b})^{n+2}}{n + 2} + C. \qquad n \neq -2$

12. $\int \dfrac{\sqrt{ax + b}}{x}\,dx = 2\sqrt{ax + b} + b\int \dfrac{dx}{x\sqrt{ax + b}}$

13. (a) $\int \dfrac{dx}{x\sqrt{ax - b}} = \dfrac{2}{\sqrt{b}}\tan^{-1}\sqrt{\dfrac{ax - b}{b}} + C$

(b) $\int \dfrac{dx}{x\sqrt{ax + b}} = \dfrac{1}{\sqrt{b}}\ln\left|\dfrac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}}\right| + C$

14. $\int \dfrac{\sqrt{ax + b}}{x^2}\,dx = -\dfrac{\sqrt{ax + b}}{x} + \dfrac{a}{2}\int \dfrac{dx}{x\sqrt{ax + b}} + C$

15. $\int \dfrac{dx}{x^2\sqrt{ax + b}} = -\dfrac{\sqrt{ax + b}}{bx} - \dfrac{a}{2b}\int \dfrac{dx}{x\sqrt{ax + b}} + C$

16. $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + C$

17. $\int \dfrac{dx}{(a^2 + x^2)^2} = \dfrac{x}{2a^2(a^2 + x^2)} + \dfrac{1}{2a^3}\tan^{-1}\dfrac{x}{a} + C$

18. $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a}\ln\left|\dfrac{x + a}{x - a}\right| + C$

19. $\int \dfrac{dx}{(a^2 - x^2)^2} = \dfrac{x}{2a^2(a^2 - x^2)} + \dfrac{1}{4a^3}\ln\left|\dfrac{x + a}{x - a}\right| + C$

20. $\int \dfrac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\dfrac{x}{a} + C = \ln(x + \sqrt{a^2 + x^2}) + C$

21. $\int \sqrt{a^2 + x^2}\,dx = \dfrac{x}{2}\sqrt{a^2 + x^2} + \dfrac{a^2}{2}\ln(x + \sqrt{a^2 + x^2}) + C$

22. $\int x^2\sqrt{a^2 + x^2}\,dx = \dfrac{x}{8}(a^2 + 2x^2)\sqrt{a^2 + x^2} - \dfrac{a^4}{8}\ln(x + \sqrt{a^2 + x^2}) + C$

23. $\int \dfrac{\sqrt{a^2 + x^2}}{x}\,dx = \sqrt{a^2 + x^2} - a\ln\left|\dfrac{a + \sqrt{a^2 + x^2}}{x}\right| + C$

24. $\int \dfrac{\sqrt{a^2 + x^2}}{x^2}\,dx = \ln(x + \sqrt{a^2 + x^2}) - \dfrac{\sqrt{a^2 + x^2}}{x} + C$

25. $\int \dfrac{x^2}{\sqrt{a^2 + x^2}}\,dx = -\dfrac{a^2}{2}\ln(x + \sqrt{a^2 + x^2}) + \dfrac{x\sqrt{a^2 + x^2}}{2} + C$

26. $\int \dfrac{dx}{x\sqrt{a^2 + x^2}} = -\dfrac{1}{a}\ln\left|\dfrac{a + \sqrt{a^2 + x^2}}{x}\right| + C$

27. $\int \dfrac{dx}{x^2\sqrt{a^2 + x^2}} = -\dfrac{\sqrt{a^2 + x^2}}{a^2 x} + C$

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28. $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\dfrac{x}{a} - C$

29. $\int \sqrt{a^2 - x^2}\, dx = \dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} + C$

30. $\int x^2\sqrt{a^2 - x^2}\, dx = \dfrac{a^2}{8}\sin^{-1}\dfrac{x}{a} - \dfrac{1}{8}x\sqrt{a^2 - x^2}(a^2 - 2x^2) + C$

31. $\int \dfrac{\sqrt{a^2 - x^2}}{x}\, dx = \sqrt{a^2 - x^2} - a\ln\left|\dfrac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

32. $\int \dfrac{\sqrt{a^2 - x^2}}{x^2}\, dx = -\sin^{-1}\dfrac{x}{a} - \dfrac{\sqrt{a^2 - x^2}}{x} + C$

33. $\int \dfrac{x^2}{\sqrt{a^2 - x^2}}\, dx = \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} - \dfrac{1}{2}x\sqrt{a^2 - x^2} + C$

34. $\int \dfrac{dx}{x\sqrt{a^2 - x^2}} = -\dfrac{1}{a}\ln\left|\dfrac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

35. $\int \dfrac{dx}{x^2\sqrt{a^2 - x^2}} = -\dfrac{\sqrt{a^2 - x^2}}{a^2 x} + C$

