UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
Academic Session 2005/2006

June 2006

IUK 191E – Mathematic I
[Matematik I]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of NINE (9) pages of printed material before you begin the examination.

Instructions:

1. Answer **ALL** questions. All questions can be answered either in Bahasa Malaysia OR English.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN (9) muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Arahan:

*1. Jawab **SEMUA** soalan. Semua soalan boleh dijawab dalam Bahasa Malaysia ATAU Bahasa Inggeris.*

1. (a) (i) Evaluate the limit for the function $\lim_{x\to 1}\frac{\frac{1}{x}-1}{\sqrt{x}-1}$

(2 marks)

(ii) Evaluate $\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ for the function f(x) = ln x

(Hint: $\lim_{h\to\infty}(1+\frac{1}{h})^h = e$ and let $\frac{1}{h}=\frac{\Delta x}{x}$)

(3 marks)

(b) (i) Find constants A and B so that the following function f(x) will be continuous for all x:

$$f(x)=\begin{cases}\dfrac{x^2-Ax-6}{x-2} & \text{if } x>2\\ x^2+B & \text{if } x\le 2\end{cases}$$

(5 marks)

(ii) Let $f(x)=-x^4+x^2+A$ for constant A. What value of A should be chosen that guarantees that if $x_0=\frac{1}{3}$ is chosen as the initial estimate, the Newton – Raphson method produces $x_1=-x_0$, $x_2=x_0$, $x_3=-x_0,\ldots$

(5 marks)

(c) (i) Find the standard form of the equation for the tangent line to the curve y = $\frac{\sin x}{x}$ at the point x = $\frac{\pi}{4}$.

(2 marks)

(ii) Show that the differential equation $\frac{dy}{dx}=x^2y^2\sqrt{4-x^3}$ has the solution $y=\dfrac{9}{2(4-x^3)^{\frac{3}{2}}+c}$

(3 marks)

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2. (a) A bucket containing 5 liters of water has a leak. After t seconds, there are $Q(t) = 5\left(1 - \frac{t}{25}\right)^2$ liters of water in the bucket.

(i) At what rate (to the nearest hundredth liter) is water leaking from the bucket after 2 seconds?

(3 marks)

(ii) How long does it take for all the water to leak out of the bucket?

(2 marks)

(iii) At what rate is the water leaking when the last drop leaks out?

(3 marks)

(b) Let f be a function for which $f'(x) = \frac{1}{x^2+1}$

(i) If g(x) = f(3x-1), what is $g'(x)$

(2 marks)

(ii) If h(x) = $f\left(\frac{1}{x}\right)$, what is $h'(x)$?

(2 marks)

(c) Find $\frac{dy}{dx}$ if $y = \sqrt{x}\sin^{-1}(3x+2)$

(2 marks)

(d) A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 6t^2 - 2t - 4$. It is known that the particle is at position $x = 6$ for $t = 2$.

(i) Write a polynomial expression for the position of the particle at any time $t \geq 0$

(2 marks)

(ii) For what values of t, $0 \leq t \leq 3$ is the particle's instantaneous velocity the same as it average velocity on the interval [0,3]?

(2 marks)

(iii) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(2 marks)

3. (a) (i) Find $\int \frac{\sin x}{\sqrt{\cos 2x}} dx$ (3 marks)

(i) Find $\int \frac{\sqrt{9x^2 - 1}}{x} dx$ (4 marks)

(b) (i) Find the volume of the solid formed by revolving about the y-axis the region bounded by the curve

$y = \frac{1}{1 + x^4}$ between $x = 0$ and $x = 4$.

(5 marks)

(ii) Find the surface area of the solid generated by revolving the region bounded by the curve $y = e^x + \frac{1}{4}e^{-x}$ on the [0,1] about the x-axis.

