
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2008/2009

April/May 2009

EKC 314 – Transport Phenomena
[Fenomena Pengangkutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of NINE pages of printed material and FOUR pages of Appendix before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak dan EMPAT muka surat Lampiran sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **FIVE** (5) questions. Answer **ALL** (3) questions from Section A. Answer **TWO** (2) questions from Section B.

Arahan: Jawab **LIMA** (5) soalan. Jawab **SEMUA** (3) soalan dari Bahagian A. Jawab **DUA** (2) soalan dari Bahagian B.]

You may answer the question either in Bahasa Malaysia or in English.

[Anda dibenarkan menjawab soalan sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.]

Section A : Answer ALL questions.

Bahagian A : Jawab SEMUA soalan.

1. Nitrogen at 1.5 atm is heated to attain an average temperature of 474.2 K through a horizontal tube of 25.4 mm inside diameter at a velocity of 7.2 m/s. The heating medium, steam at 488.7 K, is condensing on the outside of the tube. Since the heat transfer coefficient of the steam is very high, the resistance of the metal wall is assumed to be negligible and the surface temperature of the metal in contact with the nitrogen is at 488.7 K. The molar volume and heat capacity of nitrogen at 474.2 K is 0.0186m³/mole and 741.92 J/kg K respectively.

1. Nitrogen pada 1.5 atm telah dipanaskan dan mencapai suhu purata 474.2 K dengan melalui tiub mendatar dengan garispusat dalam tiub 25.4 mm pada kelajuan 7.2 m/s. Medium pemanas adalah stim pada 488.7 K sedang terkondensasi pada permukaan luar tiub. Oleh kerana pekali pemindahan haba stim adalah sangat tinggi, rintangan dinding logam boleh diabaikan dan suhu permukaan logam yang bersentuhan dengan nitrogen adalah pada 488.7 K. Isipadu molar dan kapasiti haba nitrogen pada 474.2 K adalah 0.0186 m³/mol and 741.92 J/kg K masing-masing.

- [a] Estimate the following parameters for nitrogen using the Lennard-Jones parameters:

μ_w = viscosity at the wall temperature

μ_b = viscosity at the average temperature

k = thermal conductivity of nitrogen at average temperature

- [a] Anggarkan parameter-parameter berikut untuk nitrogen dengan menggunakan parameter Lennard-Jones:

μ_w = kelikatan pada suhu dinding

μ_b = kelikatan pada suhu purata

k = kekonduksian terma nitrogen pada suhu purata

[6 marks/markah]

- [b] Determine the heat transfer coefficient, h and the heat flux of the steam by using the appropriate equation below:

- [b] Tentukan pekali pemindahan haba, h dan juga fluks haba bagi stim dengan menggunakan persamaan yang bersesuaian di bawah:

$$N_{Re} < 2100$$

$$N_{Nu} = \frac{hD}{k} = 1.86 \left(N_{Re} N_{Pr} \frac{D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

$$N_{Re} > 6000$$

$$N_{Nu} = \frac{hD}{k} = 0.027 N_{Re}^{0.8} N_{Pr}^{1/3} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

[8 marks/markah]

...3/-

- [c] If the heat resistance of metal is not assumed to be negligible, use the generalized equation of energy flux given in the appendix and show that the temperature distribution in the metal in the radial direction at steady state can be expressed either by

- [c] *Sekiranya rintangan haba logam tidak diabaikan, gunakan persamaan umum fluks tenaga yang diberi dalam lampiran dan tunjukkan bahawa taburan suhu di dalam logam pada arah jejari tiub semasa keadaan mantap adalah seperti yang ditunjukkan oleh salah satu persamaan di bawah*

$$T(r) = \frac{T_1 - T_2}{\ln(r_2/r_1)} \ln\left(\frac{r}{r_2}\right) + T_2 \quad \text{or} \quad T(r) = \frac{T_1 - T_2}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_1}\right) + T_1$$

