UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2008/2009

April 2009

JIM 201 – Linear Algebra
[*Aljabar Linear*]

Duration : 3 hours
[*Masa: 3 jam*]

Please ensure that this examination paper contains NINE printed pages before you begin the examination.

Answer ALL questions. You may answer either in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question is worth 100 marks.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab SEMUA soalan. Anda dibenarkan menjawab sama ada dalam Bahasa Malaysia atau Bahasa Inggeris.

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.]

...2/-

1. (a) Show that $(kA)^{-1} = \frac{1}{k}A^{-1}$, for a nonsingular matrix A and nonzero scalar k.

(25 marks)

(b) Solve the linear system $A^T X = B$, where A is nonsingular with

$$A^{-1} = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}.$$

(25 marks)

(c) Find the reduced row echelon form of the given matrix. Show the row operations you perform, using the notation for elementary row operations.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -2 \\ -5 & 4 & -7 \\ -2 & 6 & -5 \end{bmatrix}.$$

(25 marks)

(d) Solve the linear system, with the given augmented matrix, if it is consistent.

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right].$$

(25 marks)

2. (a) In the following linear system, determine all values of a for which the linear system has

(i) no solution,
(ii) a unique solution,
(iii) infinitely many solutions.

$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1.$$

(25 marks)

...3/-

(b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix}.$$

(25 marks)

(c) Compute the following determinant

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 2 & -3 & 0 \\ 1 & 5 & 3 & 5 \end{vmatrix}.$$

(25 marks)

(d) Let $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & 1 & -4 & -1 \\ 3 & 2 & 4 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$. Find the following cofactors:

(i) A_{12}

(ii) A_{23}.

(25 marks)

3. (a) Find the row and column ranks of the given matrices

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}.$$

(25 marks)

(b) Compute the rank and nullity of given matrix and verify

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}.$$

(25 marks)

...4/-

(c) Use the Gram-Schmidt process to construct an orthonormal basis for the subspace w of the Euchidean space R_3 spanned by

$$\left\{\begin{matrix}[1 & -1 & 1], [-2 & 2 & -2], \\ [2 & -1 & 2], [0 & 0 & 0]\end{matrix}\right\}.$$

(25 marks)

(d) Find the standard matrix representing given linear transformation

$L: R^2 \to R^2$ defined by

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} -u_1 \\ u_2 \end{bmatrix}.$$

(25 marks)

4. (a) Let $L: R^2 \to R^2$ be the linear transformation defined by

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ u_2 \end{bmatrix}$$

(i) Is $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ in kernel L?

(ii) Is $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ in range L?

(25 marks)

(b) Let $L: R^2 \to R^2$ be defined by $L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ 2u_1 + u_2 \end{bmatrix}$.

Let S be the natural basis for R^2 and let

$$T = \left\{\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}\right\}.$$

Find the representation of L with respect to S and T.

(25 marks)

...5/-

(c) Find the characteristic polynomial of the following matrix:

$$\begin{bmatrix} 2 & -3 & -4 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{bmatrix}.$$

(25 marks)

(d) Find eigen value and coresponding eigen vector of the following matrix:

$$\begin{bmatrix} 4 & 2 & -4 \\ 1 & 5 & -4 \\ 0 & 0 & 6 \end{bmatrix}.$$

(25 marks)

1. (a) Tunjukkan bahawa $(kA)^{-1} = \frac{1}{k}A^{-1}$, bagi matriks tak singular A dan skalar bukan sifar k.

