UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2008/2009

Jun 2009

EEE 228 – ISYARAT DAN SISTEM

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat dan DUA BELAS muka surat LAMPIRAN yang bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan.

Jawab **LIMA** soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah bagi setiap soalan diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam bahasa Malaysia atau bahasa Inggeris atau kombinasi kedua-duanya.

1.   Jelaskan secara ringkas,

     Briefly explain,

    (a)   Interpolasi lelurus.

          Linear interpolation.

    (b)   Teori pensampelan.

          Sampling Theorem.

    (c)   Pembahagian multiplex frekuensi.

          Frequency Division Multiplexing.

    (d)   Pengesan Segerak atau Jelas.

          Synchronous or coherent detection.   (100%)

2.   (a)   Bagi isyarat diskret

          For the discrete-time signal

$$x[n] = \left\{-1, \frac{1}{2}, 0, 1, -\frac{3}{2}, -1\right\}$$

lakarkan $x(0.5n + 1)$

sketch $x(0.5n + 1)$

    (b)   Carikan penjumlahan pelingkaran $y[n] = h[n] * x[n]$ bagi pasangan jujukan terhingga berikut:

Find the convolution summation $y[n] = h[n] * x[n]$ for the following pair of finite sequences:

$$x[n] = \{1,2,3,0,-1\}, \quad h[n] = \{2,-1,3,1,-2\}$$
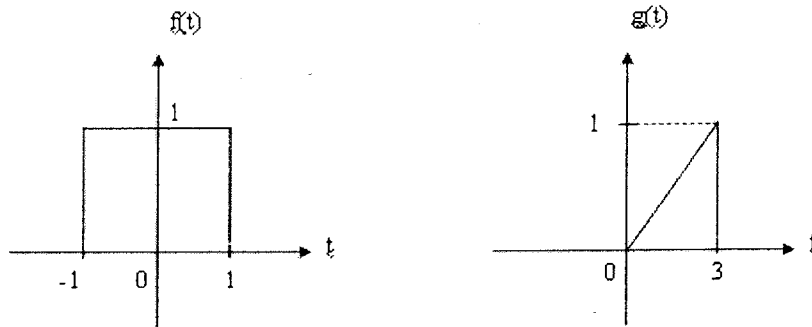
...3/-

(c)   Nyatakan tiga sifat pelingkaran.

        State three properties of convolution.

(100%)

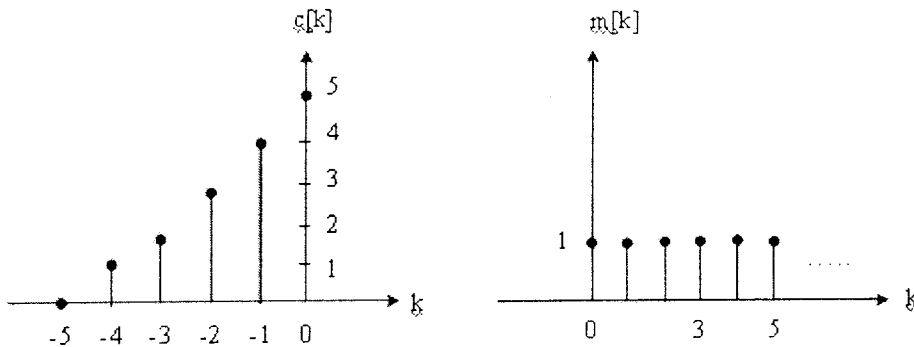3.   (a)   Cari dan lakarkan integrasi konvolusi bagi isyarat f(t) dan g(t) yang ditunjukkan dalam Rajah 3(a).

           Find and sketch the convolution integral of the signals f(t) and g(t) shown in Figure 3(a).

Rajah 3(a)
Figure 3(a)

(b)   Cari dan lakarkan hasil konvolusi bagi isyarat c[k] dan m[k] yang ditunjukkan dalam Rajah 3(b).

      Find and sketch the convolution sum of the signals c[k] and m[k] shown in Figure 3(b).

Rajah 3(b)
Figure 3(b)

(c)   Satu isyarat diskret masa diberikan oleh persamaan:-

A discrete-time signal is given by the equation:-

$$y[k] = \left(\frac{4-k}{3}\right)\{u[k-2] - 3u[k]\}$$

Lakarkan isyarat ini.
Sketch this signal.

