UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1996/97

Oktober/November 1996

MSG 466 - Analisis Multivariat

Masa : [3 jam]

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA soalan di dalam 29 halaman yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **SEMUA** soalan. Sekurang-kurangnya satu soalan mesti dijawab di dalam Bahasa Malaysia.

1. (a) Pertimbangkan enam cerapan pembolehubah x_1, x_2, dan x_3.

x_1:	3	6	8	7	4	2
x_2:	12	7	11	8	13	9
x_3:	6	5	7	8	1	3

Dapatkan $\underset{\sim}{x}$, $\underset{\sim}{S}_n$ dan $\underset{\sim}{R}_n$.

(10/100)

(b) Diberikan matriks

$$\underset{\sim}{A} = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}.$$

(i) Adakah $\underset{\sim}{A}$ bersimetri?

(ii) Adakah $\underset{\sim}{A}$ tentu positif?

(iii) Dapatkan pasangan nilai eigen - vektor eigen $\underset{\sim}{A}$.

(iv) Tentukan penghuraian spektrum $\underset{\sim}{A}$.

(v) Dapatkan $\underset{\sim}{A}^{-1}$.

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(vi) Hitung pasangan nilai eigen - vektor eigen $\underset{\sim}{A}^{-1}$.

(vii) Tentukan penghuraian spektrum $\underset{\sim}{A}^{-1}$.

(30/100)

(c) Andaikan $\mathbb{X}' = \left[X_1, \cdots, X_p\right]$ dan $\mathbb{Z}' = \left[Z_1, Z_2, \cdots, Z_q\right]$ sebagai vektor rawak. Tunjukkan bahawa jika $\mathbb{X}$ dan $\mathbb{Z}$ adalah tak bersandar, jadi setiap kumpulan $\mathbb{X}$ adalah tak bersandar dengan setiap komponen $\mathbb{Z}$.

(30/100)

(d) Diberikan matriks data

$$\mathrm{X} = \begin{bmatrix} 4 & 2 & 6 \\ 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

dan kombinasi linear

$$b'\underset{\sim}{X} = [1 \quad 2 \quad 1]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

dan

$$d'\underset{\sim}{X} = [-1 \quad 3 \quad 2]\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Hitung min-min sampel, varians-varians dan kovarians sampel $b'\mathrm{X}$ dan $d'\mathrm{X}$.

(30/100)

...3/-

1. (a) Consider six observations of variables x_1, x_2, and x_3.

x_1:	3	6	8	7	4	2
x_2:	12	7	11	8	13	9
x_3:	6	5	7	8	1	3

Find $\underset{\sim}{x}$, $\underset{\sim}{S}_n$ and $\underset{\sim}{R}_n$.

(10/100)

(b) Given the matrix

$$\underset{\sim}{A} = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}.$$

(i) Is $\underset{\sim}{A}$ symmetric?

(ii) Is $\underset{\sim}{A}$ positive definite?

(iii) Find eigenvalue-eigenvector pairs of $\underset{\sim}{A}$.

(iv) Determine the spectral decomposition of $\underset{\sim}{A}$.

(v) Find $\underset{\sim}{A}^{-1}$.

(vi) Compute the eigenvalue-eigenvector pairs of $\underset{\sim}{A}^{-1}$.

(vii) Determine the spectral decomposition of $\underset{\sim}{A}^{-1}$.

(30/100)

(c) Let $X' = [X_1, \cdots, X_p]$ and $Z' = [Z_1, Z_2, \cdots, Z_q]$ be random vectors. Show that, if X and Z are independent, then each component of X is independent of each component of Z.

(30/100)

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(d) Let the following data matrix

$$\mathbf{X} = \begin{bmatrix} 4 & 2 & 6 \\ 3 & 1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$$

and the linear combinations

$$b'\underset{\sim}{X} = [1 \quad 2 \quad 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

and

$$d'\underset{\sim}{X} = [-1 \quad 3 \quad 2] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

are given. Calculate the sample means, variances and sample covariance $b'\mathbf{X}$ dan $d'\mathbf{X}$.

(30/100)

2. (a) Andaikan $\mathbf{X}$ sebagai $N_3(\mu, \Sigma)$ dengan $\mu' = [-3, 1, 4]$ dan

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Di antara pembolehubah-pembolehubah rawak berikut, yang manakah tak bersandar? Terangkan.

(i) X_1 dan X_2.

(ii) X_2 dan X_3.

(iii) (X_1, X_2) dan X_3.

(iv) $\dfrac{X_1 + X_2}{2}$ dan X_3.

...5/-

(v) X_2 dan $X_2 - \frac{5}{2}X_1 - X_3$.

(40/100)

(b) Andaikan X sebagai $N_3(\mu, \Sigma)$ dengan $\mu' = [2, -3, 1]$ dan $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

(i) Dapatkan taburan bagi X_3.

(ii) Dapatkan taburan bagi $3X_1 - 2X_2 + X_3$.

(iii) Dapatkan taburan bagi $\begin{bmatrix} X_1 - X_2 \\ X_2 - X_3 \end{bmatrix}$.