[6 marks/markah]

2. A Newtonian fluid is in laminar flow in a narrow slit formed by two parallel walls (W is width of the wall) a distance $2B$ apart. It is understood that $B \ll W$, so that “edge effect” are unimportant. Make a differential momentum balance, and obtain the following expressions for the momentum-flux and velocity distribution:
2. *Suatu bendalir Newtonian yang berada pada aliran lamina di dalam lekah sempit yang dibina daripada dua dinding yang selari (W adalah lebar dinding) pada suatu jarak $2B$. Diketahui bahawa $B \ll W$, oleh itu kesan bucu adalah tidak penting. Buatkan satu imbangan pembezaan momentum dan terbitkan persamaan-persamaan di bawah bagi fluks-momentum dan taburan halaju:*

$$\tau_{xz} = \left(\frac{P_0 - P_L}{L} \right) x$$

$$v_z = \left(\frac{P_0 - P_L B^2}{2\mu L} \right) \left[1 - \left(\frac{x}{B} \right)^2 \right]$$

[8 marks/markah]

In these expressions $P = p + \rho gh = p - \rho gz$.

Di dalam persamaan-persamaan ini $P = p + \rho gh = p - \rho gz$.

- [a] Determine the maximum velocity of this flow

- [a] *Tentukan halaju maksimum bagi aliran ini*

[4 marks/markah]

- [b] What is the ratio of the average velocity to the maximum velocity for this flow?

- [b] *Apakah nisbah halaju purata kepada halaju maksimum bagi aliran ini?*

[5 marks/markah]

[c] Obtain the slit analogue of the Hagen-Poiseuille equation.

[c] Nyatakan analog lekah bagi persamaan Hagen-Poiseuille.

[3 marks/markah]

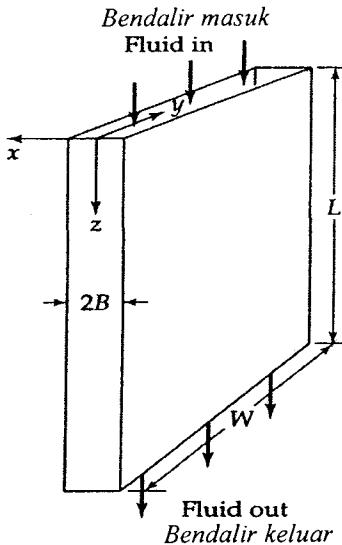


Figure Q.2
Rajah S.2

3. Figure Q.3. shows a spherical storage tank of internal radius r_i . The tank has a spherical shell wall having an outer radius of radius r_o . Sizes of the mouth and the outlet openings of the tank could be assumed to be small compared with the diameter of the tank. The tank holds a mixture having a component A. The concentration of A inside the tank can be assumed to be constant c_1 . The component A is found to diffuse slowly through the shell. As a result the concentration of A at the outer surface of the shell remains at c_2

3. Rajah S.3. menunjukkan sebuah tangki simpanan sfera dengan jejari dalam r_i . Tangki tersebut mempunyai dinding kelompang sfera dengan jejari luar r_o . Saiz bukaan untuk saluran masuk dan keluar tangki boleh diandaikan sebagai kecil jika dibandingkan dengan garispusat tangki tersebut. Tangki tersebut menakung suatu campuran yang mempunyai komponen A. Kepekatan A di dalam tangki boleh diandaikan sebagai pemalar c_1 . Komponen A didapati meresap secara perlahan melalui kelompang. Hasilnya, kepekatan A di permukaan luar kelompang kekal pada c_2

- [a] Using the generalized equations of molar flux given in the appendix, show that the differential equation governing the variation of the concentration with radius for diffusion of A in the radial direction at steady state can be expressed as