(100%)

4.   (a)   Dengan menggunakan definisi jelmaan Fourier, cari jelmaan Fourier untuk

Using the definition of Fourier Transform, find the Fourier Transform of

(i) $e^{-at}u(t)$

(ii) $f(t) = rect\left(\frac{t}{\tau}\right)$ (40%)

...5/-

(b)   Jelaskan secara ringkas,
       Explain briefly,

       (i)   Proses Modulasi.
              Modulation Process.

       (ii)   Kadar persampelan Nyquist.
              The Nyquist sampling rate.

                                                      (30%)

(c)   Tentukan kadar persampelan Nyquist dan julat persampelan Nyquist bagi
       isyarat berikut:

       *Determine the Nyquist sampling rate and the Nyquist sampling interval for the
       signal:*

$$\text{sinc}^2(100\pi t)$$

                                                      (30%)

5.   (a)   Dengan menggunakan jelmaan- z, tunjukkan bahawa
             Using the definition of z-transform, show that

$$\gamma^{k-1}u[k-1] \Leftrightarrow \frac{1}{z-\gamma}$$

                                                      (30%)

      (b)   Cari 3 ungkapan pertama f[k] jika
             Find the first three terms of f[k] if

$$F[z] = \frac{2z^3 + 13z^2 + z}{z^3 + 7z^2 + 2z + 1}$$

                                                      (30%)

(c)   Selesaikan

Solve

$$4y[k+2] + 4y[k+1] + y[k] = f[k+1]$$

With y[-1]=0, y[-2]=1 and f[k]=u[k]

(40%)

6.   (a)   Cari sambutan keadaan kosong bagi sistem LTID dengan fungsi pindah

Find the zero state response of an LTID system with transfer function
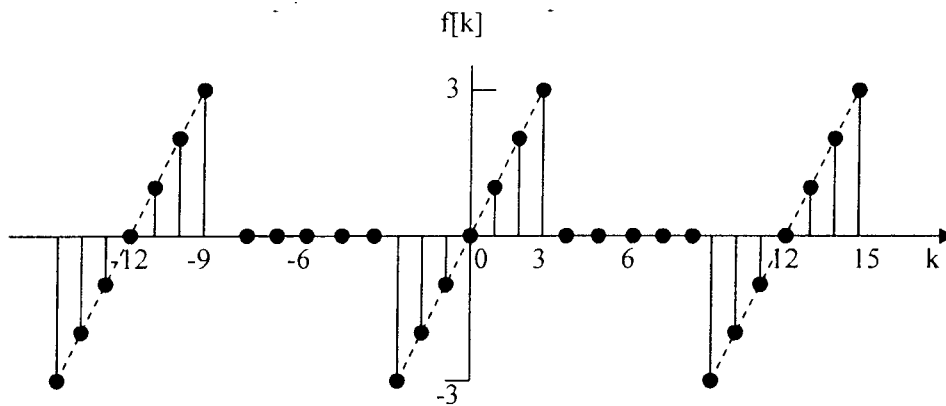
$$H[z] = \frac{2z+3}{(z-2)(z-3)}$$

Jika
If

$$f[k] = u[k]$$ (40%)

(b)   Cari siri Fourier diskret bagi f[k] seperti dalam Rajah 1.

Find the discrete-time Fourier series for the f[k] shown in Figure 1.

(60%)

ooooOOoooo

A Short Table of Fourier Transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a|t|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn}t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\,u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\,u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\,u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\,u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\displaystyle\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$	

Fourier Transform Operations

Operation	$f(t)$	$F(\omega)$
Addition	$f_1(t) + f_2(t)$	$F_1(\omega) + F_2(\omega)$
Scalar multiplication	$kf(t)$	$kF(\omega)$
Symmetry	$F(t)$	$2\pi f(-\omega)$
Scaling (a real)	$f(at)$	$\dfrac{1}{|a|}F\left(\dfrac{\omega}{a}\right)$
Time shift	$f(t - t_0)$	$F(\omega)e^{-j\omega t_0}$
Frequency shift (ω_0 real)	$f(t)e^{j\omega_0 t}$	$F(\omega - \omega_0)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(\omega)F_2(\omega)$
Frequency convolution	$f_1(t)f_2(t)$	$\dfrac{1}{2\pi}F_1(\omega) * F_2(\omega)$
Time differentiation	$\dfrac{d^n f}{dt^n}$	$(j\omega)^n F(\omega)$
Time integration	$\displaystyle\int_{-\infty}^{t} f(x)\,dx$	$\dfrac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$