(25 markah)

(b) Selesaikan sistem linear $A^T X = B$, di mana A tak singular dengan

$$A^{-1} = \begin{bmatrix} 5 & 2 \\ 2 & -1 \end{bmatrix} \text{ dan } B = \begin{bmatrix} -3 \\ 2 \end{bmatrix}.$$

(25 markah)

(c) Cari bentuk eselon baris terturun bagi matriks berikut. Tunjukkan operasi baris yang anda gunakan, menggunakan tatatanda operasi baris permulaan.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -2 \\ -5 & 4 & -7 \\ -2 & 6 & -5 \end{bmatrix}.$$

(25 markah)

(d) Selesaikan sistem linear dengan matriks imbuhan berikut, jika ianya konsisten

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 2 & 0 & 1 & 4 \\ 1 & 0 & 2 & 5 \\ 1 & 2 & 3 & 11 \\ 2 & 1 & 4 & 12 \end{array}\right].$$

(25 markah)

2. (a) Dalam sistem linear berikut, tentukan semua nilai a supaya sistem linear yang mempunyai

(i) tiada penyelesaian,
(ii) penyelesaian unik,
(iii) banyak penyelesaian secara tak terhingga.

$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$2x + 3y + (a^2 - 1)z = a + 1.$$

(25 markah)

...7/-

(b) Cari songsang bagi matrik

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 3 & 2 & 0 \end{bmatrix}.$$

(25 markah)

(c) Hitung penentu berikut

$$\begin{vmatrix} 4 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 2 & -3 & 0 \\ 1 & 5 & 3 & 5 \end{vmatrix}.$$

(25 markah)

(d) Biar $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & 1 & -4 & -1 \\ 3 & 2 & 4 & 0 \\ 0 & 3 & -1 & 0 \end{bmatrix}$. Cari kofaktor berikut:

(i) A_{12}

(ii) A_{23}.

(25 markah)

3. (a) Cari pangkat bagi lajur dan baris matriks berikut:

$$\begin{bmatrix} 1 & 2 & 3 & 2 & 1 \\ 3 & 1 & -5 & -2 & 1 \\ 7 & 8 & -1 & 2 & 5 \end{bmatrix}.$$

(25 markah)

(b) Hitung dan tentusahkan pangkat dan kenolan matriks berikut:

$$\begin{bmatrix} 1 & 3 & -2 & 4 \\ -1 & 4 & -5 & 10 \\ 3 & 2 & 1 & -2 \\ 3 & -5 & 8 & -16 \end{bmatrix}.$$

(25 markah)

...8/-

(c) Gunakan proses Gram-Schmidt untuk membina asas ortonormal bagi subruang w dari ruang Euklidan dan R_3 dijangkau oleh

$$\left\{\begin{matrix}[1 & -1 & 1], [-2 & 2 & -2], \\ [2 & -1 & 2], [0 & 0 & 0]\end{matrix}\right\}.$$

(25 markah)

(d) Hitung matriks piawai yang mewakili penjelmaan linear berikut:

$L : R^2 \to R^2$ tertakrif oleh

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} -u_1 \\ u_2 \end{bmatrix}.$$

(25 markah)

4. (a) Biar $L : R^2 \to R^2$ adalah penjelmaan linear tertakrif oleh:

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ u_2 \end{bmatrix}.$$

(i) Adakah $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ dalam inti L?

(ii) Adakah $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ dalam julat L?

(25 markah)

(b) Biar $L : R^2 \to R^2$ tertakrif oleh $L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ 2u_1 + u_2 \end{bmatrix}$.

Biar S adalah asas tabii untuk R^2 dan biar

$$T = \left\{\begin{bmatrix} -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}\right\}.$$

Dapatkan perwakilan bagi L tertakluk kepada S dan T.

(25 markah)

...9/-

(c) Dapatkan polinomial cirian matrik berikut:

$$\begin{bmatrix} 2 & -3 & -4 \\ 0 & 1 & 0 \\ 0 & -3 & 2 \end{bmatrix}.$$

(25 markah)

(d) Dapatkan nilai eigen dan vektor eigen sepadan bagi matriks berikut:

$$\begin{bmatrix} 4 & 2 & -4 \\ 1 & 5 & -4 \\ 0 & 0 & 6 \end{bmatrix}.$$

(25 markah)

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