36. $\int \dfrac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\dfrac{x}{a} + C = \ln\left|x + \sqrt{x^2 - a^2}\right| +$

37. $\int \sqrt{x^2 - a^2}\, dx = \dfrac{x}{2}\sqrt{x^2 - a^2} - \dfrac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$

38. $\int (\sqrt{x^2 - a^2})^n\, dx = \dfrac{x(\sqrt{x^2 - a^2})^n}{n + 1} - \dfrac{na^2}{n + 1}\int (\sqrt{x^2 - a^2})^{n-2}\, dx, \qquad n \neq -1$

39. $\int \dfrac{dx}{(\sqrt{x^2 - a^2})^n} = \dfrac{x(\sqrt{x^2 - a^2})^{2-n}}{(2 - n)a^2} - \dfrac{n - 3}{(n - 2)a^2}\int \dfrac{dx}{(\sqrt{x^2 - a^2})^{n-2}}, \qquad n \neq 2$

40. $\int x(\sqrt{x^2 - a^2})^n\, dx = \dfrac{(\sqrt{x^2 - a^2})^{n+2}}{n + 2} + C, \qquad n \neq -2$

41. $\int x^2\sqrt{x^2 - a^2}\, dx = \dfrac{x}{8}(2x^2 - a^2)\sqrt{x^2 - a^2} - \dfrac{a^4}{8}\ln\left|x + \sqrt{x^2 - a^2}\right| + C$

42. $\int \dfrac{\sqrt{x^2 - a^2}}{x}\, dx = \sqrt{x^2 - a^2} - a\sec^{-1}\left|\dfrac{x}{a}\right| + C$

43. $\int \dfrac{\sqrt{x^2 - a^2}}{x^2}\, dx = \ln\left|x + \sqrt{x^2 - a^2}\right| - \dfrac{\sqrt{x^2 - a^2}}{x} +$

44. $\int \dfrac{x^2}{\sqrt{x^2 - a^2}}\, dx = \dfrac{a^2}{2}\ln\left|x + \sqrt{x^2 - a^2}\right| + \dfrac{x}{2}\sqrt{x^2 - a^2} + C$

45. $\int \dfrac{dx}{x\sqrt{x^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\left|\dfrac{x}{a}\right| + C = \dfrac{1}{a}\cos^{-1}\left|\dfrac{a}{x}\right| + C$

46. $\int \dfrac{dx}{x^2\sqrt{x^2 - a^2}} = \dfrac{\sqrt{x^2 - a^2}}{a^2 x} + C$

47. $\int \dfrac{dx}{\sqrt{2ax - x^2}} = \sin^{-1}\left(\dfrac{x - a}{a}\right) + C$

48. $\int \sqrt{2ax - x^2}\, dx = \dfrac{x - a}{2}\sqrt{2ax - x^2} + \dfrac{a^2}{2}\sin^{-1}\left(\dfrac{x - a}{a}\right) + C$

49. $\int (\sqrt{2ax - x^2})^n\, dx = \dfrac{(x - a)(\sqrt{2ax - x^2})^n}{n + 1} + \dfrac{na^2}{n + 1}\int (\sqrt{2ax - x^2})^{n-2}\, dx$

50. $\int \dfrac{dx}{(\sqrt{2ax - x^2})^n} = \dfrac{(x - a)(\sqrt{2ax - x^2})^{2-n}}{(n - 2)a^2} + \dfrac{n - 3}{(n - 2)a^2}\int \dfrac{dx}{(\sqrt{2ax - x^2})^{n-2}}$

51. $\int x\sqrt{2ax - x^2}\, dx = \dfrac{(x + a)(2x - 3a)\sqrt{2ax - x^2}}{6} + \dfrac{a^3}{2}\sin^{-1}\left(\dfrac{x - a}{a}\right) + C$

52. $\int \dfrac{\sqrt{2ax - x^2}}{x}\, dx = \sqrt{2ax - x^2} + a\sin^{-1}\left(\dfrac{x - a}{a}\right) + C$

53. $\int \dfrac{\sqrt{2ax - x^2}}{x^2}\, dx = -2\sqrt{\dfrac{2a - x}{x}} - \sin^{-1}\left(\dfrac{x - a}{a}\right) + C$

54. $\int \dfrac{x\, dx}{\sqrt{2ax - x^2}} = a\sin^{-1}\left(\dfrac{x - a}{a}\right) - \sqrt{2ax - x^2} + C$

55. $\int \dfrac{dx}{x\sqrt{2ax - x^2}} = -\dfrac{1}{a}\sqrt{\dfrac{2a - x}{x}} + C$

