UNIVERSITI SAINS MALAYSIA

Peperiksaan Akhir
Sidang Akademik 2008/2009

April 2009

JIM 417 – Persamaan Pembezaan Separa

Masa: 3 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan.

Jawab SEMUA soalan.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

1. (a) Selesaikan persamaan pembezaan separa peringkat pertama berikut:

$$\frac{\partial u}{\partial x}+(x+y)\frac{\partial u}{\partial y}=(x+y)u.$$

(40 markah)

(b) Dapatkan persamaan pembezaan separa peringkat kedua jika

$u=\sin(x+y)+e^{x-y}$ adalah penyelesaiannya.

(20 markah)

(c) Tentusahkan bahawa

$$u(x,t)=(A\cos\lambda x+\beta\sin\lambda x)e^{-\lambda^2 t},$$

A dan B adalah pemalar dan $\lambda>0$ menepati persamaan pembezaan separa

$$u_t=u_{xx},\quad 0<x<2,\ t>0$$

dengan syarat sempadan

$$u(0,t)=u(2,t)=0,\quad t>0$$

dan syarat awal

$$u(x,0)=2\sin 3\pi x.$$

(40 markah)

2. Fungsi berkala $f(x)$ ditakrifkan seperti berikut:

$$f(x)=\begin{cases}1, & -1<x<0\\ 3, & 0<x<1\end{cases}$$

dan $f(x)=f(x+2)$.

(a) Lakar graf bagi fungsi f pada selang $-3<x<3$.

(10 markah)

(b) Dapatkan koefisien Fourier bagi fungsi f.

(60 markah)

(c) Nyatakan siri Fourier bagi fungsi f.

(10 markah)

(d) Dengan menggunakan (c), buktikan bahawa $\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$

(20 markah)

...3/-

3. Dengan menggunakan jelmaan Laplace, selesaikan masalah nilai awal-sempadan berikut:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - 2\sin \pi x, \quad 0 < x < 1, \ t > 0,$$

$$u(x,0) = 0, \qquad \frac{\partial u}{\partial t}(x,0) = 0,$$

$$u(0,t) = 0, \qquad u(1,t) = 0.$$

(100 markah)

4. Diberi persamaan pembezaan separa berikut:

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial y^2} = y^2 e^y.$$

(a) Tentukan jenis persamaan di atas.

(10 markah)

(b) Dapatkan persamaan cirian.

(15 markah)

(c) Dengan menggunakan transformasi

$$\xi = \frac{y}{x}, \eta = y,$$

dapatkan bentuk berkanun.

(40 markah)

(d) Cari penyelesaian am persamaan pembezaan separa tersebut.

(35 markah)

5. Dengan menggunakan kaedah pemisahan pembolehubah, selesaikan persamaan pembezaan separa berikut:

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$$

$$u(0,t) = u(\pi,t) = 0, \ t > 0$$

$$u(x,0) = x(\pi - x), \quad 0 < x < \pi.$$

(100 markah)

Senarai Rumus

$$u_x = u_\xi \xi_x + u_\eta \eta_x$$

$$u_y = u_\xi \xi_y + u_\eta \eta_y$$

$$u_{xx} = u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi \xi_{xx} + u_\eta \eta_{xx}$$

$$u_{xy} = u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}\left(\xi_x\eta_y + \xi_y\eta_x\right) + u_{\eta\eta}\eta_x\eta_y + u_\xi \xi_{xy} + u_\eta \eta_{xy}$$

$$u_{yy} = u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_y\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi \xi_{yy} + u_\eta \eta_{yy}.$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

dengan

$$a_o = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x) \operatorname{kos}\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$$