(60/100)

2. (a) Let X be $N_3(\mu, \Sigma)$ with $\mu' = [-3, 1, 4]$ and

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Which of the following random variables are independent. Explain.

(i) X_1 and X_2.

(ii) X_2 and X_3.

(iii) (X_1, X_2) and X_3.

(iv) $\frac{X_1 + X_2}{2}$ and X_3.

(v) X_2 and $X_2 - \frac{5}{2}X_1 - X_3$.

(40/100)

...6/-

(b) Let X be $N_3(\mu, \Sigma)$ with $\mu' = [2, -3, 1]$ and $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$.

(i) Find the distribution of X_3.

(ii) Find the distribution of $3X_1 - 2X_2 + X_3$.

(iii) Find the distribution of $\begin{bmatrix} X_1 - X_2 \\ X_2 - X_3 \end{bmatrix}$.

(60/100)

3. (a) Pertimbangkan kadar pulangan tahunan (termasuk dividen) untuk Purata Perindustrian Dow Jones bagi tahun 1963-1972. Datanya, didarab dengan 100, ialah:

tahun	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
kadar	20.6	18.7	14.2	−15.7	19.0	7.7	−11.6	8.8	9.8	18.2

Guna 10 cerapan ini untuk menjawab soalan berikut.

(i) Binakan suatu plot Q–Q. Adakah data ini kelihatan tertabur secara normal? Terangkan.

(ii) Jalankan suatu ujian kenormalan berdasarkan pekali korelasi

$$r_Q = \frac{\sum_{i=1}^{n} \left(x_{(i)} - \bar{x}\right)\left(q_{(i)} - \bar{q}\right)}{\sqrt{\sum_{i=1}^{n} \left(x_{(i)} - \bar{x}\right)^2} \cdot \sqrt{\sum_{i=1}^{n} \left(q_{(i)} - \bar{q}\right)^2}}$$

Andaikan aras keertian sebagai $\alpha = .10$.
Gunakan jadual kuantiti normal piawai $q(i)$ berikut:

$\frac{i-0.5}{10}$	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
$q(i)$	−1.645	−1.036	−.674	−.385	−.125	.125	.385	.674	1.036	1.645

dan nilai $f(n, \alpha) = f(10, .10) = .935$ daripada jadual titik genting bagi plot Q–Q.

(30/100)

...7/-

(b) Suatu surat khabar menyenaraikan harga kereta terpakai jenis kecil dengan pembolehubah umur x_1, dalam tahun, dan harga jualan x_2, dalam ribuan ringgit.

x_1	3	5	5	7	7	7	8	9	10	11
x_2	2.30	1.90	1.00	.70	.30	1.00	1.05	.45	.70	.30

(i) Binakan suatu plot sebaran data dan gambarajah titik sut.

(ii) Buat kesimpulan tentang tanda kovarians sampel S_{12} daripada plot sebaran tersebut.

(iii) Hitung min sampel $\bar{x}_1, \bar{x}_2$ dan varians sampel S_{11}, S_{22}. Hitung kovarians sampel S_{12} dan pekali korelasi sampel r_{12}. Tafsirkan kuantiti-kuantiti tersebut.

(iv) Pamerkan vektor min sampel $\underset{\sim}{\bar{x}}$, matriks varians dan kovarians sampel S_n dan matriks korelasi $\underset{\sim}{R}$.

(30/100)

(c) Dengan menggunakan data dari Bahagian *b*, dengan x_1 merupakan umur dan x_2 merupakan harga jualan dalam ribuan ringgit bagi $n = 10$ kereta terpakai,

(i) Hitungkan jarak kuasa dua teritlak

$$\left(\underset{\sim}{x}_j - \underset{\sim}{\bar{x}}\right)\underset{\sim}{S}^{-1}\left(\underset{\sim}{x}_j - \underset{\sim}{\bar{x}}\right), \; j = 1, 2, \ldots, 10, \text{ dengan } \underset{\sim}{x}_j = \begin{bmatrix} x_{1j} \\ x_{2j} \end{bmatrix},$$

dengan menggunakan jawapan yang diperolehi dalam Bahagian b.

(ii) Dengan menggunakan jarak-jarak dalam Bahagian (i), tentukan kadaran cerapan yang terletak di dalam kontur kebarangkalian 50% yang dianggar bagi suatu taburan normal bivariat. Guna nilai $\chi_2^2(.5) = 1.39$ untuk titik genting X^2.

...8/-

(iii) Susun jarak-jarak dalam Bahagian (i) dan bina suatu plot khi kuasa dua. Guna jadual persentil khi kuasa dua berikut bagi $p = 2,\ n = 10$:

j	1	2	3	4	5	6	7	8	9	10
$\chi_2^2\left(\frac{j-\frac{1}{2}}{10}\right)$	.10	.33	.58	.86	1.20	1.60	2.10	2.77	3.79	5.99

(iv) Diberikan jawapan dalam Bahagian (ii) dan (iii), adakah data ini bertaburan normal bivariat secara hampiran? Terangkan.

