UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2005/2006

June 2006

IUK 291E – Mathematic II
[Matematik II]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of FIVE (5) pages of printed material before you begin the examination.

Instructions:

1. Answer **ALL** questions. All questions can be answered either in Bahasa Malaysia OR English.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA (5) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Arahan:

1. *Jawab **SEMUA** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia ATAU Bahasa Inggeris.*

Answer all questions.

1. (a) If f(x,y) = $\sin^{-1} xy$, verify that $f_{xy} = f_{yx}$

(10 marks)

(b) Find all points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane 2x + y - 3z = 2.

(10 marks)

2. (a) Find the volume of the solid region common to the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder r = 2 cos θ.

(10 marks)

(b) Evaluate the double integral $\int_0^{2\pi} \int_0^a \frac{1}{a+r}$ r dr dθ

(10 marks)

3. (a) Use variation of parameters to solve the differential equation $y'' - y' = 2\cos^2 x$ that satisfies the initial conditions y(0) = 0, $y'(0) = 1$.

(7 marks)

(b) A dairy product produces whole milk and skim milk in quantities x and y liters, respectively. Suppose the price (in ringgit) of whole milk is p(x) = 100 – x and that of skim milk is q(y) = 100 – y, and also assume that C(x, y) = $x^2 + xy + y^2$ is the joint-cost function of the commodities. Maximize the profit

$$P(x, y) = p\,x + q\,y - C(x, y)$$

(7 marks)

(c) Use the Lagrange multipliers to find the minimum distance from the origin to the ellipse $5x^2 - 6xy + 5y^2 = 4$

(6 marks)