- [a] Dengan menggunakan persamaan-persamaan umum fluks molar yang diberikan dalam lampiran, tunjukkan bahawa persamaan pembezaan yang mengatur perubahan kepekatan dengan jejari bagi resapan A arah jejarian pada keadaan mantap boleh diungkapkan sebagai

$$\frac{\partial}{\partial r} \left(r^2 \frac{\partial c_A}{\partial r} \right) = 0$$

[6 marks/markah]

...5/-

- [b] Hence, show that the distribution of concentration c_A within the shell can be expressed as

- [b] Maka, tunjukkan bahawa taburan kepekatan c_A dalam kelompang boleh diungkapkan sebagai

$$c_A = \left(\frac{r_i c_2 (r_o - r) + r_o c_1 (r - r_i)}{r(r_o - r_i)} \right)$$

[7 marks/markah]

- [c] Using the above expression estimate the diffusion flux of water through the wall of a spherical vessel holding a mixture having a concentration of water $c_1 = 30.2 \text{ kmol/m}^3$. r_i and r_o can be assumed to be 2.0 m and 2.05 m respectively. The concentration of water at the outer surface of the vessel may be assumed to be negligible and zero at steady state. The Diffusion coefficient of water through the wall of the vessel is given as $2.41 \times 10^{-10} \text{ m}^2/\text{s}$

- [c] Dengan menggunakan ungkapan di atas, anggarkan fluks resapan air melalui dinding bekas sfera yang menakung suatu campuran dengan kepekatan air $c_1 = 30.2 \text{ kmol/m}^3$. r_i dan r_o masing-masing boleh diandaikan sebagai 2.0 m dan 2.05 m. Kepekatan air pada permukaan luar bekas boleh diabaikan dan bernilai sifar pada keadaan mantap. Pekali resapan air melalui dinding bekas diberi sebagai $2.41 \times 10^{-10} \text{ m}^2/\text{s}$

[7 marks/markah]

Supplementary equations are given in Appendix.
Persamaan-persamaan tambahan di beri dalam Lampiran.

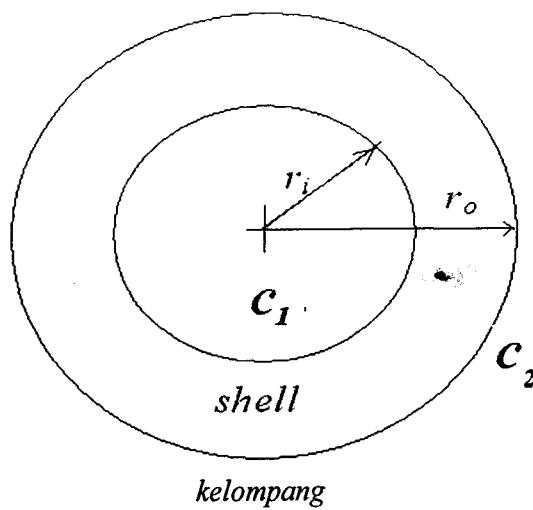


Figure Q.3.
Rajah S.3.

Section B : Answer any TWO questions.

Bahagian B : Jawab mana-mana DUA soalan.

4. The outer surface of a vertical tube (1 m long and 80 mm in diameter) as shown in Figure Q.4. is exposed to saturated steam at atmospheric pressure and is maintained at an average temperature of 50 °C by the flow of cool water through the tube.
4. *Permukaan luar suatu tiub menegak (1 m panjang dan garispusat 80 mm) seperti yang ditunjukkan dalam Gambarajah S.4 didedahkan pada stim tepu pada tekanan atmosfera dan dikekalkan pada suhu purata 50 °C dengan aliran air sejuk melalui tiub tersebut.*