(40/100)

3. (a) Consider the annual rates of return (including dividends) on the Dow Jones Industrial Average for the years 1963-1972. These data, multiplied by 100 are:

tahun	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
kadar	20.6	18.7	14.2	−15.7	19.0	7.7	−11.6	8.8	9.8	18.2

Use these 10 observations to answer the following.

(i) Construct a Q–Q plot. Do the data seem to be normally distributed? Explain.

(ii) Carry out a test of normality based on the correlation coefficient

$$r_Q = \frac{\sum_{i=1}^{n}\left(x_{(i)}-\bar{x}\right)\left(q_{(i)}-\bar{q}\right)}{\sqrt{\sum_{i=1}^{n}\left(x_{(i)}-\bar{x}\right)^2}\cdot\sqrt{\sum_{i=1}^{n}\left(q_{(i)}-\bar{q}\right)^2}}$$

Let the significance level be $\alpha = .10$.
Use the following table of standard normal quantities $q(i)$:

$\frac{i-0.5}{10}$	.05	.15	.25	.35	.45	.55	.65	.75	.85	.95
$q(i)$	−1.645	−1.036	−.674	−.385	−.125	.125	.385	.674	1.036	1.645

and the value $f(n, \alpha) = f(10, .10) = .935$ from the table critical points for the Q–Q plot.

(30/100)

(b) A morning newspaper lists the following used car prices for foreign compact with age x_1 measured in years and selling price x_2 measured in thousands of dollars.

x_1	3	5	5	7	7	7	8	9	10	11
x_2	2.30	1.90	1.00	.70	.30	1.00	1.05	.45	.70	.30

(i) Construct a scatterplot of the data and marginal dot diagrams.

(ii) Infer the sign of the sample covariance S_{12} from the scatterplot.

(iii) Compute the sample means $\bar{x}_1, \bar{x}_2$ and sample variances S_{11}, S_{22}. Compute the sample covariance S_{12} and the sample correlation coefficient r_{12}. Interpret these quantities.

(iv) Display the sample mean vector $\bar{\underset{\sim}{x}}$, the matrix of sample variance and covariances S_n and the matrix of correlations $\underset{\sim}{R}$.

(30/100)

(c) Using the data from Part *b*, where x_1 the age and x_2 is the selling price measured in thousands of dollars for $n = 10$ used cars.

(i) Calculate the squared generalized distance

$$\left(\underset{\sim}{x}_j - \bar{\underset{\sim}{x}}\right)' S^{-1}\left(\underset{\sim}{x}_j - \bar{\underset{\sim}{x}}\right), \; j = 1, 2, \ldots, 10, \text{ dengan } \underset{\sim}{x}_j = \begin{bmatrix} x_{1j} \\ x_{2j} \end{bmatrix},$$

using results obtained in Part b.

(ii) Using the distances in Part (i), determine the proportion of the observations falling within the estimated 50% probability contour of a bivariate normal distribution Use the value $\chi_2^2(.5) = 1.39$ of the X^2 critical point.

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(iii) Order the distances in Part (i) and construct a chi-square plot. Use the following table of chi-square percentiees for $p = 2$, $n = 10$:

j	1	2	3	4	5	6	7	8	9	10
$\chi_2^2\left(\frac{j-\frac{1}{2}}{10}\right)$	.10	.33	.58	.86	1.20	1.60	2.10	2.77	3.79	5.99

(iv) Given the results in Parts (ii) and (iii), are these data approximately bivariate normal? Explain.

(40/100)

4. (a) Andaikan matriks data bagi suatu sampel rawak bersaiz $n = 4$ daripada suatu populasi normal bivariat sebagai

$$X = \begin{bmatrix} 2 & 8 & 6 & 8 \\ 12 & 9 & 9 & 10 \end{bmatrix}.$$

(i) Nilaikan T^2, bagi menguji H_0: $\mu = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

(ii) Tentukan taburan T^2 bagi keadaan dalam bahagian (i).

(iii) Dengan menggunakan (i) dan (ii), uji H_0 pada aras $\alpha = .05$. Apakah kesimpulan yang boleh dibuat? Gunakan jadual bagi titik genting taburan-F ($\alpha = .05$).

(60/100)

(b) Pertimbangkan matriks data

$$X = \begin{bmatrix} 2 & -1 & 5 \\ 4 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

(i) Hitungkan matriks sisihan $\underset{(3\times3)}{X} - \underset{(3\times1)}{\bar{x}}\underset{(1\times3)}{1'}$.

(ii) Tentukan S dan hitung varians sampel teritlak $|S|$.

(iii) Hitungkan jumlah varians sampel.

(40/100)

4. (a) Let the data matrix for a random sample of size $n = 4$ from a bivariate normal population be

$$X = \begin{bmatrix} 2 & 8 & 6 & 8 \\ 12 & 9 & 9 & 10 \end{bmatrix}.$$

(i) Evaluate T^2, for testing H_0: $\mu = \begin{bmatrix} 7 \\ 11 \end{bmatrix}$.

(ii) Specify the distribution of T^2 for the situation in (i).