Appendix

Convolution Table

No	$f_1(t)$	$f_2(t)$	$f_1(t) * f_2(t) = f_2(t) * f_1(t)$
1	$f(t)$	$\delta(t-T)$	$f(t-T)$
2	$e^{\lambda t}u(t)$	$u(t)$	$\dfrac{1-e^{\lambda t}}{-\lambda}u(t)$
3	$u(t)$	$u(t)$	$tu(t)$
4	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_1-\lambda_2}u(t)\qquad \lambda_1 \neq \lambda_2$
5	$e^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$te^{\lambda t}u(t)$
6	$te^{\lambda t}u(t)$	$e^{\lambda t}u(t)$	$\dfrac{1}{2}t^2 e^{\lambda t}u(t)$
7	$t^n u(t)$	$e^{\lambda t}u(t)$	$\dfrac{n!\,e^{\lambda t}}{\lambda^{n+1}}u(t) - \displaystyle\sum_{j=o}^{n}\dfrac{n!\,t^{n-j}}{\lambda^{j+1}(n-j)!}u(t)$
8	$t^m u(t)$	$t^n u(t)$	$\dfrac{m!\,n!}{(m+n+1)!}t^{m+n+1}u(t)$
9	$te^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(t)$	$\dfrac{e^{\lambda_2 t}-e^{\lambda_1 t}+(\lambda_1-\lambda_2)te^{\lambda_1 t}}{(\lambda_1-\lambda_2)^2}u(t)$
10	$t^m e^{\lambda t}u(t)$	$t^n e^{\lambda t}u(t)$	$\dfrac{m!\,n!}{(n+m+1)!}t^{m+n+1}e^{\lambda t}u(t)$
11	$t^m e^{\lambda_1 t}u(t)$	$t^n e^{\lambda_2 t}u(t)$	$\displaystyle\sum_{j=0}^{m}\dfrac{(-1)^j m!(n+j)!\,t^{m-j}e^{\lambda_1 t}}{j!(m-j)!(\lambda_1-\lambda_2)^{n+j+1}}u(t)$
	$\lambda_1 \neq \lambda_2$		$+\displaystyle\sum_{k=0}^{n}\dfrac{(-1)^k n!(m+k)!\,t^{n-k}e^{\lambda_2 t}}{k!(n-k)!(\lambda_2-\lambda_1)^{m+k+1}}u(t)$
12	$e^{-\alpha t}\cos(\beta t+\theta)u(t)$	$e^{\lambda t}u(t)$	$\dfrac{\cos(\theta-\phi)e^{\lambda t}-e^{-\alpha t}\cos(\beta t+\theta-\phi)}{\sqrt{(\alpha+\lambda)^2+\beta^2}}u(t)$
			$\phi=\tan^{-1}[-\beta/(\alpha+\lambda)]$
13	$e^{\lambda_1 t}u(t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}u(t)+e^{\lambda_2 t}u(-t)}{\lambda_2-\lambda_1}\quad \mathrm{Re}\,\lambda_2 > \mathrm{Re}\,\lambda_1$
14	$e^{\lambda_1 t}u(-t)$	$e^{\lambda_2 t}u(-t)$	$\dfrac{e^{\lambda_1 t}-e^{\lambda_2 t}}{\lambda_2-\lambda_1}u(-t)$

