UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2008/2009

April 2009

JIM 215 – Introduction to Numerical Analysis
[Pengantar Analisis Berangka]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains TWELVE printed pages before you begin the examination.

Answer ALL questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.]

1. (a) Evaluate the polynomial $2.75x^3 - 2.95x^2 + 3.16x - 4.67$ for $x = 1.07$, using both three-digit rounding arithmetic and three-digit chopping arithmetic. Hence, calculate the absolute error and relative error in each case.

(40 marks)

(b) Use graphical method to obtain an approximation of the root of the equation $2 + 3x + \sin x = 0$ correct to one decimal place.

(30 marks)

(c) Use Newton's method with $x_0 = -0.5$ to determine the root (correct to 6 decimal places) of the equation $e^x - 3x^2 = 0$.

(30 marks)

2. (a) Based on the table below:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0.125	0.791 68				
		− 0.018 34			
0.250	0.773 34		− 0.011 29		
		− 0.029 63		0.001 34	
0.375	0.743 71		− 0.009 95		0.000 38
		− 0.039 58		0.001 72	
0.500	0.704 13		− 0.008 23		0.000 28
		− 0.047 81		0.002 00	
0.625	0.656 32		− 0.006 23		
		− 0.054 04			
0.750	0.602 28				

Evaluate $f(0.158)$ and $f(0.636)$ by using Newton-Gregory interpolation formula.

(40 marks)

(b) For the four points (1, 0), (– 2, 15), (– 1, 0) and (2, 9), use the Lagrange interpolating polynomial to construct a polynomial equation of degree three, $P_3(x)$, of a curve which passes through them.
Hence, find the y-intercept of the curve.

(30 marks)

(c) Based on the table below:

x	0.0	0.3	0.6	0.9	1.2	1.5
$f(x)$	0.003	0.067	0.148	0.248	0.370	0.518

Use forward-difference formula, central-difference formula and backward-difference formula to evaluate $f'(0.6)$.

(30 marks)

3. (a) Given the following initial-value problem

$$y' = x + y \text{ and } y(0) = 1.$$

Use Taylor's method of order four to evaluate $y(0.1)$ correct to five decimal places.

(35 marks)

(b) Solve $y' = \sin x + y$, $y(0) = 2$, by the modified Euler method to approximate $y(0.1)$ correct to four decimal places.

(25 marks)

(c) Determine $y(0.2)$ correct to four decimal places by the Runge-Kutta method of order four with $h = 0.2$, given that $\frac{dy}{dx} = \frac{1}{x+y}$ and $y(0) = 2$.

(40 marks)

4. (a) Solve the system below by using Gaussian elimination.

$$\begin{pmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 10 \\ 12 \\ -1 \end{pmatrix}.$$

(40 marks)

...4/-

(b) Use LU decomposition to solve the matrix equation below:

$$\begin{pmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 10 \\ 12 \\ -1 \end{pmatrix}.$$

(40 marks)

(c) If $A = \begin{pmatrix} 5 & -1 & -7 \\ -3 & 2 & 0 \\ -7 & -4 & 5 \end{pmatrix}$, determine $\|A\|_1$ and $\|A\|_\infty$.

(20 marks)

5. (a) Solve the following system by using Jacobi method and Gauss-Seidel method correct to 3 decimal places.

$$\begin{aligned} 8x + y - z &= 8 \\ 2x + y + 9z &= 12 \\ x - 7y + 2z &= -4. \end{aligned}$$

(50 marks)

(b) Use the *Geršgorin* Circle Theorem to determine bounds for the eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 3 & 2 \end{pmatrix}$.

(20 marks)

(c) Use power method with $X^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ to find the dominant eigenvalue and its corresponding eigenvector of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ correct to three decimal places.