(iii) Using (i) and (ii), test H_0 at the $\alpha = .05$ level. What conclusion is reached? Use the table for F distribution critical points ($\alpha = .05$).

(60/100)

(b) Consider the data matrix

$$X = \begin{bmatrix} 2 & -1 & 5 \\ 4 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

(i) Calculate the matrix of deviations $\underset{(3\times3)}{X} - \underset{(3\times1)(1\times3)}{\bar{x}\ 1'}$.

(ii) Determine S and calculate the generalized sample variance $|S|$.

(iii) Calculate the total sample variance.

(40/100)

5. (a) Cerapan terhadap dua pembolehubah sambutan telah dikumpul bagi tiga rawatan. Vektor-vektor cerapannya ialah

Rawatan 1: $\begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

Rawatan 2: $\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Rawatan 3: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

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(i) Pecahkan cerapan-cerapan ini kepada komponen min, rawatan dan reja. Bina tatasusunan yang sepadan bagi setiap pembolehubah.

(ii) Dengan menggunakan maklumat dalam Bahagian (i), bina satu jadual MANOVA satu-hala.

(iii) Nilaikan lambda Λ^* Wilk, dan uji kesan-kesan rawatan. Tetapkan $\alpha = .01$ dan guna jadual taburan Lambda Λ^* Wilk.

(50/100)

(b) Pertimbangkan matriks kovarians

$$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}.$$

(i) Tentukan komponen prinsipal populasi Y_1 dan Y_2.

(ii) Hitung kadaran jumlah varians populasi yang diterangkan oleh komponen prinsipal pertama.

(iii) Tukarkan matriks kovarians Σ kepada suatu matriks korelasi ρ dan tentukan komponen prinsipal Y_1 dan Y_2 daripada ρ. Hitung kadaran jumlah varians populasi yang diterangkan oleh Y_1.

(iv) Bandingkan komponen-komponen tersebut dengan komponen-komponen yang diperolehi dalam (i). Adakah kedua-duanya sama?

(v) Hitung korelasi ρ_{Y_1,Z_1}, ρ_{Y_1,Z_2} dan ρ_{Y_2,Z_2}.

(50/100)

5. (a) Observations on two responses are collected for three treatments. The observation vectors are

Treatments 1: $\begin{bmatrix} 6 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 9 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix}, \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

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Treatments 2: $\begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Treatments 3: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(i) Break up the observations into mean, treatment and residual components. Construct the corresponding arrays for each variable.

(ii) Using the information in Part (i), construct the one-way MANOVA table.

(iii) Evaluate Wilk's lambda Λ^*, and test treatment effects. Set $\alpha = .01$ and use the table of distribution of Wilk's Lambda Λ^*.

(50/100)

(b) Consider the covariance matrix

$$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}.$$

(i) Determine the population principal components Y_1 and Y_2.

(ii) Calculate the proportion of total population variance explained by the first principal component.

(iii) Convert the covariance matrix Σ to a correlation matrix ρ and determine the principal components Y_1 and Y_2 from ρ. Compute the proportion of total population variance explained Y_1.

(iv) Compare the components with those obtained in (i). Are they the same?

(v) Compute the correlations ρ_{Y_1,Z_1}, ρ_{Y_1,Z_2} and ρ_{Y_2,Z_2}.

(50/100)

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MSG 466 - ANALISIS MULTIVARIAT

LAMPIRAN

Tatatanda adalah seperti di dalam kuliah.

1. Penguraian spektrum bagi suatu matriks simetrik $k \times k$, $\underset{\sim}{A}$ diberikan oleh

$$\underset{\sim}{A} = \lambda_1 \underset{\sim}{e}_1 \underset{\sim}{e}'_1 + \lambda_2 \underset{\sim}{e}_2 \underset{\sim}{e}'_2 + \ldots + \lambda_k \underset{\sim}{e}_k \underset{\sim}{e}'_k$$

di mana $\lambda_1, \lambda_2, \cdots \lambda_k$ adalah nilai-nilai eigen $\underset{\sim}{A}$ dan $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \ldots, \underset{\sim}{e}_k$ adalah vektor-vektor eigen terpiawai yang berkaitan.

2. Katakan $\underset{\sim}{X}$ mempunyai $E\left(\underset{\sim}{X}\right) = \underset{\sim}{\mu}$ dan $\text{Kov}\left(\underset{\sim}{X}\right) = \Sigma$. Maka $\underset{\sim}{c}' \underset{\sim}{X}$ mempunyai min, $\underset{\sim}{c}' \underset{\sim}{\mu}$ dan varians $\underset{\sim}{c}' \underset{\sim}{\Sigma} \underset{\sim}{c}$.