(3 marks)

(iii) A scientist has discovered a radioactive substance that disintegrates in such a way that at time t, the rate of disintegration is propotional to the square of the amount present. If a 100g sample of the substance dwindles to only 80 g in 1 day, how much will be left after 6 days? When will only 10 g be left?

(5 marks)

4. (a) The standard equation for a straight line in a plane is ax + by + c = 0. Two points determine a straight line. Find the equation of the straight line that passes through two points (1,2) and (5,7).
[Do not use the method you learned in geometry coordinate. Use the fact that homogeneous system has the solution $\bar{x} = \bar{0}$ if and only if $|A| = 0$]

(10 marks)

(b) A company has factories in Ipoh and Seremban which produces computer desks and printer desks. The productions in units for the month of February and January is given in matrix J and F respesctively

$$J = \begin{array}{c} \begin{array}{cc} \textit{Ipoh} & \textit{Seremban} \end{array} \\ \begin{bmatrix} 1500 & 1650 \\ 850 & 700 \end{bmatrix} \end{array}$$

$$F = \begin{array}{c} \begin{array}{cc} \textit{Ipoh} & \textit{Seremban} \end{array} \\ \begin{bmatrix} 1700 & 1810 \\ 930 & 740 \end{bmatrix} \end{array}$$

(i) Find the average production for the month of January and February

(3 marks)

(ii) Determine the increase in production from January to February

(3 marks)

(iii) Determine J = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and give an explanation for the matrix produced.

(4 marks)

1. (a) (i) *Kirakan had bagi fungsi* $\lim_{x \to 1} \dfrac{\frac{1}{x} - 1}{\sqrt{x} - 1}$

(2 markah)

(ii) *Guna* $\lim_{\Delta x \to 0} \dfrac{f(x + \Delta x) - f(x)}{\Delta x}$ *untuk mengira fungsi f(x) = ln x*

(Petunjuk: $\lim_{h \to 0} (1 + \frac{1}{h})^h = \infty$ *dan biar* $\frac{1}{h} = \frac{\Delta x}{x}$*)*

(3 markah)

(b) (i) *Cari pemalar A dan B supaya fungsi f(x) adalah selanjar bagi semua x:*

$$f(x) = \begin{cases} \dfrac{x^2 - Ax - 6}{x - 2} & \text{if } x > 2 \\ x^2 + B & \text{if } x \le 2 \end{cases}$$

(5 markah)

(ii) *Biar* $f(x) = -x^4 + x^2 + A$ *bagi pemalar A. Apakah nilai A yang sepatut dipilh untuk memastikan jika* $x_0 = \frac{1}{3}$ *dipilih sebagai anggaran awal, kaedah Newton-Raphson akan menghasilkan* $x_1 = -x_0$, $x_2 = x_0$, $x_3 = -x_0, \ldots$

(5 markah)

(c) (i) *Cari persamaan piawai bagi garis tangen pada lengkung* $y = \frac{\sin x}{x}$ *pada titik* $x = \frac{\pi}{4}$.

(2 markah)

(ii) *Tunjukkan bahawa persamaan pembezaan* $\frac{dy}{dx} = x^2 y^2 \sqrt{4 - x^3}$ *mempunyai penyelesaian* $y = \dfrac{9}{2(4 - x^3)^{\frac{3}{2}} + c}$

(3 markah)

2 (a) *Sebuah baldi yang bocor mengandungi 5 liter air. Selepas t saat, air yang tinggal didalam baldi ialah* $Q(t) = 5\left(1 - \frac{t}{25}\right)^2$

(i) *Pada kadar berapakah (peperatus liter yang hampir), air keluar daripada baldi selepa 2 saat?*

(3 markah)

(ii) *Berapa lamakah yang diambil untuk semua air itu keluar daripada baldi?*

(2 markah)

(iii) *Apakah kadar air yang keluar daripada baldi apabila titisan terakhir*
mengalir keluar?

