UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2008/2009

April 2009

JIM 311 – Vector Analysis
[Analisis Vektor]

Duration : 3 hours
[Masa: 3 jam]

Please ensure that this examination paper contains SEVEN printed pages before you begin the examination.

Answer ALL questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.]

1. (a) If $\underline{a} \times \underline{b} = \underline{a} \times \underline{c}$, what can you conclude about the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$?

(30 marks)

(b) If $\underline{u} + \underline{v} + \underline{w} = \underline{0}$, show that

$$\underline{u}.\underline{v} = \frac{1}{2}\left(|\underline{w}|^2 - |\underline{u}|^2 + |\underline{v}|^2\right).$$

(35 marks)

(c) Find the value α that makes vectors

$\underline{a} = \underline{i} + \underline{j} - \underline{k}$, $\underline{b} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{c} = \alpha\underline{i} - \underline{j} + \alpha\underline{k}$

coplanar.

(35 marks)

2. The four distinct points A, B, C and D are non-collinear and such that $\overrightarrow{AB} = \underline{u}$, $\overrightarrow{BC} = \underline{v}$ and $\overrightarrow{DA} = \underline{w}$. Suppose that

$\underline{u} = \underline{i} - \underline{j} + \underline{k}$, $\underline{v} = \underline{i} - 3\underline{j} - \underline{k}$ and $\underline{w} = 3\underline{i} - 2\underline{j} + 4\underline{k}$.

(a) Using the vectors $\underline{u}$ and $\underline{v}$, show that a unit vector normal to the plane containing the points A, B and C is given by

$$\hat{\underline{n}} = \frac{2\underline{i} + \underline{j} - \underline{k}}{\sqrt{6}}.$$

(20 marks)

(b) Show by explicitly calculating the scalar products that

$\hat{\underline{n}} \cdot \underline{u} = 0$ and $\hat{\underline{n}} \cdot \underline{v} = 0$.

(20 marks)

(c) Show that A, B, C and D lie in the same plane.

(20 marks)

...3/-

(d) Suppose that A is the point (2, 3, 1). What is the Cartesian equation of the plane through A, B, C and D?

(20 marks)

(e) Find where the straight line

$$\underline{r} = \underline{i} - 2\underline{j} + 4\underline{k} + \lambda\left(\underline{i} - 3\underline{j} + 4\underline{k}\right)$$

intersects the plane through A, B, C and D.

(20 marks)

3. (a) Compute the directional derivative at position $(2,1,-1)$ of the scalar field

$$\phi = x^2 yz^3$$

in the direction $\underline{a} = \underline{i} + \underline{j} + \underline{k}$.

In which direction is the directional derivative at $(2,1,-1)$ a maximum?

(40 marks)

(b) Find an equation of the tangent plane to the parametric surface

$$x = u + v, \quad y = 3u^2, \quad z = u - v$$

at the point $(2,3,0)$.

(60 marks)

4. (a) Let $\underline{F} = xy\underline{i} + yz\underline{j} + zy\underline{k}$ and $\phi = xyz^2$. Compute

(i) $\underline{\nabla}\phi$
(ii) $\underline{\nabla} \times \underline{F}$.

Hence verify the identities

$curl\, grad\, \phi = \underline{0}$ and $div\, curl\, \underline{F} = 0$.

(40 marks)

(b) A vector field is given by

$$\underline{F} = (y^2 + 2xz^2 - 1)\underline{i} + 2xy\underline{j} + (2x^2z + z^3)\underline{k}.$$

(i) Show that $\underline{F}$ is a conservative field.

(ii) Find a potential function ϕ such that $\underline{F} = \underline{\nabla}\phi$.

(iii) Use the potential function to evaluate the line integral

$$\int_C \underline{F}.d\underline{r},$$

where C is a curve from point $(0,-1,2)$ to $(1,2,4)$.

(60 marks)

5. (a) State Green's Theorem.

Let C be the circle of radius 1 centered at the origin and oriented counterclockwise. Use Green's Theorem to evaluate the following integral

$$\oint_C \left(e^y - y^3\right)dx + \left(xe^y + x^3\right)dy.$$

(40 marks)

(b) State Stokes' Theorem.

Let

$$\underline{F} = (z-y)\underline{i} + (x+z)\underline{j} - (x+y)\underline{k}$$

be a vector field over the surface S of the paraboloid

$$z = 4 - x^2 - y^2, \quad 0 \le z \le 4.$$

Using Stokes' Theorem, compute the flux

$$\iint_S (\underline{\nabla} \times \underline{F}).d\underline{S}$$

over the surface S.

(60 marks)

1. (a) Jika $\underline{a}\times\underline{b}=\underline{a}\times\underline{c}$, apa yang dapat anda simpulkan tentang vektor $\underline{a}$, $\underline{b}$ dan $\underline{c}$?

