UNIVERSITI SAINS MALAYSIA

Final Examination
Academic Session 2008/2009

April 2009

JIF 318 – Quantum Mechanics
[JIF 318 – Mekanik Kuantum]

Time : 3 hours
[Masa : 3 jam]

Please ensure that this examination paper contains **SEVEN** printed pages before you begin the examination.

Answer **ALL** questions. You may answer **either** in Bahasa Malaysia or in English.

Read the instructions carefully before answering.

Each question carries 100 marks.

*Sila pastikan bahawa kertas peperiksaan ini mengandungi **TUJUH** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.*

*Jawab **SEMUA** soalan. Anda dibenarkan menjawab soalan **sama ada** dalam Bahasa Malaysia atau Bahasa Inggeris.*

Baca arahan dengan teliti sebelum anda menjawab soalan.

Setiap soalan diperuntukkan 100 markah.

...2/-

1. (a) What is Black Body Radiation? Why did the classical physics fail to explain the properties of Black Body Radiation?

(30 marks)

(b) How did Planck overcome the problems in explaining the experimental properties of the Black Body radiation?

(30 marks)

(c) Describe two experimental evidences that indicate the wave properties of electron.

(40 marks)

2. (a) What is the characteristic of a linear operator?

(10 marks)

(b) Show that the sum and product of two linear operators are linear operators.

(10 marks)

(c) Calculate $\left(\frac{1+i}{1-i}\right)^5$ using complex number Cartesian coordinate and polar coordinate.

(20 marks)

(d) If the wave function of a wave travelling in one dimension in the positive x direction is given by

$$\Psi = A\cos(2\pi x/\lambda - 2\pi vt),$$

show that the Schrödinger equation can be given as

$$\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}.$$

(60 marks)

...3/-

3. (a) One dimensional Schrödinger equation for a infinite square well potential is given by

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Find the solution of this equation if the potential $V(x)$ is given by

$$V(x) = 0 \text{ for } 0 \le x \le a$$
$$V(x) = \infty \text{ for } x < 0 \text{ or } x > a$$

where a is the width of the square well potential.

(80 marks)

(b) Prove that the mean value for the position of the particle in the infinite square well is

$$\langle x \rangle = \frac{a}{2}$$

(20 marks)

4. The Schrödinger equation of a bound state for potential $V(x) = 0$ for $0 \le x \le a$ and $V(x) = \infty$ for $x < 0$ or $x > a$ is

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

By taking into consideration the appropriate boundary conditions, show that

(a) the wave function is given by

$$\psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

(40 markah)

...4/-

(b) the energy of the bound state E is given by

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

where $n = 1, 2, 3, ...$

(30 marks)

Plot $\psi_n(x)^* \psi_n(x)$ for $n = 1, 2, 3$. Describe the physical interpretation of these plots.

(30 marks)

5. (a) If one dimensional Hamiltonian operator H is $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ and one- dimensional momentum operator P is $-i\hbar\frac{d\psi}{dx}$, show that

$$[H, P] = i\hbar\frac{\partial V}{\partial x}$$

(40 marks)

(b) Explain the following:

(i) Vector space
(ii) Inner product
(iii) Adjoint Operator
(iv) Hermitian operator
(v) Basis set
(vi) Commutator
(vii) Orthonormal Basis

(60 marks)

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1. (a) *Apakah Sinaran Jasad Hitam? Kenapa fizik klasikal menjelaskan sifat-sifat sinaran hitam?*

(30 markah)

(b) *Bagaimana Planck mengatasi masalah dalam menjelaskan sifat-sifat ujikaji Sinaran Jasad Hitam?*

(30 markah)

(c) *Jelaskan dua bukti ujikaji yang menunjukkan sifat gelombang elektron.*

(40 markah)

2. (a) *Apakah ciri operator linear?*

(10 markah)

(b) *Tunjuk bahawa hasil campur dan hasil darab dua operator linear ialah operator linear.*

(10 markah)

(c) *Kira* $\left(\frac{1+i}{1-i}\right)^5$ *menggunakan nombor kompleks koordinat cartesian dan koordinat pola.*

(20 markah)

(d) *Jika fungsi gelombang suatu gelombang bergerak dalam satu dimensi dalam arah x positif x diberikan oleh*

$$\Psi = A\cos\left(2\pi x/\lambda - 2\pi\nu t\right),$$

tunjukkan persamaan Schrödinger boleh diberikan oleh

$$\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}.$$

(60 markah)

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3. (a) *Persamaan Schrödinger satu dimensi untuk satu telaga keupayaan segi empat tak terhingga diberikan oleh*

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Cari penyelesaian persamaan ini jika keupayaan V (x) diberikan oleh

$$V(x) = 0 \text{ for } 0 \leq x \leq a$$
$$V(x) = \infty \text{ for } x < 0 \text{ or } x > a$$

di mana a ialah kelebaran keupayaan segiempat.

(80 markah)

(b) *Buktikan nilai nin untuk kedudukan zarah dalam telaga segiempat tak terhingga ialah*

$$\langle x \rangle = \frac{a}{2}$$

(20 markah)

4. *Persamaan Schrödinger untuk keadaan terikat bagi keupayaan V(x) = 0 untuk* $0 \leq x \leq a$ *dan V(x) =* ∞ *untuk* $x < 0$ *atau* $x > a$ *ialah*

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$

Dengan mengambil kira syarat-syarat batasan yang sesuai, tunjukkan bahawa

(a) *fungsi gelombang diberikan oleh*

$$\psi_n = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$$

(40 markah)

...7/-

(b) *Tenaga keadaan terikat E diberikan oleh*

$$E_n = \frac{\hbar^2 \pi^2}{2ma^2} n^2$$

di mana n = 1, 2, 3, ...

(30 marks)

Lakarkan $\psi_n(x)^* \psi_n(x)$ *untuk n = 1, 2, 3. Jelaskan interpretasi fizikal lakaran-lakaran ini.*

(30 markah)

5. (a) *Jika operator Hamiltonian satu dimensi H ialah* $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$ *dan operator momentum satu dimensi, P ialah* $-i\hbar\frac{d\psi}{dx}$, *tunjukkan*

$$[H, P] = i\hbar\frac{\partial V}{\partial x}$$

(40 markah)

(b) *Jelaskan perkara-perkara berikut:*

(i) *Ruang vektor*
(ii) *Hasil darab berkedalaman*
(iii) *Operator Adjoint*
(iv) *Operator Hermitian*
(v) *Set asas*
(vi) *Kommutator*
(vii) *Asas Orthonormal*

(60 markah)

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