(30 marks)

...5/-

[APPENDIX]

Formulae

1. $e = x - \bar{x}$

2. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

3. $q = \dfrac{x - x_0}{h}$

4. $P_n(x) = f_0 + q\Delta f_0 + \dfrac{q(q-1)}{2!}\Delta^2 f_0 + \ldots + \dfrac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n f_0$

5. $P_n(x) = f_0 + q\Delta f_{-1} + \dfrac{q(q+1)}{2!}\Delta^2 f_{-2} + \ldots + \dfrac{q(q+1)\ldots(q+n-1)}{n!}\Delta^n f_{-n}$

6. $P_n(x) = \sum_{j=0}^{n} f(x_j) \prod_{\substack{i=0 \\ i \neq j}}^{n} \dfrac{(x - x_i)}{(x_j - x_i)}$

7. $f'(x_0) = \dfrac{f(x_0 + h) - f(x_0)}{h}$

8. $f'(x_0) = \dfrac{f(x_0) - f(x_0 - h)}{h}$

9. $f'(x_0) = \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$

10. $y(x) = y(x_0 + h) = y(x_0) + y'(x_0)h + \dfrac{y''(x_0)}{2!}h^2 + \ldots$

11. Predictor : $y_{n+1}^{(1)} = y_n + hy_n'$

    Corrector : $y_{n+1}^{(2)} = y_n + \dfrac{h}{2}(y_n' + y_{n+1}^{(1)}{}')$

12. $y_{i+1} = y_i + \dfrac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

    with $k_1 = f(x_i,\ y_i)$

    $k_2 = f(x_i + \dfrac{h}{2},\ y_i + \dfrac{h}{2}k_1)$

    $k_3 = f(x_i + \dfrac{h}{2},\ y_i + \dfrac{h}{2}k_2)$

    $k_4 = f(x_i + h,\ y_i + hk_3)$

    and $y' = f(x, y)$

13. $\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} = A$

    with $LU = A$, $Ly = b$ and $Ux = y$

14. $\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} |a_{ij}|$

15. $\|A\|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{n} |a_{ij}|$

16. $R_i = \left\{ z \in C \,\middle|\, |z - a_{ii}| \le \sum_{\substack{j=1 \\ j \ne i}}^{n} |a_{ii}| \right\}$

...7/-

1. (a) Cari nilai polinomial $2.75x^3 - 2.95x^2 + 3.16x - 4.67$ untuk $x = 1.07$, dengan menggunakan kedua-dua pembundaran aritmetik 3-digit dan pemangkasan aritmetik 3-digit.
Dengan ini, dapatkan ralat mutlak dan ralat relatif untuk kedua-dua kes itu.

(40 markah)

(b) Gunakan kaedah graf untuk mencari anggaran punca persamaan $2 + 3x + \sin x = 0$ tepat kepada satu tempat perpuluhan.

(30 markah)

(c) Guna kaedah Newton dengan $x_0 = -0.5$ untuk menentukan punca (tepat kepada 6 tempat perpuluhan) bagi persamaan $e^x - 3x^2 = 0$.

(30 markah)

2. (a) Berdasarkan jadual di bawah:

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0.125	0.791 68				
		- 0.018 34			
0.250	0.773 34		- 0.011 29		
		- 0.029 63		0.001 34	
0.375	0.743 71		- 0.009 95		0.000 38
		- 0.039 58		0.001 72	
0.500	0.704 13		- 0.008 23		0.000 28
		- 0.047 81		0.002 00	
0.625	0.656 32		- 0.006 23		
		- 0.054 04			
0.750	0.602 28				

Nilaikan $f(0.158)$ dan $f(0.636)$ dengan menggunakan rumus interpolasi Newton-Gregory.

(40 markah)

(b) Bagi keempat-empat titik ini (1, 0), (– 2, 15), (– 1, 0) dan (2, 9), gunakan polynomial interpolasi Lagrange untuk membina satu persamaan polynomial peringkat tiga, $P_3(x)$, bagi suatu lengkung yang melaluinya. Dengan ini, carikan pintasan-y untuk lengkung ini.

(30 markah)

(c) Berdasarkan jadual berikut:

x	0.0	0.3	0.6	0.9	1.2	1.5
$f(x)$	0.003	0.067	0.148	0.248	0.370	0.518

Gunakan rumus beza ke depan, rumus beza pusat dan rumus beza ke belakang untuk mencari nilai $f'(0.6)$.

(30 markah)

3. (a) Diberi masalah nilai awal berikut

$$y' = x + y \text{ dan } y(0) = 1.$$

Gunakan kaedah Taylor peringkat empat untuk menilai $y(0.1)$ tepat kepada lima tempat perpuluhan.

(35 markah)

(b) Selesaikan $y' = \sin x + y$, $y(0) = 2$, dengan menggunakan kaedah Euler terubahsuai untuk menganggar $y(0.1)$ tepat kepada empat tempat perpuluhan.