56. $\int \sin ax\, dx = -\dfrac{1}{a}\cos ax + C$

57. $\int \cos ax\, dx = \dfrac{1}{a}\sin ax + C$

58. $\int \sin^2 ax\, dx = \dfrac{x}{2} - \dfrac{\sin 2ax}{4a} + C$

59. $\int \cos^2 ax\, dx = \dfrac{x}{2} + \dfrac{\sin 2ax}{4a} + C$

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60. $\int \sin^n ax \, dx = -\dfrac{\sin^{n-1} ax \cos ax}{na} + \dfrac{n-1}{n} \int \sin^{n-2} ax \, dx$

61. $\int \cos^n ax \, dx = \dfrac{\cos^{n-1} ax \sin ax}{na} - \dfrac{n-1}{n} \int \cos^{n-2} ax \, dx$

62. (a) $\int \sin ax \cos bx \, dx = -\dfrac{\cos(a+b)x}{2(a+b)} - \dfrac{\cos(a-b)x}{2(a-b)} + C. \qquad a^2 \neq b^2$

(b) $\int \sin ax \sin bx \, dx = \dfrac{\sin(a-b)x}{2(a-b)} - \dfrac{\sin(a+b)x}{2(a+b)} + C. \qquad a^2 \neq b^2$

(c) $\int \cos ax \cos bx \, dx = \dfrac{\sin(a-b)x}{2(a-b)} + \dfrac{\sin(a+b)x}{2(a+b)} + C. \qquad a^2 \neq b^2$

63. $\int \sin ax \cos ax \, dx = -\dfrac{\cos 2ax}{4a} + C$

64. $\int \sin^n ax \cos ax \, dx = \dfrac{\sin^{n-1} ax}{(n+1)a} + C. \qquad n \neq -1$

65. $\int \dfrac{\cos ax}{\sin ax} \, dx = \dfrac{1}{a} \ln |\sin ax| + C$

66. $\int \cos^n ax \sin ax \, dx = -\dfrac{\cos^{n-1} ax}{(n+1)a} - C. \qquad n \neq -1$

67. $\int \dfrac{\sin ax}{\cos ax} \, dx = -\dfrac{1}{a} \ln |\cos ax| + C$

68. $\int \sin^n ax \cos^m ax \, dx = -\dfrac{\sin^{n-1} ax \cos^{m-1} ax}{a(m+n)} + \dfrac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \, dx. \qquad n \neq -m \qquad \text{(reduces } \sin^n ax\text{)}$

69. $\int \sin^n ax \cos^m ax \, dx = \dfrac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \dfrac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \, dx. \qquad m \neq -n \qquad \text{(reduces } \cos^m ax\text{)}$

70. $\int \dfrac{dx}{b + c \sin ax} = \dfrac{-2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\dfrac{b-c}{b+c}} \tan\left(\dfrac{\pi}{4} - \dfrac{ax}{2} \right) \right] + C. \qquad b^2 > c^2$

71. $\int \dfrac{dx}{b + c \sin ax} = \dfrac{-1}{a\sqrt{c^2 - b^2}} \ln \left| \dfrac{c + b \sin ax + \sqrt{c^2 - b^2} \cos ax}{b + c \sin ax} \right| + C. \qquad b^2 < c^2$

72. $\int \dfrac{dx}{1 + \sin ax} = -\dfrac{1}{a} \tan\left(\dfrac{\pi}{4} - \dfrac{ax}{2} \right) + C$

73. $\int \dfrac{dx}{1 - \sin ax} = \dfrac{1}{a} \tan\left(\dfrac{\pi}{4} + \dfrac{ax}{2} \right) + C$

74. $\int \dfrac{dx}{b + c \cos ax} = \dfrac{2}{a\sqrt{b^2 - c^2}} \tan^{-1}\left[\sqrt{\dfrac{b-c}{b+c}} \tan \dfrac{ax}{2} \right] + C. \qquad b^2 > c^2$

75. $\int \dfrac{dx}{b + c \cos ax} = \dfrac{1}{a\sqrt{c^2 - b^2}} \ln \left| \dfrac{c + b \cos ax + \sqrt{c^2 - b^2} \sin ax}{b + c \cos ax} \right| + C. \qquad b^2 < c^2$