(3 markah)

(b) *Biar f ialah fungsi dimana* $f'(x) = \frac{1}{x^2 + 1}$

(i) *Jika g(x) = f(3x-1), cari* $g'(x)$

(2 markah)

(ii) *Jika h(x) =* $f\left(\frac{1}{x}\right)$, *cari* $h'(x)$?

(2 markah)

(c) *Dapatkan* $\frac{dy}{dx}$ *jika* $y = \sqrt{x}\sin^{-1}(3x + 2)$

(2 markah)

2. (d) *Suatu zarah bergerak diatas paksi x dimana halaju pada sebarang masa* $t \geq 0$ *diberi oleh* $v(t) = 6t^2 - 2t - 4$. *Diketahui bahawa kedudukan zarah ialah pada* $x = 6$ *apabila* $t = 2$

(i) *Tuliskan pernyataan polinimial bagi kedudukan zarah pada sebarang masa* $t \geq 0$

(2 markah)

(ii) *Bagi nilai* t, $0 \leq t \leq 3$, *tentukan sama ada halaju seketika zarah sama dengan halaju purata dalam selang* $[0,3]$

(2 markah)

(iii) *Cari jumlah jarak yang dilalui zarah dari* $t = 0$ *ke* $t = 3$.

(2 markah)

3. (a) (i) *Tentukan* $\int \frac{\sin x}{\sqrt{\cos 2x}} dx$

(3 markah)

(ii) *Tentukan* $\int \frac{\sqrt{9x^2 - 1}}{x} dx$

(4 markah)

(b) (i) *Cari isipadu yang dibatasi oleh lengkung* $y = \frac{1}{1+x^4}$ *yang terjana melalui putaran pada paksi y antara* $x = 0$ *dan* $x = 4$

(5 markah)

(ii) *Cari luas permukaan yang dibatasi oleh lengkung* $y = e^x + \frac{1}{4}e^{-x}$ *yang dijana oleh putaran pada* $[0,1]$

(3 markah)

(iii) *Seorang saintis telah menemui satu bahan radioaktif yang mula reput sehingga pada satu masa t, kadar pereputan berkadar langsung dengan kuasa dua jumlah yang sedia ada.Sekiranya 100g sampel bahan radioaktif mereput kepada 80g dalam masa satu hari,berapakah jumlah yang tinggal selepas 6 hari? Bilakah hanya 10g yang tinggal?*

(5 markah)

4. (a) *Persamaan am bagi suatu garis lurus di dalam satah ialah $ax + bx + c = 0$. Dua titik menentukan persamaan garis lurus. Cari persamaan garis lurus yang melalui titik-titik (1,2) dan (5.7).*
[Jangan gunakan kaedah geometri koordinat. Guna fakta bahawa suatu sistem homogen mempunyai penyelesaian $\bar{x} = \bar{0}$ jika dan hanya jika $|A| = 0$]

(10 markah)

(b) *Sebuah syarikat mempunyai kilang di Ipoh dan Seremban yang menghasilkan meja computer dan meja pencetak. Pengeluaran (dalam unit) pada bulan Januari dan Februari masing-masing diberikan di dalam matriks J dan F berikut:*

$$J = \begin{bmatrix} Ipoh & Seremban \\ 1500 & 1650 \\ 850 & 700 \end{bmatrix}$$

$$F = \begin{bmatrix} Ipoh & Seremban \\ 1700 & 1810 \\ 930 & 740 \end{bmatrix}$$

(i) *Dapatkan purata pengeluaran pada bulan Januari dan Februari*

(3 markah)

(ii) *Tentukan peningkatan pengeluaran daripada bulan Januari ke Februari*

(3 markah)

(iii) *Tentukan $J = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ dan beri makna matriks yang dihasilkan.*

(4 markah)

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4. (a) *Guna siri binomial untuk mendapatkan kembangan siri kuasa bagi* $\dfrac{1}{\sqrt{(1-x^2)}}$

(10 markah)

