UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2008/2009

November 2008

ZCA 110/4 – Calculus and Linear Algebra
[Kalkulus dan Aljabar Linear]

Duration: 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of **SEVEN** printed pages before the examination begins.
*[Sila Pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

Instruction: Answer ALL (**FOUR**) questions in Section A. Please indicate the chosen questions clearly on the front page of each answer booklet. Answer BOTH (**TWO**) questions in Section B.
*[**Arahan**: Jawab KESEMUA (**EMPAT**) soalan yang diberikan dalam Seksyen A. Sila tunjukkan soalan-soalan pilihan anda dengan jelas di muka surat depan tiap-tiap buku jawapan. Jawab **KEDUA-DUA** soalan dalam Seksyen B.]*

...2/-

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(a) Consider the function

$$f(x) = x^4 - 4x^3 + 10.$$

i. Identify where the extrema of f occur. [*Tentukan di mana extremum-extremum f berlaku.*]

ii. Find the intervals on which f is increasing and the intervals on which f is decreasing. [*Carikan selang di mana f bertambah dan selang di mana ia berkurangan.*]

iii. Find where the graph of f is concave up and where it is concave down. [*Carikan selang di mana graf f berkecekungan atas and selang di mana ia berkecekungan bawah.*]

(b)

(i) Sketch the graph of $\sinh x$. Label your sketch clearly. *[Lakarkan graf untuk* $\sinh x$ *pada graf yang sama.]*

(ii) State the domains and ranges of $\sinh x$. *[Nyatakan domain dan banjaran* $\sinh x$*.]*

(15/100)

3.

(a) Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the line $y = x/4$, above by the curve $y = 1 + \sqrt{x}$, and above right by the curve $y = 2/\sqrt{x}$. Draw using graph paper all the boundaries and indicate the area calculated by shading.

[Cari luas kawasan dalam kuadrant pertama yang di batas oleh paksi-y dari kiri, garis $y = x/4$*, dari bawah, lengkungan* $y = 1 + \sqrt{x}$*, dari atas, dan lengkungan* $y = 2/\sqrt{x}$*, dari atas kanan. Lukis dengan menggunakan kertas graf semua*

batasan dan tunjuk luas yang dicari dengan menggelapkannya.]

(b) Find the following integrals. *[Cari kamiran-kamiran berikut.]*

(i)

$$\int_1^e \frac{1}{x}(1+7\ln x)^{-1/3}\,dx$$

(ii)

$$\int \frac{dx}{x(3\sqrt{x+1})}$$

(iii)

$$\int \frac{x^5\,dx}{x^4-16}$$

(iv)

$$\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$$

(15/100)

4.

Given the linear transformation, $Y_W = AX_W$, relative to the W-basis: $W_1 = [0,-1,2]^T$, $W_2 = [4,1,0]^T$, and $W_3 = [-2,0,-4]^T$, where $W = [W_1, W_2, W_3]$, and

[Diberi transformasi linear, $Y_W = AX_W$, relatif kepada W-basis: $W_1 = [0,-1,2]^T$, $W_2 = [4,1,0]^T$, dan $W_3 = [-2,0,-4]^T$, di mana $W = [W_1, W_2, W_3]$, dan]

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Find the representation, $Y_Z = BX_Z$, relative to the Z-basis: $Z_1 = [1,-1,1]^T$, $Z_2 = [1,0,-1]^T$, and $Z_3 = [1,2,1]^T$, where $Z = [Z_1, Z_2, Z_3]$. That is, find the matrix, B. Do this by answering the following questions.

[Cari perwakilan, $Y_Z = BX_Z$, relatif kepada Z-basis: $Z_1 = [1,-1,1]^T$, $Z_2 =$

...5/-

$[1, 0, -1]^T$, *dan* $Z_3 = [1, 2, 1]^T$, *di mana* $Z = [Z_1, Z_2, Z_3]$. *Iaitu, cari matriks, B. Buat ini dengan menjawab soalan-soalan berikut.]*

i. By using the elementary row transfomations, find the inverse matrix W^{-1}.

[Dengan menggunakan transformasi baris asas, cari matriks songsang, W^{-1}*.]*

ii. Find the matrix Q where $X_W = QX_Z$.

[Cari matriks Q *di mana* $X_W = QX_Z$*.]*

iii, Find the inverse matrix Q^{-1}.

[Cari matriks songsang, Q^{-1}*.]*

iv. Finally, find the matrix B.

[Akhirnya, cari matriks B*.]*

(15/100)

SECTION B

Answer BOTH questions from this section. [*Jawab kedua-dua soalan dalam Seksyen ini.*]

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(a) Find equations for the tangent and normal to the curve $y = 1 + \cos x$ at the point (π/2,1). Sketch the curve, tangent and normal together, labelling each with its equation. [*Carikan persamaan tangen and normal kepada lengkungan* $y = 1 + \cos x$ *pada titik* (π/2, 1)*. Lakarkan lengkungan ini, bersama-sama dengan normal dan tangen tersebut, serta melabelkan mereka dengan persamaannya.*]

(b)

i. By differentiating $x^2 - y^2 = 1$ implicitly, show that $dy/dx = x/y$. *[Dengan membezakan* $x^2 - y^2 = 1$ *secara terserat, tunjukkan bahawa* $dy/dx = x/y$*.]*

ii. Then show that $d^2y/dx^2 = -1/y^3$ *[Selanjutnya, tunjukkan bahawa* $d^2y/dx^2 = -1/y^3$*.]*

(c) Find the height and radius of the largest right circular cylinder that can be put in a sphere of radius $\sqrt{3}$. *[Carikan tinggi and radius silinder tegak lurus yang paling besar yang boleh dimasukkan ke dalam suatu sfera yang radiusnya $\sqrt{3}$.]*

(d)

(i) Use logarithmic differentiation to find the derivative of y with respect to the variable t. *[Guna pembezaan logaritma untuk memperolehi terbitan y terhadap pembolehubah t.]*

$$y = \left(\frac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5, t > 2$$

(ii) Find the limit of the following. *[Dapatkan limit bagi yang berikut.]*

$$\lim_{y \to 0^+} e^{-1/y} \ln y$$

(20/100)

6

(a) Find the sum of the following series: *[Cari jumlah siri-siri berikut:]*

i.

$$\sum_{n=1}^{\infty} \frac{40n}{(2n-1)^2(2n+1)^2}$$

ii.

$$\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$$

iii.

$$\sum_{n=0}^{\infty} 3\left(\frac{x-1}{2}\right)^n$$

In part iii, find the values of x for which the series converges. *[Dalam bahagian iii, cari nilai x di mana siri ini menumpu.]*