Convolution Sums

No.	$f_1[k]$	$f_2[k]$	$f_1[k] * f_2[k] = f_2[k] * f_1[k]$	
1	$\delta[k-j]$	$f[k]$	$f[k-j]$	
2	$\gamma^k u[k]$	$u[k]$	$\left[\dfrac{1-\gamma^{k+1}}{1-\gamma}\right]u[k]$	
3	$u[k]$	$u[k]$	$(k+1)u[k]$	
4	$\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\left[\dfrac{\gamma_1^{k+1}-\gamma_2^{k+1}}{\gamma_1-\gamma_2}\right]u[k]$ $\quad \gamma_1 \neq \gamma_2$	
5	$\gamma_1^k u[k]$	$\gamma_2^k u[-(k+1)]$	$\dfrac{\gamma_1}{\gamma_2-\gamma_1}\gamma_1^k u[k] + \dfrac{\gamma_2}{\gamma_2-\gamma_1}\gamma_2^k u[-(k+1)]$ $\qquad |\gamma_2| > |\gamma_1|$	
6	$k\gamma_1^k u[k]$	$\gamma_2^k u[k]$	$\dfrac{\gamma_1\gamma_2}{(\gamma_1-\gamma_2)^2}\left[\gamma_2^k - \gamma_1^k + \dfrac{\gamma_1-\gamma_2}{\gamma_2}k\gamma_1^k\right]u[k]$ $\quad \gamma_1 \neq \gamma_2$	
7	$ku[k]$	$ku[k]$	$\dfrac{1}{6}k(k-1)(k+1)u[k]$	
8	$\gamma^k u[k]$	$\gamma^k u[k]$	$(k+1)\gamma^k u[k]$	
9	$\gamma^k u[k]$	$ku[k]$	$\left[\dfrac{\gamma(\gamma^k-1)+k(1-\gamma)}{(1-\gamma)^2}\right]u[k]$	
10	$|\gamma_1|^k \cos(\beta k + \theta)u[k]$	$\gamma_2^k u[k]$	$\dfrac{1}{R}\left[|\gamma_1|^{k+1}\cos[\beta(k+1)+\theta-\phi] - \gamma_2^{k+1}\cos(\theta-\phi)\right]u[k]$	γ_2 real
			$R = \left[|\gamma_1|^2 + \gamma_2^2 - 2|\gamma_1|\gamma_2\cos\beta\right]^{1/2}$	
			$\phi = \tan^{-1}\left[\dfrac{(|\gamma_1|\sin\beta)}{(|\gamma_1|\cos\beta - \gamma_2)}\right]$	

4

Appendix

$\mathcal{Z}$- Transform Operations

Operation	$f[k]$	$F[z]$
Addition	$f_1[k] + f_2[k]$	$F_1[z] + F_2[z]$
Scalar multiplication	$af[k]$	$aF[z]$
Right-shift	$f[k-m]u[k-m]$	$\frac{1}{z^m}F[z]$
	$f[k-m]u[k]$	$\frac{1}{z^m}F[z] + \frac{1}{z^m}\sum_{k=1}^{m} f[-k]z^k$
	$f[k-1]u[k]$	$\frac{1}{z}F[z] + f[-1]$
	$f[k-2]u[k]$	$\frac{1}{z^2}F[z] + \frac{1}{z}f[-1] + f[-2]$
	$f[k-3]u[k]$	$\frac{1}{z^3}F[z] + \frac{1}{z^2}f[-1] + \frac{1}{z}f[-2] + f[-3]$
Left-shift	$f[k+m]u[k]$	$z^m F[z] - z^m \sum_{k=0}^{m-1} f[k]z^{-k}$
	$f[k+1]u[k]$	$zF[z] - zf[0]$
	$f[k+2]u[k]$	$z^2 F[z] - z^2 f[0] - zf[1]$
	$f[k+3]u[k]$	$z^3 F[z] - z^3 f[0] - z^2 f[1] - zf[2]$
Multiplication by γ^k	$\gamma^k f[k]u[k]$	$F\left[\frac{z}{\gamma}\right]$
Multiplication by k	$kf[k]u[k]$	$-z\frac{d}{dz}F[z]$
Time Convolution	$f_1[k] * f_2[k]$	$F_1[z]F_2[z]$
Frequency Convolution	$f_1[k]f_2[k]$	$\frac{1}{2\pi j}\oint F_1[u]F_2\left[\frac{z}{u}\right]u^{-1}du$
Initial value	$f[0]$	$\lim_{z\to\infty} F[z]$
Final value	$\lim_{N\to\infty} f[N]$	$\lim_{z\to 1}(z-1)F[z]$ poles of $(z-1)F[z]$ inside the unit circle.