(b) *Guna pecahan separa untuk mencari siri Maclaurin bagi fungsi*

$$f(x) = \frac{2x-3}{x^2-3x+2}$$

(10 markah)

5. (a) *Guna Ujian Nisbah untuk menguji siri* $\sum_{k=1}^{\infty} \dfrac{k^k}{k!}$ *bagi penumpuan.*

(Petunjuk: $\lim_{k\to\infty}\left(1+\dfrac{1}{k}\right)^k = e$ *).*

(6 markah)

(b) *Terangkan siri -p? Dapatkan nilai p dan nyatakan samada siri-p bagi* $\sum_{k=1}^{\infty} \dfrac{1}{2k\sqrt{k}}$ *menumpu atau mencapah.*

(6 markah)

(c) *Suatu fungsi pengeluaran Cobb-Douglas ialah suatu fungsi output dalam bentuk* $Q(x,y) = cx^{\alpha}y^{\beta}$ *dengan* $\alpha+\beta=1$. *Tunjukkan fungsi tersebut dimaksimumkan terhadap kos tetap* $px + qy = k$ *bila* $x = \dfrac{\alpha k}{p}$ *dan* $y = \dfrac{\beta k}{q}$.

(8 markah)

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4. (a) Use the binomial series to obtain the power series expansion of $\dfrac{1}{\sqrt{(1-x^2)}}$

(10 marks)

(b) Use partial fractions to find the Maclaurin series for the function

$$f(x)=\frac{2x-3}{x^2-3x+2}$$

(10 marks)

5. (a) Use the Ratio Test to test the series $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ for convergence.

(Hint: $\lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k=e$).

(6 marks)

(b) What is a p-series? Specify the value p and tell whether the p-series $\sum_{k=1}^{\infty}\frac{1}{2k\sqrt{k}}$ converges or diverges.

(6 marks)

(c) A Cobb-Douglas production function is an output function of the form Q(x,y) = $cx^{\alpha}y^{\beta}$ with $\alpha+\beta=1$. Show that such a function is maximized with respect to the fixed cost px + qy = k when x = $\dfrac{\alpha k}{p}$ and y = $\dfrac{\beta k}{q}$.

(8 marks)

Jawab semua soalan.

1. (a) *Jika* $f(x,y) = \sin^{-1} xy$, *tentusahkan bahawa* $f_{xy} = f_{yx}$

(10 markah)

(b) *Cari semua titik diatas sfera* $x^2 + y^2 + z^2 = 1$ *dimana satah tangen adalah selari dengan satah* $2x + y - 3z = 2$.

(10 markah)

2. (a) *Cari isipadu pepejal bagi rantau yang sama pada sfera* $x^2 + y^2 + z^2 = 4$ *dan*
silinder $r = 2\cos\theta$.

(10 markah)

(b) *Nilaikan kamiran berganda* $\int_0^{2\pi}\int_0^{a} \frac{1}{a+r}\, r\, dr\, d\theta$

(10 markah)

3. (a) *Dengan menggunakan ubahan parameter, selesaikan persamaan pembezaan* $y'' - y' = 2\cos^2 x$ *yang memenuhi syarat awal* $y(0) = 0$, $y'(0) = 1$.

(7 markah)

(b) *Satu hasil tenusu mengeluarkan susu pekat dan susu cair dalam kuantiti x dan y liter, masing-masing. Katakan harga (dalam ringgit) bagi susu pekat ialah* $p(x) = 100 - x$ *dan bagi susu cair ialah* $q(y) = 100 - y$, *dan anggapkan juga bahawa* $C(x, y) = x^2 + xy + y^2$ *ialah kos bersama bagi kedua-dua komoditi. Maksimumkan keuntungan*

$$P(x, y) = p\,x + q\,y - C(x, y)$$

(7 markah)

(c) *Guna pendarab Lagrange untuk mencari jarak minimum dari asalan ke elips* $5x^2 - 6xy + 5y^2 = 4$

(6 markah)