3. f.k.k. normal bivariat:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}\left(1 - \rho_{12}^2\right)}} \times \exp\left\{ - \frac{1}{2\left(1 - \rho_{12}^2\right)} \right.$$

$$\left. \left[\left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right)^2 + \left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right)^2 - 2\rho_{12} \left(\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right)\left(\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right) \right] \right.$$

4. f.k.k. normal multivariat:

$$f(x) = \frac{1}{(2\pi)^{p/2} \, |\Sigma|^{1/2}} \, e^{-(1/2)(\underset{\sim}{x} - \underset{\sim}{\mu})'\Sigma^{-1}(\underset{\sim}{x} - \underset{\sim}{\mu})}$$

5. Jika $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right)$, maka $\underset{\sim}{A}\underset{\sim}{X} \sim N_q(\underset{\sim}{A}\underset{\sim}{\mu}\,,\, A\,\underset{\sim}{\Sigma}\,\underset{\sim}{A}')$

...2/-

6. Satu sampel:

(a) $T^2 = n\left(\underset{\sim}{X} - \underset{\sim}{\mu}\right)' \underset{\sim}{S}^{-1}\left(\overline{X} - \underset{\sim}{\mu}\right)$

$$\overline{X} = \frac{1}{n}\sum_{j=1}^{n} \underset{\sim}{X}_j, \quad \underset{\sim}{S} = \frac{1}{n-1}\sum_{j=1}^{n}\left(\underset{\sim}{X}_j - \overline{X}\right)\left(\underset{\sim}{X}_j - \overline{X}\right)'$$

$$T^2 \sim \frac{(n-1)\rho}{n-\rho} F_{\rho,n-\rho}$$

(b) Lambda Wilks $\Lambda^{2/n} = \left|\hat{\underset{\sim}{\Sigma}}\right| \Big/ \left|\hat{\underset{\sim}{\Sigma}}_0\right| = \left(1 + \frac{T^2}{(n-1)}\right)^{-1}$

(c) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\underset{\sim}{\ell}'\underset{\sim}{\mu}$:

$$\underset{\sim}{\ell}' \overline{X} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p,n-p}(\alpha)\, \underset{\sim}{\ell}'\underset{\sim}{S}\underset{\sim}{\ell}}$$

(d) Selang keyakinan serentak Bonferroni $100(1-\alpha)\%$ bagi

$$\mu_i,\ i = 1, \cdots, p:$$

$$\overline{X}_i + t_{n-1}\left(\frac{\alpha}{2p}\right)\sqrt{\frac{S_{11}}{n}}$$

7. Dua sampel tak bersandar:

(a) $T^2 = \left[\overline{X}_1 - \overline{\underset{\sim}{X}}_2 - \left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)\right]' \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\underset{\sim}{S}_p\right]^{-1}$

$$\left[\overline{\underset{\sim}{X}}_1 - \overline{\underset{\sim}{X}}_2 - \left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)\right]$$

$$T^2 \sim \frac{(n_1 + n_2 - 2)\,p}{(n_1 + n_2 - p - 1)} F_{p,\, n_1 + n_2 - p - 1}$$

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(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\underset{\sim}{\ell}'\left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)$:

$$\underset{\sim}{\ell}'\left[\overline{\underset{\sim}{X}}_1 - \overline{\underset{\sim}{X}}_2\right] \pm c\sqrt{\underset{\sim}{\ell}'\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\underset{\sim}{S}_p\,\underset{\sim}{\ell}}$$

di mana $c^2 = \dfrac{(n_1 + n_2 - 2)p}{n_1 + n_2 - p - 1} F_{p,\, n_1 + n_2 - p - 1}$

8. <u>MANOVA satu-hala</u>:

(a) $$\underset{\sim}{B} = \sum_{\ell=1}^{g} n_\ell\left(\overline{\underset{\sim}{x}}_\ell - \overline{\underset{\sim}{x}}\right)\left(\overline{\underset{\sim}{x}}_\ell - \overline{\underset{\sim}{x}}\right)'$$

$$\underset{\sim}{W} = \sum_{\ell=1}^{g}\sum_{j=1}^{n_\ell}\left(\underset{\sim}{x}_{\ell j} - \overline{\underset{\sim}{x}}_\ell\right)\left(\underset{\sim}{x}_{\ell j} - \overline{\underset{\sim}{x}}_\ell\right)'$$

$$\Lambda^* = \frac{\left|\underset{\sim}{W}\right|}{\left|\underset{\sim}{B} + \underset{\sim}{W}\right|}$$

(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\tau_{ki} - \tau_{\ell i}$:

$$\overline{X}_{ki} - \overline{X}_{\ell i} \pm t_{n-g}\left(\frac{\alpha}{pq(g-1)}\right)\sqrt{\frac{W_{ii}}{n-g}\left(\frac{1}{n_k} + \frac{1}{n_\ell}\right)}.$$

$i = 1, 2, \ldots, p, \quad \ell < k = 1, 2, \ldots, g$.

9. Andaikan $\underset{\sim}{E}$ mempunyai d.k. m_E dan H mempunyai d.k. m_H. Katakan $\Lambda = \dfrac{\left|\underset{\sim}{E}\right|}{\left|\underset{\sim}{E} + \underset{\sim}{H}\right|}$.