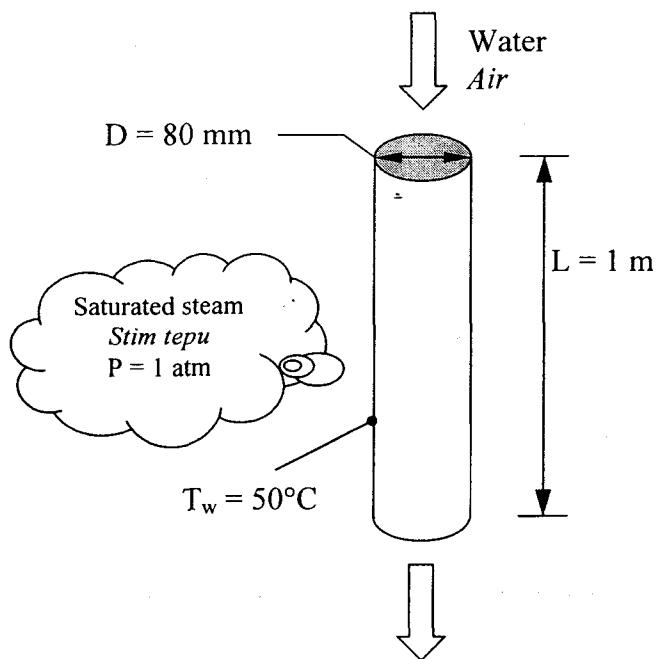


Figure Q.4.
Gambarajah S.4.

- [a] Sketch the boundary layer conditions (liquid velocity boundary layer and liquid thermal boundary layer) of the wall of the vertical tube assuming laminar film condensation is occurring on its surface.

- [a] *Lakarkan keadaan lapisan sempadan (lapisan sempadan halaju cecair dan lapisan sempadan terma cecair) pada dinding tiub menegak tersebut dengan anggapan kondensasi saput lamina terbentuk pada permukaannya.*

[4 marks/markah]

- [b] Show that the expression for the mass flow rate of film condensate and the average heat transfer coefficient are given by the equations below. Specify all the assumptions made.

- [b] *Tunjukkan bahawa persamaan bagi kadar aliran jisim kondensasi saput dan purata pekali pemindahan haba adalah seperti yang diberikan oleh persamaan-persamaan di bawah. Senaraikan semua anggapan yang dibuat.*

$$m = \frac{\rho_l g (\rho_l - \rho_v) \delta^3}{3\mu_l}$$

$$h = 0.943 \left[\frac{\rho_l g (\rho_l - \rho_v) h_{fg} k_l^3}{\mu L (T_{sat} - T_w)} \right]^{1/4}$$

[13 marks/markah]

- [c] What is the rate of steam condensed at the surface?

Given that $\rho_v = 0.596 \text{ kg/m}^3$, $\rho_l = 975 \text{ kg/m}^3$, $h_{fg} = 2257 \text{ kJ/kg}$, $k_l = 0.668 \text{ W/mK}$, $\mu_l = 375 \times 10^{-6} \text{ Ns/m}$ and $g = 9.8066 \text{ m}^2/\text{s}$

- [c] Tentukan kadar stim yang terkondensasi pada permukaan tiub?

Diberi $\rho_v = 0.596 \text{ kg/m}^3$, $\rho_l = 975 \text{ kg/m}^3$, $h_{fg} = 2257 \text{ kJ/kg}$, $k_l = 0.668 \text{ W/mK}$, $\mu_l = 375 \times 10^{-6} \text{ Ns/m}$ and $g = 9.8066 \text{ m}^2/\text{s}$

[3 marks/markah]

5. [a] The Navier-Stokes equation may be derived by considering a momentum balance on an element of volume $\Delta x \Delta y \Delta z$ as shown in the sketch.

- [a] Persamaan Navier-Stokes boleh diterbitkan dengan mempertimbangkan imbangan momentum bagi suatu elemen dengan isipadu $\Delta x \Delta y \Delta z$ seperti yang diberikan di dalam lakaran di bawah.

