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UNIVERSITI SAINS MALAYSIA

Final Examination  
Academic Session 2008/2009

April 2009

**JIM 101 – CALCULUS**  
**[KALKULUS]**

Duration : 3 hours  
*[Masa: 3 jam]*

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Please ensure that this examination paper contains NINE printed pages before you begin the examination.

Answer ALL questions.

Read the instructions carefully before answering.

Each question is worth 100 marks.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi SEMBILAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab SEMUA soalan.*

*Baca arahan dengan teliti sebelum anda menjawab soalan.*

*Setiap soalan diperuntukkan 100 markah.*

...2/-

1. (a) Given that

$$\lim_{x \rightarrow 2} f(x) = 2 \text{ and } \lim_{x \rightarrow 2} g(x) = 27.$$

Find the limits if exist,

(i)  $\lim_{x \rightarrow 2} [f(x) \times \sqrt[3]{g(x)} + g(x)],$

(ii)  $\lim_{x \rightarrow 2} \left[ \frac{g(x)}{[f(x)]^2 - 4} \right].$

(20 marks)

(b) Solve the following inequalities:

(i)  $\frac{x+1}{x-2} < 3,$

(ii)  $-\frac{2}{x^2} + 2 \geq 0.$

(30 marks)

(c) Evaluate the limits if exist,

(i)  $\lim_{x \rightarrow \infty} \frac{x + 4x^3}{1 - x^2 + 7x^3},$

(ii)  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}.$

(30 marks)

(d) (i) Find  $\frac{dy}{dx}$  if  $y = \frac{x}{x-1}.$

(ii) Find the second derivative of  $f(x) = 4 + 2x - x^2 + \sqrt{x}.$

(20 marks)

...3/-

2. (a) Use De Moivre's Theorem to simplify the following expressions.

(i)  $(\cos 2\theta + i \sin 2\theta)(\cos 5\theta + i \sin 5\theta),$

(ii)  $\frac{\cos 7\theta + i \sin 7\theta}{\cos 2\theta - i \sin 2\theta}.$

(40 marks)

(b) Find  $\frac{dy}{dx}$  for each of the following:

(i)  $y = x(3^x - 1),$

(ii)  $xy - y^3 \ln(x) = 4x - 10,$

(iii)  $y = \sin^3(1 - 2x).$

(40 marks)

(c) Use the first principle of derivatives to find  $\frac{dy}{dx}$  for  $y = \frac{1}{x}.$

(20 marks)

3. (a) Given that

$$f(x) = \begin{cases} x + a & , \quad x < \frac{\pi}{3} \\ -\frac{1}{2} & , \quad x = \frac{\pi}{3} \\ -b + \sin x & , \quad x > \frac{\pi}{3} \end{cases}$$

is continuous at  $x = \frac{\pi}{3}$ , find  $a$  and  $b$ .

(30 marks)

...4/-

- (b) Given the function  $g(x) = x^3 - 3x + 3$ .

Determine

- (i) the maximum, minimum and inflection points,
- (ii) the intervals on which  $g$  is concave up and concave down,
- (iii) sketch the graph of  $g$ .

(50 marks)

- (c) Sketch the graph of  $r = \cos(2\theta)$ ,  $0 \leq \theta \leq 2\pi$  in polar form.

(20 marks)

4. (a) Evaluate the following integrals:

(i)  $\int \frac{6x^2 - 4x + 2}{x^3 - x^2 + x - 1} dx,$

(ii)  $\int \frac{3}{\sqrt{16 - 9x^2}} dx,$

(iii)  $\int_0^{\frac{\pi}{4}} (x+1) \sin 4x dx,$

(iv)  $\int_1^2 \frac{4}{x^2(x+2)} dx.$

(70 marks)

- (b) Find the area of the region bounded by the curve  $x = y^2$  and the line  $y = x - 2$ .