Maka (1) Untuk $p = 1$.

$$\left(\frac{1-\Lambda}{\Lambda}\right)\frac{m_E}{m_H} \sim F_{m_H, m_E} \text{ bagi sebarang } m_H.$$

...4/-

(2) Untuk $m_H = 1$.

$$\left(\frac{1-\Lambda}{\Lambda}\right)\frac{m_E+1-p}{P} \sim F_{P,\, m_E+1-p} \text{ bagi sebarang } P.$$

(3) Untuk $p = 2$.

$$\left(\frac{1-\Lambda^{1/2}}{\Lambda^{1/2}}\right)\left(\frac{m_E-1}{m_H}\right) \sim F_{2m_H,\, 2(m_E-1)}.$$

(4) Untuk $m_H = 2$.

$$\left(\frac{1-\Lambda^{1/2}}{\Lambda^{1/2}}\right)\left(\frac{m_E+1-p}{P}\right) \sim F_{2p,\, 2(m_E+1-p)}$$

untuk $p \geq 2$.

Pembetulan Bartlett: Katakan $n_0 = m_E + m_H$.

Bagi m_E besar.

$$-f \log \Lambda \sim X^2_{pm_H}$$

di mana $f = m_E - \frac{1}{2}(p - m_H + 1)$

$$= n_0 - \frac{1}{2}(p + m_H + 1)$$

10. MANOVA dua-hala:

$$SSP_{\text{faktor 1}} = \sum_{\ell=1}^{g} bn\left(\underset{\sim}{\bar{x}}_{\ell} - \underset{\sim}{\bar{x}}\right)\left(\underset{\sim}{\bar{x}}_{\ell} - \underset{\sim}{\bar{x}}\right)'$$

$$SSP_{\text{faktor 2}} = \sum_{k=1}^{b} gn\left(\underset{\sim}{\bar{x}}_{.k} - \underset{\sim}{\bar{x}}\right)\left(\underset{\sim}{\bar{x}}_{.k} - \underset{\sim}{\bar{x}}\right)'$$

$$SSP_{\substack{\text{tindakan}\\ \text{bersaling}}} = \sum_{\ell=1}^{q}\sum_{k=1}^{b} n\left(\underset{\sim}{\bar{x}}_{\ell k} - \underset{\sim}{\bar{x}}_{\ell .} - \underset{\sim}{\bar{x}}_{.k} + \underset{\sim}{\bar{x}}\right)\left(\underset{\sim}{\bar{x}}_{\ell k} - \underset{\sim}{\bar{x}}_{\ell .} - \underset{\sim}{\bar{x}}_{.k} + \underset{\sim}{\bar{x}}\right)'$$

$$SSP_{\text{residual}} = \sum_{\ell=1}^{q}\sum_{k=1}^{b}\sum_{r=1}^{n}\left(\underset{\sim}{x}_{\ell kr} - \underset{\sim}{\bar{x}}_{\ell k}\right)\left(\underset{\sim}{x}_{\ell kr} - \underset{\sim}{\bar{x}}_{\ell k}\right)'$$

...5/-

11. Komponen Prinsipal

(a) $\underset{\sim}{Y_i} = \underset{\sim}{e'_i} \, \underset{\sim}{X}, \quad i = 1, 2, \ldots, p.$

$$\rho_{Y_i, X_k} = \frac{e_{ki} \sqrt{\lambda_i}}{\sqrt{\sigma_{kk}}}, \qquad i, k = 1, 2, \ldots, p.$$

(b) $\underset{\sim}{Y_i} = \underset{\sim}{e'_i} \, \underset{\sim}{Z}$

$$\rho_{Y_i, Z_k} = e_{ki} \sqrt{\lambda_i}, \qquad i, k = 1, 2, \ldots, p.$$

12. Analisis Faktor

(a) $\underset{\sim}{X} - \underset{\sim}{\mu} = \underset{\sim}{L} \, \underset{\sim}{F} + \underset{\sim}{\epsilon}$

(b) $\text{Kov}\left(\underset{\sim}{X}\right) = \underset{\sim}{L} \, \underset{\sim}{L}' + \psi$

$\text{Kov}\left(\underset{\sim}{X}, \underset{\sim}{F}\right) = L$

(c) $h_i^2 = \ell_{i1}^2 + \ell_{i2}^2 + \ldots + \ell_{im}^2, \quad i = 1, 2, \ldots, p.$

$\sigma_{1i} = h_i^2 + \psi_i, \quad i = 1, 2, \ldots, p.$

(d) Kriterium varimax: Pilih transformasi ortogon T yang menjadikan

$$V = \frac{1}{p} \sum_{j=1}^{m} \left[\sum_{i=1}^{p} \tilde{\ell}_{ij}^{*4} - \left(\sum_{i=1}^{p} \tilde{\ell}_{ij}^{*2} \right)^2 \Big/ p \right]$$

sebesar yang mungkin.