5

Lampiran

Appendix

(Unilateral) z-Transform Pairs

$f[k]$	$F[z]$
1 $\delta[k-j]$	z^{-j}
2 $u[k]$	$\dfrac{z}{z-1}$
3 $ku[k]$	$\dfrac{z}{(z-1)^2}$
4 $k^2 u[k]$	$\dfrac{z(z+1)}{(z-1)^3}$
5 $k^3 u[k]$	$\dfrac{z(z^2+4z+1)}{(z-1)^4}$
6 $\gamma^{k-1} u[k-1]$	$\dfrac{1}{z-\gamma}$
7 $\gamma^k u[k]$	$\dfrac{z}{z-\gamma}$
8 $k\gamma^k u[k]$	$\dfrac{\gamma z}{(z-\gamma)^2}$
9 $k^2 \gamma^k u[k]$	$\dfrac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10 $\dfrac{k(k-1)(k-2)\cdots(k-m+1)}{\gamma^m m!}\gamma^k u[k]$	$\dfrac{z}{(z-\gamma)^{m+1}}$
11a $\lvert\gamma\rvert^k \cos\beta k\, u[k]$	$\dfrac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b $\lvert\gamma\rvert^k \sin\beta k\, u[k]$	$\dfrac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\dfrac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$ $\gamma=\lvert\gamma\rvert e^{j\beta}$	$\dfrac{(0.5re^{j\theta})z}{z-\gamma}+\dfrac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c $r\lvert\gamma\rvert^k \cos(\beta k+\theta)u[k]$	$\dfrac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$

$$r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$$

$$\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert},\quad \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$$

The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\dfrac{df}{dt}$	$sF(s) - f(0^-)$
	$\dfrac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - \dot{f}(0^-)$
	$\dfrac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\displaystyle\int_{0^-}^{t} f(\tau)\, d\tau$	$\dfrac{1}{s} F(s)$
	$\displaystyle\int_{-\infty}^{t} f(\tau)\, d\tau$	$\dfrac{1}{s} F(s) + \dfrac{1}{s}\displaystyle\int_{-\infty}^{0^-} f(t)\, dt$
Time shift	$f(t - t_0)u(t - t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0 t}$	$F(s - s_0)$
Frequency differentiation	$-tf(t)$	$\dfrac{dF(s)}{ds}$
Frequency integration	$\dfrac{f(t)}{t}$	$\displaystyle\int_{s}^{\infty} F(z)\, dz$
Scaling	$f(at), \ a \geq 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\dfrac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s \to \infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s \to 0} sF(s) \ \text{(poles of } sF(s) \text{ in LHP)}$

B.7 Miscellaneous

B.7-1 L'Hôpital's Rule

If $\lim f(x)/g(x)$ results in the indeterministic form $0/0$ or ∞/∞, then

$$\lim \frac{f(x)}{g(x)} = \lim \frac{\dot{f}(x)}{\dot{g}(x)}$$

B.7-2 The Taylor and Maclaurin Series

$$f(x) = f(a) + \frac{(x-a)}{1!}\dot{f}(a) + \frac{(x-a)^2}{2!}\ddot{f}(a) + \cdots$$

$$f(x) = f(0) + \frac{x}{1!}\dot{f}(0) + \frac{x^2}{2!}\ddot{f}(0) + \cdots$$

B.7-3 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \cdots \qquad x^2 < \pi^2/4$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots + \binom{n}{k}x^k + \cdots + x^n$$

$$\approx 1 + nx \qquad |x| \ll 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \qquad |x| < 1$$

B.7-4 Sums

$$\sum_{m=0}^{k} r^m = \frac{r^{k+1}-1}{r-1} \qquad r \neq 1$$

$$\sum_{m=M}^{N} r^m = \frac{r^{N+1}-r^M}{r-1} \qquad r \neq 1$$

$$\sum_{m=0}^{k} \left(\frac{a}{b}\right)^m = \frac{a^{k+1}-b^{k+1}}{b^k(a-b)} \qquad a \neq b$$

B.7-5 Complex Numbers

$$e^{\pm j\pi/2} = \pm j$$

$$e^{\pm jn\pi} = \begin{cases} 1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$$

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$a + jb = re^{j\theta} \qquad r = \sqrt{a^2 + b^2}, \qquad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$(re^{j\theta})^k = r^k e^{jk\theta}$$