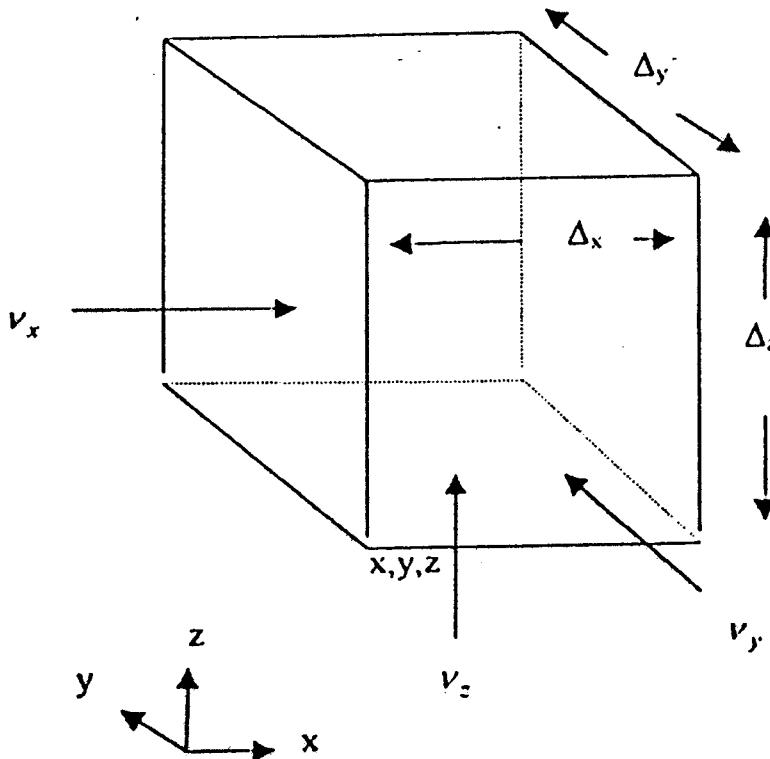


Figure Q. 5.
Rajah S. 5.

The rate of momentum accumulation in the volume element is given by the balance between the rates at which momentum is transferred into and out of the box and the sum of the forces (pressure and gravitational) on the box. Momentum balances are written in each of the three directions x , y and z as shown in the sketch.

Derive the Navier- Stokes equation for the x -direction as follows:

Kadar penumpukan momentum di dalam isipadu elemen diberikan olehimbangan di antara kadar di mana momentum dipindahkan ke dalam dan ke luar daripada kotak dan hasil tambah daya (tekanan dan graviti) ke atas kotak. Imbangang momentum ditulis untuk setiap arah x , y dan z seperti yang diberikan di dalam lakaran di atas.

Terbitkan persamaan Navier- Stokes bagi arah-x seperti berikut:

$$\rho \left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) = \mu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) - \frac{\partial p}{\partial x} + \rho g_x \quad (1)$$

where ρ is the fluid density and g_x the acceleration due to gravity in the direction of x . Note that you should use the continuity expression of the form given by;

di mana ρ adalah ketumpatan bendalir dan g_x adalah pecutan graviti pada arah-x. Anda boleh menggunakan terbitan keterusan seperti di bawah;

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

[15 marks/markah]

[b] Write down the equivalent forms of equation (1) for the y and z -directions.

[b] Tuliskan persamaan yang serupa seperti persamaan (1) pada arah-y dan z.

[5 marks/markah]

6. Component A in a binary mixture undergoes diffusion through a film of thickness δ . The concentration of A at a point x from one edge of the film is represented by C_A . While undergoing diffusion, the component A undergoes a first order reaction too within the film. The rate of reaction is first order represented by $r_A = -k_C C_A$.

Starting with the generalized equations of molar flux given in the Appendix, show that under steady state conditions, the differential equation governing the concentration C_A with distance x for uni-directional diffusion within the film in the direction of x can be expressed as

$$\frac{\partial^2 C_A}{\partial x^2} = \frac{k_1}{D} C_A$$

...9/-

If at $x = 0$, $C_A = C_{A1}$ and at $x = \delta$, $C_A = C_{A2}$, show that at steady state,

$$C_A = \frac{C_{A2} \sinh(\alpha x) + C_{A1} \sinh[\alpha(\delta - x)]}{\sinh(\alpha\delta)}$$

where $\alpha^2 = k/D$ and D = Diffusion coefficient of A in the film.