13. Analisis Pembezalayan

(a) $Y = \underset{\sim}{\ell}' \, \underset{\sim}{X} = \left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)' \Sigma^{-1} \underset{\sim}{X}$

$$m = \frac{1}{2}\left(\underset{\sim}{\mu}_1 - \underset{\sim}{\mu}_2\right)' \Sigma^{-1} \left(\underset{\sim}{\mu}_1 + \underset{\sim}{\mu}_2\right)$$

...6/-

(b) $y = \hat{\underset{\sim}{\ell}}' \underset{\sim}{x} = \left(\bar{\underset{\sim}{x}}_1 - \bar{\underset{\sim}{x}}_2\right)' S_p^{-1} \underset{\sim}{x}$

$$\hat{m} = \frac{1}{2}\left(\bar{\underset{\sim}{x}}_1 - \bar{\underset{\sim}{x}}_2\right)' S_p^{-1}\left(\bar{\underset{\sim}{x}}_1 + \bar{\underset{\sim}{x}}_2\right)$$

(c) Petua peruntukan:

$$\text{Untukkan } \underset{\sim}{x}_0 \text{ kepada } \begin{cases} \pi_1 & \text{jika } y_0 \geq \hat{m} \\ \pi_2 & \text{jika } y_0 < \hat{m} \end{cases}$$

(d) $\underset{\sim}{B}_0 = \sum_{i=1}^{g} \left(\underset{\sim}{\mu}_i - \bar{\underset{\sim}{\mu}}\right)\left(\underset{\sim}{\mu}_i - \bar{\underset{\sim}{\mu}}\right)'$

$\lambda_1, \ldots, \lambda_s$ nilai eigen dan

$\underset{\sim}{e}_1, \ldots, \underset{\sim}{e}_s$ vektor eigen $\underset{\sim}{\Sigma}^{-1} \underset{\sim}{B}_0$.

$\underset{\sim}{\ell}_i \underset{\sim}{X} = \underset{\sim}{e}_i \underset{\sim}{X}$ pembezalayan ke-i, $i = 1, 2, \ldots, s$.

(e) $\hat{\underset{\sim}{B}}_0 = \sum_{i=1}^{g} \left(\bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}\right)\left(\bar{\underset{\sim}{x}}_i - \bar{\underset{\sim}{x}}\right)'$

$$\underset{\sim}{W} = \sum_{i=1}^{g} \sum_{j=1}^{n_i} \left(\underset{\sim}{x}_{ij} - \bar{\underset{\sim}{x}}_i\right)\left(\underset{\sim}{x}_{ij} - \bar{\underset{\sim}{x}}_i\right)'$$

$\hat{\underset{\sim}{\ell}}_i \underset{\sim}{x} = \hat{\underset{\sim}{e}}_i \underset{\sim}{x}$ pembezalayan sampel ke-i, $i = 1, \ldots, s$.

(f) Petua peruntukan:

Untukkan $\underset{\sim}{x}$ kepada π_k jika

$$\sum_{j=1}^{r} \left(\hat{y}_j - \bar{y}_{kj}\right)^2 = \sum_{j=1}^{r} \left[\hat{\underset{\sim}{\ell}}'_j \left(\underset{\sim}{x} - \bar{\underset{\sim}{x}}_k\right)\right]^2$$

$$\leq \sum_{j=1}^{r} \left[\hat{\underset{\sim}{\ell}}'_j \left(\underset{\sim}{x} - \bar{\underset{\sim}{x}}_i\right)\right]^2$$

bagi semua $i \neq k$, $r \leq s$.

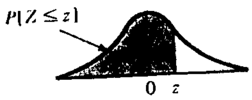
- oooOooo -

Table. Distribution of Wilk's Lambda Λ^*

No. of variables	No. of groups	Sampling distribution for multivariate normal data
$p = 1$	$g \geq 2$	$\left(\frac{n-g}{g-1}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \underline{\underline{d}}\ F_{g-1,\, n-g}$
$p = 2$	$g \geq 2$	$\left(\frac{n-g-1}{g-1}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \underline{\underline{d}}\ F_{2(g-1),2(n-g-1)}$
$p \geq 1$	$g = 2$	$\left(\frac{n-p-1}{p}\right)\left(\frac{1-\Lambda^*}{\Lambda^*}\right) \underline{\underline{d}}\ F_{p,n-p-1}$
$p \geq 1$	$g = 3$	$\left(\frac{n-p-2}{p}\right)\left(\frac{1-\sqrt{\Lambda^*}}{\sqrt{\Lambda^*}}\right) \underline{\underline{d}}\ F_{2p,2(n-p-2)}$

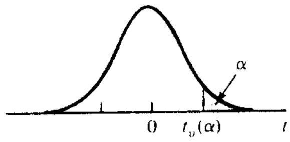
Here $n = \sum_{e=1}^{g} n_e$.