76. $\int \dfrac{dx}{1 + \cos ax} = \dfrac{1}{a} \tan \dfrac{ax}{2} + C$

77. $\int \dfrac{dx}{1 - \cos ax} = -\dfrac{1}{a} \cot \dfrac{ax}{2} + C$

78. $\int x \sin ax \, dx = \dfrac{1}{a^2} \sin ax - \dfrac{x}{a} \cos ax + C$

79. $\int x \cos ax \, dx = \dfrac{1}{a^2} \cos ax + \dfrac{x}{a} \sin ax + C$

80. $\int x^n \sin ax \, dx = -\dfrac{x^n}{a} \cos ax + \dfrac{n}{a} \int x^{n-1} \cos ax \, dx$

81. $\int x^n \cos ax \, dx = \dfrac{x^n}{a} \sin ax - \dfrac{n}{a} \int x^{n-1} \sin ax \, dx$

82. $\int \tan ax \, dx = \dfrac{1}{a} \ln |\sec ax| + C$

83. $\int \cot ax \, dx = \dfrac{1}{a} \ln |\sin ax| + C$

84. $\int \tan^2 ax \, dx = \dfrac{1}{a} \tan ax - x + C$

85. $\int \cot^2 ax \, dx = -\dfrac{1}{a} \cot ax - x + C$

86. $\int \tan^n ax \, dx = \dfrac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx. \qquad n \neq 1$

87. $\int \cot^n ax \, dx = -\dfrac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax \, dx. \qquad n \neq 1$

88. $\int \sec ax \, dx = \dfrac{1}{a} \ln |\sec ax + \tan ax| + C$

89. $\int \csc ax \, dx = -\dfrac{1}{a} \ln |\csc ax + \cot ax| + C$

90. $\int \sec^2 ax \, dx = \dfrac{1}{a} \tan ax + C$

91. $\int \csc^2 ax \, dx = -\dfrac{1}{a} \cot ax + C$

92. $\int \sec^n ax \, dx = \dfrac{\sec^{n-2} ax \tan ax}{a(n-1)} + \dfrac{n-2}{n-1} \int \sec^{n-2} ax \, dx. \qquad n \neq 1$

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93. $\int \csc^n ax\, dx = -\dfrac{\csc^{n-2} ax\, \cot ax}{a(n-1)} + \dfrac{n-2}{n-1} \int \csc^{n-2} ax\, dx.$ $n \neq 1$

94. $\int \sec^n ax\, \tan ax\, dx = \dfrac{\sec^n ax}{na} + C,$ $n \neq 0$

95. $\int \csc^n ax\, \cot ax\, dx = -\dfrac{\csc^n ax}{na} + C,$ $n \neq 0$

96. $\int \sin^{-1} ax\, dx = x \sin^{-1} ax - \dfrac{1}{a}\sqrt{1-a^2 x^2} + C$

97. $\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \dfrac{1}{a}\sqrt{1-a^2 x^2} + C$

98. $\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \dfrac{1}{2a} \ln(1+a^2 x^2) + C$

99. $\int x^n \sin^{-1} ax\, dx = \dfrac{x^{n+1}}{n+1} \sin^{-1} ax - \dfrac{a}{n+1} \int \dfrac{x^{n+1}\, dx}{\sqrt{1-a^2 x^2}},$ $n \neq -1$

100. $\int x^n \cos^{-1} ax\, dx = \dfrac{x^{n+1}}{n+1} \cos^{-1} ax + \dfrac{a}{n+1} \int \dfrac{x^{n+1}\, dx}{\sqrt{1-a^2 x^2}},$ $n \neq -1$

✱ 101. $\int x^n \tan^{-1} ax\, dx = \dfrac{x^{n+1}}{n+1} \tan^{-1} ax - \dfrac{a}{n+1} \int \dfrac{x^{n+1}\, dx}{\sqrt{1+a^2 x^2}},$ $n \neq -1$ p. 509   2.851

102. $\int e^{ax}\, dx = \dfrac{1}{a} e^{ax} + C$

103. $\int b^{ax}\, dx = \dfrac{1}{a}\dfrac{b^{ax}}{\ln b} + C,$ $b > 0,$ $b \neq 1$

104. $\int xe^{ax}\, dx = \dfrac{e^{ax}}{a^2}(ax-1) + C$

105. $\int x^n e^{ax}\, dx = \dfrac{1}{a} x^n e^{ax} - \dfrac{n}{a} \int x^{n-1} e^{ax}\, dx$

106. $\int x^n b^{ax}\, dx = \dfrac{x^n b^{ax}}{a \ln b} - \dfrac{n}{a \ln b} \int x^{n-1} b^{ax}\, dx,$ $b > 0,$ $b \neq 1$

107. $\int e^{ax} \sin bx\, dx = \dfrac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + C$

108. $\int e^{ax} \cos bx\, dx = \dfrac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$