(25 markah)

(c) Tentukan $y(0.2)$ tepat kepada empat tempat perpuluhan dengan menggunakan kaedah Runge-Kutta peringkat empat dan $h = 0.2$, jika $\frac{dy}{dx} = \frac{1}{x+y}$ dan $y(0) = 2$.

(40 markah)

4. (a) Selesaikan sistem berikut dengan menggunakan kaedah penghapusan Gauss.

$$\begin{pmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 10 \\ 12 \\ -1 \end{pmatrix}.$$

(40 markah)

(b) Gunakan penghuraian LU untuk menyelesaikan persamaan matriks berikut:

$$\begin{pmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{pmatrix} X = \begin{pmatrix} 10 \\ 12 \\ -1 \end{pmatrix}.$$

(40 markah)

(c) Jika $A = \begin{pmatrix} 5 & -1 & -7 \\ -3 & 2 & 0 \\ -7 & -4 & 5 \end{pmatrix}$, tentukan $\|A\|_1$ dan $\|A\|_\infty$.

(20 markah)

5. (a) Selesaikan sistem persamaan berikut dengan menggunakan kaedah Jacobi dan kaedah Gauss-Seidel tepat kepada 3 tempat perpuluhan.

$$8x + y - z = 8$$
$$2x + y + 9z = 12$$
$$x - 7y + 2z = -4.$$

(50 markah)

(b) Gunakan teorem bulatan *Geršgorin* untuk menentukan sempadan bagi nilai-nilai eigen bagi matriks $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 3 & 2 \end{pmatrix}$.

(20 markah)

(c) Gunakan kaedah kuasa $X^{(0)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ untuk mencari nilai eigen yang paling dominan dan vektor eigen yang sepadan bagi matriks $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$ tepat kepada tiga tempat perpuluhan.

(30 markah)

[LAMPIRAN]

Rumus-Rumus

1. $e = x - \bar{x}$

2. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

3. $q = \dfrac{x - x_0}{h}$

4. $P_n(x) = f_0 + q\Delta f_0 + \dfrac{q(q-1)}{2!}\Delta^2 f_0 + \ldots + \dfrac{q(q-1)\ldots(q-n+1)}{n!}\Delta^n f_0$

5. $P_n(x) = f_0 + q\Delta f_{-1} + \dfrac{q(q+1)}{2!}\Delta^2 f_{-2} + \ldots + \dfrac{q(q+1)\ldots(q+n-1)}{n!}\Delta^n f_{-n}$

6. $P_n(x) = \sum_{j=0}^{n} f(x_j) \prod_{\substack{i=0 \\ i \neq j}}^{n} \dfrac{(x - x_i)}{(x_j - x_i)}$

7. $f'(x_0) = \dfrac{f(x_0 + h) - f(x_0)}{h}$

8. $f'(x_0) = \dfrac{f(x_0) - f(x_0 - h)}{h}$

9. $f'(x_0) = \dfrac{f(x_0 + h) - f(x_0 - h)}{2h}$

10. $y(x) = y(x_0 + h) = y(x_0) + y'(x_0)h + \dfrac{y''(x_0)}{2!}h^2 + \ldots$

11. Peramal : $y_{n+1}^{(1)} = y_n + hy_n'$

    Pembetul : $y_{n+1}^{(2)} = y_n + \dfrac{h}{2}(y_n' + y_{n+1}^{(1)}{}')$

12. $y_{i+1} = y_i + \dfrac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

    dengan $k_1 = f(x_i,\ y_i)$

    $k_2 = f(x_i + \dfrac{h}{2},\ y_i + \dfrac{h}{2}k_1)$

    $k_3 = f(x_i + \dfrac{h}{2},\ y_i + \dfrac{h}{2}k_2)$

    $k_4 = f(x_i + h,\ y_i + hk_3)$

    dan $y' = f(x, y)$

13. $\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} = A$

    dengan $LU = A$, $Ly = b$ dan $Ux = y$

14. $\| A \|_1 = \max_{1 \le j \le n} \sum_{i=1}^{n} | a_{ij} |$

15. $\| A \|_\infty = \max_{1 \le i \le n} \sum_{j=1}^{n} | a_{ij} |$

16. $R_i = \left\{ z \in C \,\middle|\, |z - a_{ii}| \le \sum_{\substack{j=1 \\ j \ne i}}^{n} |a_{ii}| \right\}$

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