(30 marks)

...5/-

5. (a) For the parabola  $y^2 - 4y - x + 2 = 0$ , find its focus and directrix, then sketch the parabola. Label its focus and directrix.  
(40 marks)
- (b) Given the hyperbola  $16(x+1)^2 - 8(y-3)^2 = 16$ . Find its foci and asymptotes. Then sketch this hyperbola. Label its foci and asymptotes.  
(40 marks)
- (c) Find the volume of the solid generated when the region enclosed by  $y = \sqrt{x}$ ,  $y = 2$ , and  $x = 0$  is revolved about the  $y$ -axis.  
(20 marks)

1. (a) Diberi

$$\lim_{x \rightarrow 2} f(x) = 2 \text{ dan } \lim_{x \rightarrow 2} g(x) = 27.$$

Tentukan had berikut jika wujud,

(i)  $\lim_{x \rightarrow 2} [f(x) \times \sqrt[3]{g(x)} + g(x)],$

(ii)  $\lim_{x \rightarrow 2} \left[ \frac{g(x)}{[f(x)]^2 - 4} \right].$

(20 markah)

(b) Selesaikan ketaksamaan berikut:

(i)  $\frac{x+1}{x-2} < 3,$

(ii)  $-\frac{2}{x^2} + 2 \geq 0.$

(30 markah)

(c) Nilaikan had jika wujud,

(i)  $\lim_{x \rightarrow \infty} \frac{x + 4x^3}{1 - x^2 + 7x^3},$

(ii)  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}.$

(30 markah)

(d) (i) Dapatkan  $\frac{dy}{dx}$  jika  $y = \frac{x}{x-1},$

(ii) Tentukan terbitan kedua bagi  $f(x) = 4 + 2x - x^2 + \sqrt{x}.$

(20 markah)

2. (a) Ringkaskan yang berikut dengan menggunakan Teorem De Moivre:

(i)  $(\cos 2\theta + i \sin 2\theta)(\cos 5\theta + i \sin 5\theta),$

(ii)  $\frac{\cos 7\theta + i \sin 7\theta}{\cos 2\theta - i \sin 2\theta}.$

(40 markah)

(b) Cari  $\frac{dy}{dx}$  bagi setiap yang berikut:

(i)  $y = x(3^x - 1),$

(ii)  $xy - y^3 \ln(x) = 4x - 10,$

(iii)  $y = \sin^3(1 - 2x).$

(40 markah)

(c) Gunakan prinsip pertama pembezaan untuk mendapatkan  $\frac{dy}{dx}$  bagi fungsi

$$y = \frac{1}{x}.$$

(20 markah)

3. (a) Diberi

$$f(x) = \begin{cases} x + a & , \quad x < \frac{\pi}{3} \\ -\frac{1}{2} & , \quad x = \frac{\pi}{3} \\ -b + \sin x & , \quad x > \frac{\pi}{3} \end{cases}$$

adalah selanjar pada  $x = \frac{\pi}{3}$ , dapatkan nilai  $a$  dan  $b$ .

(30 markah)

...8/-

(b) Diberi fungsi  $g(x) = x^3 - 3x + 3$ .

Tentukan

- (i) titik-titik maksimum, minimum dan lengkok balas,
- (ii) selang pada mana  $g$  cekung ke atas dan cekung ke bawah,
- (iii) lakar graf  $g$ .

(50 markah)

(c) Lakar graf  $r = \cos 2\theta$ ,  $0 \leq \theta \leq 2\pi$  dalam koordinat kutub.

(20 markah)

4. (a) Cari nilai kamiran berikut:

(i)  $\int \frac{6x^2 - 4x + 2}{x^3 - x^2 + x - 1} dx,$

(ii)  $\int \frac{3}{\sqrt{16 - 9x^2}} dx,$

(iii)  $\int_0^{\frac{\pi}{4}} (x+1) \sin 4x dx,$

(iv)  $\int_1^2 \frac{4}{x^2(x+2)} dx.$

(70 markah)

(b) Dapatkan luas rantau yang dibatasi lengkung  $x = y^2$  dan garis  $y = x - 2$ .

(30 markah)

5. (a) Bagi parabola  $y^2 - 4y - x + 2 = 0$ , cari fokus dan direktriks, kemudian lakar parabola tersebut. Tandakan fokus dan direktriks.

(40 markah)

- (b) Diberi hiperbola  $16(x+1)^2 - 8(y-3)^2 = 16$ . Cari fokus dan asimptot, kemudian lakar hiperbola tersebut.

(40 markah)

- (c) Cari isipadu pepejal yang terjana apabila rantau yang dibatasi lengkung  $y = \sqrt{x}$ , garis  $y = 2$  dan  $x = 0$ , dikisarkan pada paksi-y.

(20 markah.)

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