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \operatorname{kos}\left(\frac{n\pi x}{L}\right)$$

dengan $a_n = \frac{2}{L}\int_{0}^{L} f(x) \operatorname{kos}\left(\frac{n\pi x}{L}\right)dx,\ n = 1, 2, 3, \ldots$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

dengan

$$b_n = \frac{2}{L}\int_0^L f(x)\left(\frac{n\pi x}{L}\right)dx,\ \ n = 1, 2, 3, \ldots$$

$$f(x) = \frac{1}{2}\sum_{-\infty}^{\infty} c_n e^{inx}$$

dengan

$$c_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, e^{-inx}\, dx,\ \ n = 0, \pm 1, \pm 2, \ldots .$$

$\frac{d^2y}{dx^2} - \alpha^2 y = 0$ mempunyai penyelesaian

$$y = Ae^{\alpha x} + Be^{-\alpha x}.$$

$\frac{d^2y}{dx^2} + \alpha^2 y = 0$ mempunyai penyelesaian

$$y = A \text{ kosh } \alpha x + B \sinh \alpha x.$$

$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} - n^2R = 0$ mempunyai penyelesaian

$$R_n = C_n r^n + \frac{D_n}{r^n}.$$

$r\frac{d^2R}{dr^2} + r\frac{dR}{dr} = 0$ mempunyai penyelesaian

$$R = A + B \ln r.$$

$$\mathfrak{F}\left[f(t)\right] = F(\alpha) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\, e^{i\alpha t} dt$$

...6/-

$$f(x) = \mathfrak{F}^{-1}[F(\alpha)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\alpha) e^{-i\alpha x}\, dx$$

$$\mathfrak{F}[f(x)] = F_s(n) = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx,\ n = 1, 2, \ldots$$

$$f(x) = \mathfrak{F}^{-1}[F_s(n)] = \sum_{n=1}^{\infty} F_s(n) \sin \frac{n\pi x}{L}$$

$$\mathfrak{F}[f(x)] = F_c(n) = \frac{2}{L} \int_0^L f(x) \operatorname{kos} \frac{n\pi x}{L} dx,\quad n = 1, 2, \ldots$$

$$f(x) = \mathfrak{F}^{-1}[F_c(n)] = \frac{F_c(0)}{2} + \sum_{n=1}^{\infty} F_c(n) \operatorname{kos} \frac{n\pi x}{L}$$

$$\mathfrak{F}[f''(x)] = \frac{2n}{\pi}\left[f(0) - (-1)^n f(\pi)\right] - n^2 F_s(n)$$

$$\mathfrak{F}[f''(x)] = \frac{2}{\pi}\left[(-1)^n f'(\pi) - f'(0)\right] - n^2 F_c(n)$$

$$\mathscr{L}[f(t)] = F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$$

$$\mathscr{L}[e^{\alpha t} f(t)] = F(s-\alpha).$$

Jika $g(t) = \begin{cases} 0 &, \quad t < \alpha \\ f(t-\alpha), & \quad t > 0 \end{cases}$

maka $[f(t)] = e^{-\alpha s} F(s)$

$$\mathscr{L}[f^n(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\mathscr{L}[t f(t)] = -F'(s) = \frac{d}{ds} \mathscr{L}[f(t)]$$

$$\mathscr{L}\left[\int_0^t f(u)\, du\right] = \frac{F(s)}{s}$$

$$\mathscr{L}^{-1}[F(s)G(s)] = \int_0^t f(u) g(t-u)\, du = f * g$$

Jadual Jelmaan Laplace

f(t)	$\mathscr{L}\{f(t)\} = F(s)$
1	$\dfrac{1}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
kos at	$\dfrac{s}{s^2+a^2}$
sin at	$\dfrac{a}{s^2+a^2}$
kosh at	$\dfrac{s}{s^2-a^2}$
sinh at	$\dfrac{a}{s^2-a^2}$
t kos bt	$\dfrac{s^2-a^2}{\left(s^2+b^2\right)^2}$
t sin bt	$\dfrac{2bs}{\left(s^2+b^2\right)^2}$
e^{at} kos bt	$\dfrac{s-a}{(s-a)^2+b^2}$
e^{at} sin bt	$\dfrac{b}{(s-a)^2+b^2}$

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