$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

B.7-6 Trigonometric Identities

$$e^{\pm jx} = \cos x \pm j\sin x$$

$$\cos x = \frac{1}{2}[e^{jx} + e^{-jx}]$$

$$\sin x = \frac{1}{2j}[e^{jx} - e^{-jx}]$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x$$

$$2\sin x \cos x = \sin 2x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)$$

$$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2}[\sin(x - y) + \sin(x + y)]$$

$$a\cos x + b\sin x = C\cos(x + \theta)$$

in which $C = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

B.7-7 Indefinite Integrals

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)\dot{g}(x)\,dx = f(x)g(x) - \int \dot{f}(x)g(x)\,dx$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax \qquad \int \cos ax\,dx = \frac{1}{a}\sin ax$$

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x\sin ax\,dx = \frac{1}{a^2}(\sin ax - ax\cos ax)$$

$$\int x\cos ax\,dx = \frac{1}{a^2}(\cos ax + ax\sin ax)$$

$$\int x^2\sin ax\,dx = \frac{1}{a^3}(2ax\sin ax + 2\cos ax - a^2 x^2\cos ax)$$

$$\int x^2\cos ax\,dx = \frac{1}{a^3}(2ax\cos ax - 2\sin ax + a^2 x^2\sin ax)$$

$$\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int \sin ax\cos bx\,dx = -\left[\frac{\cos(a-b)x}{2(a-b)} + \frac{\cos(a+b)x}{2(a+b)}\right] \qquad a^2 \neq b^2$$

$$\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \qquad a^2 \neq b^2$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a^2}(ax - 1)$$

$$\int x^2 e^{ax}\,dx = \frac{e^{ax}}{a^3}(a^2 x^2 - 2ax + 2)$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx)$$

$$\int \frac{1}{x^2 + a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2 + a^2}\,dx = \frac{1}{2}\ln(x^2 + a^2)$$

B.7-8 Differentiation Table

$$\frac{d}{dx}f(u) = \frac{d}{du}f(u)\frac{du}{dx} \qquad \frac{d}{dx}a^{bx} = b(\ln a)a^{bx}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\sin ax = a\cos ax$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \frac{d}{dx}\cos ax = -a\sin ax$$

$$\frac{dx^n}{dx} = nx^{n-1} \qquad \frac{d}{dx}\tan ax = \frac{a}{\cos^2 ax}$$

$$\frac{d}{dx}\ln(ax) = \frac{1}{x} \qquad \frac{d}{dx}(\sin^{-1} ax) = \frac{a}{\sqrt{1 - a^2 x^2}}$$

$$\frac{d}{dx}\log(ax) = \frac{\log e}{x} \qquad \frac{d}{dx}(\cos^{-1} ax) = \frac{-a}{\sqrt{1 - a^2 x^2}}$$

$$\frac{d}{dx}e^{bx} = be^{bx} \qquad \frac{d}{dx}(\tan^{-1} ax) = \frac{a}{1 + a^2 x^2}$$

B.7-9 Some Useful Constants

$\pi \approx 3.1415926535$

$e \approx 2.7182818284$

$\frac{1}{e} \approx 0.3678794411$

$\log_{10} 2 = 0.30103$

$\log_{10} 3 = 0.47712$

B.7-10 Solution of Quadratic and Cubic Equations

Any quadratic equation can be reduced to the form

$$ax^2 + bx + c = 0$$

The solution of this equation is provided by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A general cubic equation

$$y^3 + py^2 + qy + r = 0$$

may be reduced to the depressed cubic form

$$x^3 + ax + b = 0$$

by substituting

$$y = x - \tfrac{p}{3}$$

This yields

$$a = \tfrac{1}{3}(3q - p^2) \qquad b = \tfrac{1}{27}(2p^3 - 9pq + 27r)$$

Now let

$$A = \sqrt[3]{-\tfrac{b}{2} + \sqrt{\tfrac{b^2}{4} + \tfrac{a^3}{27}}}, \qquad B = \sqrt[3]{-\tfrac{b}{2} - \sqrt{\tfrac{b^2}{4} + \tfrac{a^3}{27}}}$$

The solution of the depressed cubic is

$$x = A + B, \qquad x = -\tfrac{A+B}{2} + \tfrac{A-B}{2}\sqrt{-3}, \qquad x = -\tfrac{A+B}{2} - \tfrac{A-B}{2}\sqrt{-3}$$

and

$$y = x - \tfrac{p}{3}$$

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