109. $\int \ln ax\, dx = x \ln ax - x + C$

110. $\int x^n (\ln ax)^m\, dx = \dfrac{x^{n+1}(\ln ax)^m}{n+1} - \dfrac{m}{n+1} \int x^n (\ln ax)^{m-1}\, dx,$ $n \neq -1$

111. $\int x^{-1}(\ln ax)^m\, dx = \dfrac{(\ln ax)^{m+1}}{m+1} + C,$ $m \neq -1$

112. $\int \dfrac{dx}{x \ln ax} = \ln |\ln ax| + C$

113. $\int \sinh ax\, dx = \dfrac{1}{a} \cosh ax + C$

114. $\int \cosh ax\, dx = \dfrac{1}{a} \sinh ax + C$

115. $\int \sinh^2 ax\, dx = \dfrac{\sinh 2ax}{4a} - \dfrac{x}{2} + C$

116. $\int \cosh^2 ax\, dx = \dfrac{\sinh 2ax}{4a} + \dfrac{x}{2} + C$

117. $\int \sinh^n ax\, dx = \dfrac{\sinh^{n-1} ax \cosh ax}{na} - \dfrac{n-1}{n} \int \sinh^{n-2} ax\, dx,$ $n \neq 0$

118. $\int \cosh^n ax\, dx = \dfrac{\cosh^{n-1} ax \sinh ax}{na} + \dfrac{n-1}{n} \int \cosh^{n-2} ax\, dx,$ $n \neq 0$

119. $\int x \sinh ax\, dx = \dfrac{x}{a} \cosh ax - \dfrac{1}{a^2} \sinh ax + C$

120. $\int x \cosh ax\, dx = \dfrac{x}{a} \sinh ax - \dfrac{1}{a^2} \cosh ax + C$

121. $\int x^n \sinh ax\, dx = \dfrac{x^n}{a} \cosh ax - \dfrac{n}{a} \int x^{n-1} \cosh ax\, dx$

122. $\int x^n \cosh ax\, dx = \dfrac{x^n}{a} \sinh ax - \dfrac{n}{a} \int x^{n-1} \sinh ax\, dx$

123. $\int \tanh ax\, dx = \dfrac{1}{a} \ln(\cosh ax) + C$

124. $\int \coth ax\, dx = \dfrac{1}{a} \ln |\sinh ax| + C$

125. $\int \tanh^2 ax\, dx = x - \dfrac{1}{a} \tanh ax + C$

126. $\int \coth^2 ax\, dx = x - \dfrac{1}{a} \coth ax + C$

127. $\int \tanh^n ax\, dx = -\dfrac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax\, dx,$ $n \neq 1$

128. $\int \coth^n ax\, dx = -\dfrac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax\, dx,$ $n \neq 1$

129. $\int \operatorname{sech} ax\, dx = \dfrac{1}{a} \sin^{-1}(\tanh ax) + C$

130. $\int \operatorname{csch} ax\, dx = \dfrac{1}{a} \ln \left| \tanh \dfrac{ax}{2} \right| + C$

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31. $\int \operatorname{sech}^2 ax\, dx = \frac{1}{a}\tanh ax + C$

132. $\int \operatorname{csch}^2 ax\, dx = -\frac{1}{a}\coth ax + C$

33. $\int \operatorname{sech}^n ax\, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2} ax\, dx. \qquad n \neq 1$

34. $\int \operatorname{csch}^n ax\, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1}\int \operatorname{csch}^{n-2} ax\, dx. \qquad n \neq 1$

35. $\int \operatorname{sech}^n ax \tanh ax\, dx = -\frac{\operatorname{sech}^n ax}{na} + C. \qquad n \neq 0$

136. $\int \operatorname{csch}^n ax \coth ax\, dx = -\frac{\operatorname{csch}^n ax}{na} + C. \qquad n \neq 0$

37. $\int e^{ax}\sinh bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b}\right] + C. \qquad a^2 \neq b^2$

138. $\int e^{ax}\cosh bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b}\right] + C. \qquad a^2 \neq b^2$

39. $\int_0^z x^{n-1}e^{-x}\, dx = \Gamma(n) = (n-1)!. \qquad n > 0$

140. $\int_0^z e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}. \qquad a > 0$

41. $\int_0^{\pi/2}\sin^n x\, dx = \int_0^{\pi/2}\cos^n x\, dx = \begin{cases} \dfrac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots n}\cdot\dfrac{\pi}{2}, & \text{if } n \text{ is an even integer} \geq 2 \\[2ex] \dfrac{2\cdot 4\cdot 6\cdots(n-1)}{3\cdot 5\cdot 7\cdots n}, & \text{if } n \text{ is an odd integer} \geq 3 \end{cases}$

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