[20 marks]

Data:

- (a) For additional supplementary equations see Appendix.
(b) The solution of the differential equation $\partial^2 y / \partial x^2 = \alpha^2 \cdot y$ may be assumed as $y = K_1 \cdot \sinh(\alpha x) + K_2 \cdot \cosh(\alpha x)$ where K_1 and K_2 are constants.

6. Komponen A dalam suatu campuran binari melalui resapan menerusi suatu saput dengan ketebalan δ . Kepekatan A pada suatu titik x dari suatu pinggir saput diwakili oleh C_A . Semasa melalui resapan, komponen A turut melalui suatu tindakbalas tertib pertama dalam saput tersebut. Kadar tindakbalas ialah tertib pertama yang diwakili oleh $r_A = -k \cdot C_A$.

Bermula dengan persamaan-persamaan umum bagi fluks molar yang diberi dalam Lampiran. Tunjukkan bahawa di dalam keadaan mantap, persamaan pembezaan yang menentukan kepekatan C_A dengan jarak x bagi resapan ke arah dalam saput untuk arah x boleh diungkapkan sebagai

$$\frac{\partial^2 C_A}{\partial x^2} = \frac{k_1}{D} C_A$$

Jika pada $x = 0$, $C_A = C_{A1}$ dan pada $x = \delta$, $C_A = C_{A2}$, tunjukkan bahawa,

$$C_A = \frac{C_{A2} \sinh(\alpha n + C_{A1} \sinh[\alpha(\delta - x)])}{\sinh(\alpha n)}$$

pada keadaan mantap di mana $\alpha^2 = k/D$ dan D = pekali resapan A dalam saput.

[20 markah]

Data:

- (a) Untuk persamaan-persamaan tambahan lihat Lampiran.
(b) Penyelesaian bagi persamaan pembezaan $\partial^2 y / \partial x^2 = \alpha^2 \cdot y$ boleh diandaikan sebagai $y = K_1 \cdot \sinh(\alpha x) + K_2 \cdot \cosh(\alpha x)$ di mana K_1 dan K_2 adalah pemalar.

Appendix
Lampiran

Collision Integrals for Use with the Lennard-Jones (6-12) Potential for the Prediction of Transport Properties of Gases at Low Densities^{a,b,c}