(30 markah)

(b) Jika $\underline{u}+\underline{v}+\underline{w}=\underline{0}$, tunjukkan bahawa

$$\underline{u}.\underline{v}=\frac{1}{2}\left(|\underline{w}|^2-|\underline{u}|^2+|\underline{v}|^2\right).$$

(35 markah)

(c) Cari nilai α supaya vektor

$\underline{a}=\underline{i}+\underline{j}-\underline{k}$, $\underline{b}=2\underline{i}-\underline{j}+\underline{k}$ dan $\underline{c}=\alpha\underline{i}-\underline{j}+\alpha\underline{k}$

adalah sesatah.

(35 markah)

2. Empat titik berbeza A, B, C dan D adalah tidak segaris dan $\overrightarrow{AB}=\underline{u}$, $\overrightarrow{BC}=\underline{v}$ dan $\overrightarrow{DA}=\underline{w}$. Andaikan

$\underline{u}=\underline{i}-\underline{j}+\underline{k}$, $\underline{v}=\underline{i}-3\underline{j}-\underline{k}$ dan $\underline{w}=3\underline{i}-2\underline{j}+4\underline{k}$.

(a) Dengan menggunakan vektor $\underline{u}$ dan $\underline{v}$, tunjukkan bahawa normal vektor unit kepada satah yang mengandungi titik A, B dan C diberi oleh

$$\hat{\underline{n}}=\frac{2\underline{i}+\underline{j}-\underline{k}}{\sqrt{6}}.$$

(20 markah)

(b) Tunjukkan secara pengiraan hasildarab skalar bahawa

$\hat{\underline{n}}\cdot\underline{u}=0$ dan $\hat{\underline{n}}\cdot\underline{v}=0$.

(20 markah)

(c) Tunjukkan bahawa A, B, C dan D terletak dalam satah yang sama.

(20 markah)

...6/-

(d) Andaikan A adalah titik (2, 3, 1). Apakah persamaan Cartesian bagi satah yang melalui A, B, C dan D?

(20 markah)

(e) Cari di mana garis lurus

$$\underline{r} = \underline{i} - 2\underline{j} + 4\underline{k} + \lambda\left(\underline{i} - 3\underline{j} + 4\underline{k}\right)$$

bersilang dengan satah yang melalui A, B, C dan D.

(20 markah)

3. (a) Kirakan terbitan berarah di titik $(2,1,-1)$ bagi medan skalar

$$\phi = x^2 yz^3$$

dalam arah $\underline{a} = \underline{i} + \underline{j} + \underline{k}$.

Dalam arah manakah terbitan berarah di $(2,1,-1)$ memberi maksimum?

(40 markah)

(b) Cari persamaan satah tangen kepada permukaan parametrik

$$x = u + v, \quad y = 3u^2, \quad z = u - v$$

di titik $(2,3,0)$.

(60 markah)

4. (a) Katakan $\underline{F} = xy\underline{i} + yz\underline{j} + zy\underline{k}$ dan $\phi = xyz^2$. Kirakan

(i) $\underline{\nabla}\phi$

(ii) $\underline{\nabla} \times \underline{F}$.

Seterusnya tentusahkan identiti

$curl\ grad\ \varphi = \underline{0}$ dan $div\ curl\ \underline{F} = 0$.

(40 markah)

(b) Suatu medan vektor diberi oleh

$$\underline{F} = (y^2 + 2xz^2 - 1)\underline{i} + 2xy\underline{j} + (2x^2z + z^3)\underline{k}.$$

(i) Tunjukkan bahawa $\underline{F}$ adalah medan abadi.

(ii) Cari fungsi keupayaan ϕ supaya $\underline{F} = \underline{\nabla}\phi$.

(iii) Gunakan fungsi keupayaan untuk menilai kamiran garis

$$\int_C \underline{F}.d\underline{r},$$

di mana C adalah lengkung dari titik $(0,-1,2)$ ke $(1,2,4)$.

(60 markah)

5. (a) Nyatakan Teorem Green.

Katakan C adalah bulatan berjejari 1 yang berpusat di asalan dan diorientasikan mengikut lawan arah jam. Guna Teorem Green untuk menilai kamiran berikut :

$$\oint_C \left(e^y - y^3\right)dx + \left(xe^y + x^3\right)dy.$$

(40 markah)

(b) Nyatakan Teorem Stokes.

Katakan

$$\underline{F} = (z-y)\underline{i} + (x+z)\underline{j} - (x+y)\underline{k}$$

adalah medan vektor ke atas permukaan paraboloid S

$$z = 4 - x^2 - y^2, \quad 0 \le z \le 4.$$

Dengan menggunakan Teorem Stokes, kirakan fluks

$$\iint_S (\underline{\nabla} \times \underline{F}).d\underline{S}$$

ke atas permukaan S.

(60 markah)

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