$\kappa T/\epsilon$ or $\kappa T/\epsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{D,AB}$ (for diffusivity)	$\kappa T/\epsilon$ or $\kappa T/\epsilon_{AB}$	$\Omega_\mu = \Omega_k$ (for viscosity and thermal conductivity)	$\Omega_{D,AB}$ (for diffusivity)
0.30	2.840	2.649	2.7	1.0691	0.9782
0.35	2.676	2.468	2.8	1.0583	0.9682
0.40	2.531	2.314	2.9	1.0482	0.9588
0.45	2.401	2.182	3.0	1.0388	0.9500
0.50	2.284	2.066	3.1	1.0300	0.9418
0.55	2.178	1.965	3.2	1.0217	0.9340
0.60	2.084	1.877	3.3	1.0139	0.9267
0.65	1.999	1.799	3.4	1.0066	0.9197
0.70	1.922	1.729	3.5	0.9996	0.9131
0.75	1.853	1.667	3.6	0.9931	0.9068
0.80	1.790	1.612	3.7	0.9868	0.9008
0.85	1.734	1.562	3.8	0.9809	0.8952
0.90	1.682	1.517	3.9	0.9753	0.8897
0.95	1.636	1.477	4.0	0.9699	0.8845
1.00	1.593	1.440	4.1	0.9647	0.8796
1.05	1.554	1.406	4.2	0.9598	0.8748
1.10	1.518	1.375	4.3	0.9551	0.8703
1.15	1.485	1.347	4.4	0.9506	0.8659
1.20	1.455	1.320	4.5	0.9462	0.8617
1.25	1.427	1.296	4.6	0.9420	0.8576
1.30	1.401	1.274	4.7	0.9380	0.8537
1.35	1.377	1.253	4.8	0.9341	0.8499
1.40	1.355	1.234	4.9	0.9304	0.8463
1.45	1.334	1.216	5.0	0.9268	0.8428
1.50	1.315	1.199	6.0	0.8962	0.8129
1.55	1.297	1.183	7.0	0.8727	0.7898
1.60	1.280	1.168	8.0	0.8538	0.7711
1.65	1.264	1.154	9.0	0.8380	0.7555
1.70	1.249	1.141	10.0	0.8244	0.7422
1.75	1.235	1.128	12.0	0.8018	0.7202
1.80	1.222	1.117	14.0	0.7836	0.7025
1.85	1.209	1.105	16.0	0.7683	0.6878
1.90	1.198	1.095	18.0	0.7552	0.6751
1.95	1.186	1.085	20.0	0.7436	0.6640
2.00	1.176	1.075	25.0	0.7198	0.6414
2.10	1.156	1.058	30.0	0.7010	0.6235
2.20	1.138	1.042	35.0	0.6854	0.6088
2.30	1.122	1.027	40.0	0.6723	0.5964
2.40	1.107	1.013	50.0	0.6510	0.5763
2.50	1.0933	1.0006	75.0	0.6140	0.5415
2.60	1.0807	0.9890	100.0	0.5887	0.5180

Chapman-Enskog formula - Monoatomic gases

$$k = 1.9891 \times 10^{-4} \frac{\sqrt{T/M}}{\sigma^2 \Omega_\mu}$$

Eucken formula - Polyatomic gas

$$k = \left(\hat{C}_p + \frac{5}{4} \frac{R}{M} \right) \mu$$

Viscosity

$$\mu = 2.6693 \times 10^{-5} \frac{\sqrt{TM}}{\sigma^2 \Omega_\mu}$$

Useful formulae for Mass transport analysis

NOTATION AND DEFINITIONS

A. Notation

- c_A = concentration of A (kmol of A/m³)
- D_{AB} =Diffusion coefficient of A in B (m²/s)
- i,j,k are unit vectors in three perpendicular directions associated with the system considered.
- J_A =Diffusional flux in kmol/(m².s) = $J_x.i+J_y.j+J_z.k$ (vector)
- J_x,J_y,J_z = Diffusional flux in kmol/(m².s)in the directions i,j,k respectively
- n_A = Overall mass transfer flux in kg/(m².s)= $Nx.i+Ny.j+Nz.k$ (vector)
- n_x,n_y,n_z =Overall mass transfer fluxes in kg/(m².s)in directions of the unit vectors i,j,k
- N_A = Overall mass transfer flux in kmol/(m².s)= $Nx.i+Ny.j+Nz.k$ (vector)
- Nx,Ny,Nz =Overall mass transfer fluxes in kmol/(m².s) in directions of the unit vectors i,j,k
- r_A = rate of reaction in kg/(m³.s)
- R_A =rate of reaction in kmoles/(m³.s)
- s = scalar
- w_A =weight fraction of component A (kg of A /total kg) w_A =weight fraction of component A (kg of A /total kg)
- x,y,z = distances (m) in directions of the unit vectors i,j,k
- u,v,w = velocities (m/s) in i, j,k directions represented by $v^*=u.i+v.j+w.k$ (vector)
- \tilde{v} = a vector
- ρ =density of medium kg/m³

B. Definition of the operator ∇ Cartesian Co-ordinates
VECTOR

$$\nabla.v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

SCALAR

$$\nabla s = \frac{\partial s}{\partial x} i + \frac{\partial s}{\partial y} j + \frac{\partial s}{\partial z} k$$

C. Definition of the operator ∇ Cylindrical co-ordinates

$$\nabla.\tilde{v} = \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$\nabla s = \frac{\partial s}{\partial r} i + \frac{1}{r} \frac{\partial s}{\partial \theta} j + \frac{\partial s}{\partial z} k$$

D. Definition of the operator ∇ Spherical co-ordinates

$$\nabla \cdot v = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r \right) + \frac{1}{r \sin \theta} \frac{\partial (v_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\nabla s = \frac{\partial s}{\partial r} i + \frac{1}{r} \frac{\partial s}{\partial \theta} j + \frac{1}{r \sin \theta} \frac{\partial s}{\partial \phi} k$$

E. Definition of the operator ∇^2 Cartesian Co-ordinates

$$\nabla^2 s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

F. Definition of the operator ∇^2 Cylindrical co-ordinates

$$\nabla^2 s = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

G. Definition of the operator ∇^2 Spherical co-ordinates

$$\nabla^2 s = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial s}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial s}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 s}{\partial \phi^2}$$

DIFFERENTIAL EQUATIONS FOR MASS TRANSPORT ANALYSIS

A. Equivalent forms of Fick's law of binary diffusion and Mass/Molar flux equations

(mass flux)

$$n_A = \rho w_A \cdot v - D_{AB} \nabla \cdot (\rho w_A)$$

(molar flux)

$$N_A = c_A \cdot v^* - D_{AB} \nabla \cdot c_A$$

B. The equations of continuity for a multi-component mixture

(mass flux)

$$\frac{\partial (\rho w_A)}{\partial t} = -(\nabla \cdot n_A) + r_A$$

(molar flux)

$$\frac{\partial c_A}{\partial t} = -(\nabla \cdot N_A) + R_A$$

Table E.1 Lennard-Jones (6-12) Potential Parameters and Critical Properties

Substance	Lennard-Jones parameters				Critical properties ^{a,b}				
	Molecular Weight M	σ (Å)	ϵ/K (K)	Ref.	T_c (K)	p_c (atm)	\tilde{V}_c (cm ³ /g-mole)	$\mu_c \times 10^6$ (g/cm · s)	$k_c \times 10^6$ (cal/cm · s ¹ · K)
Light gases:									
H ₂	2.016	2.915	38.0	a	33.3	12.80	65.0	34.7	79.2
He	4.003	2.576	10.2	a	5.26	2.26	57.8	25.4	71.0
Noble gases:									—
Ne	20.180	2.789	35.7	a	44.5	26.9	41.7	156.	49.4
Ar	39.948	3.432	122.4	b	150.7	48.0	75.2	264.	40.2
Kr	83.80	3.675	170.0	b	209.4	54.3	92.2	396.	—
Xe	131.29	4.009	234.7	b	289.8	58.0	118.8	490.	—
Simple polyatomic gases:									—
Air	28.964 ^c	3.617	97.0	a	132.4 ⁱ	37.0 ⁱ	86.7 ⁱ	193.	90.8
N ₂	28.013	3.667	99.8	b	126.2	33.5	90.1	180.	86.8
O ₂	31.999	3.433	113.	a	154.4	49.7	74.4	250.	105.3
CO	28.010	3.590	110.	a	132.9	34.5	93.1	190.	86.5
CO ₂	44.010	3.996	190.	a	304.2	72.8	94.1	343.	122.
NO	30.006	3.470	119.	a	180.	64.	57.	258.	118.2
N ₂ O	44.012	3.879	220.	a	309.7	71.7	96.3	332.	131.
SO ₂	64.065	4.026	363.	c	430.7	77.8	122.	411.	98.6
F ₂	37.997	3.653	112.	a	—	—	—	—	—
Cl ₂	70.905	4.115	357.	a	417.	76.1	124.	420.	97.0
Br ₂	159.808	4.268	520.	a	584.	102.	144.	—	—
I ₂	253.809	4.982	550.	a	800.	—	—	—	—