TABLE 1 STANDARD NORMAL PROBABILITIES



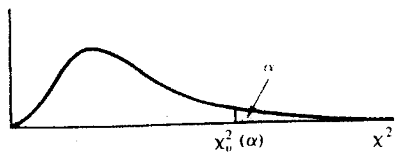
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

TABLE 2 STUDENT'S *t*-DISTRIBUTION CRITICAL POINTS



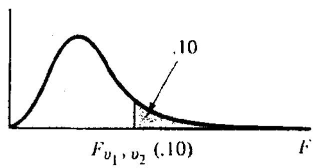
d.f. ν	α .250	.100	.050	.025	.010	.00833	.00625	.005
1	1.000	3.078	6.314	12.706	31.821	38.190	50.923	63.657
2	.816	1.886	2.920	4.303	6.965	7.649	8.860	9.925
3	.765	1.638	2.353	3.182	4.541	4.857	5.392	5.841
4	.741	1.533	2.132	2.776	3.747	3.961	4.315	4.604
5	.727	1.476	2.015	2.571	3.365	3.534	3.810	4.032
6	.718	1.440	1.943	2.447	3.143	3.287	3.521	3.707
7	.711	1.415	1.895	2.365	2.998	3.128	3.335	3.499
8	.706	1.397	1.860	2.306	2.896	3.016	3.206	3.355
9	.703	1.383	1.833	2.262	2.821	2.933	3.111	3.250
10	.700	1.372	1.812	2.228	2.764	2.870	3.038	3.169
11	.697	1.363	1.796	2.201	2.718	2.820	2.981	3.106
12	.695	1.356	1.782	2.179	2.681	2.779	2.934	3.055
13	.694	1.350	1.771	2.160	2.650	2.746	2.896	3.012
14	.692	1.345	1.761	2.145	2.624	2.718	2.864	2.977
15	.691	1.341	1.753	2.131	2.602	2.694	2.837	2.947
16	.690	1.337	1.746	2.120	2.583	2.673	2.813	2.921
17	.689	1.333	1.740	2.110	2.567	2.655	2.793	2.898
18	.688	1.330	1.734	2.101	2.552	2.639	2.775	2.878
19	.688	1.328	1.729	2.093	2.539	2.625	2.759	2.861
20	.687	1.325	1.725	2.086	2.528	2.613	2.744	2.845
21	.686	1.323	1.721	2.080	2.518	2.601	2.732	2.831
22	.686	1.321	1.717	2.074	2.508	2.591	2.720	2.819
23	.685	1.319	1.714	2.069	2.500	2.582	2.710	2.807
24	.685	1.318	1.711	2.064	2.492	2.574	2.700	2.797
25	.684	1.316	1.708	2.060	2.485	2.566	2.692	2.787
26	.684	1.315	1.706	2.056	2.479	2.559	2.684	2.779
27	.684	1.314	1.703	2.052	2.473	2.552	2.676	2.771
28	.683	1.313	1.701	2.048	2.467	2.546	2.669	2.763
29	.683	1.311	1.699	2.045	2.462	2.541	2.663	2.756
30	.683	1.310	1.697	2.042	2.457	2.536	2.657	2.750
40	.681	1.303	1.684	2.021	2.423	2.499	2.616	2.704
60	.679	1.296	1.671	2.000	2.390	2.463	2.575	2.660
120	.677	1.289	1.658	1.980	2.358	2.428	2.536	2.617
∞	.674	1.282	1.645	1.960	2.326	2.394	2.498	2.576

TABLE 3 χ^2 CRITICAL POINTS



d.f. ν	.990	.950	.900	α .500	.100	.050	.025	.010	.005
1	.0002	.004	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.02	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.11	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.30	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.55	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.87	1.64	2.20	5.35	10.64	12.59	14.45	16.81	18.55
7	1.24	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.65	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.95
9	2.09	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.56	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	3.05	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.57	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	4.11	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.66	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	5.23	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.81	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	6.41	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	7.01	9.39	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	7.63	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	8.26	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.90	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	9.54	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	10.20	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	10.86	13.85	15.66	23.34	33.20	36.42	39.36	[illegible]	45.56
25	11.52	14.61	16.47	24.34	34.38	37.65	40.65	[illegible]	46.93
26	12.20	15.38	17.29	25.34	35.56	38.89	41.92	[illegible]	48.29
27	12.88	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	13.56	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	14.26	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	14.95	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	22.16	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
[illegible]	29.71	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
[illegible]	[illegible]	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
[illegible]	[illegible]	51.74	55.33	69.33	85.53	90.53	95.02	100.43	104.21
[illegible]	[illegible]	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
[illegible]	[illegible]	69.13	73.29	89.33	107.57	113.15	118.14	124.12	128.30
[illegible]	70.06	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

TABLE 4 *F*-DISTRIBUTION CRITICAL POINTS (α = .10)



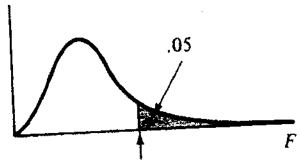
ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.79
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.76
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.51
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.21
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.11
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03

Lampiran 12

12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.96
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.86
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.82
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.75
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.72
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.70
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.68
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.66
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.62
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.59
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.58
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.57
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.56
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.55
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.54
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.40
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24

Lampiran 13

TABLE 5 *F*-DISTRIBUTION CRITICAL POINTS ($\alpha = .05$)

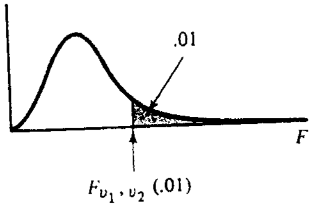


$F_{v_1, v_2}(.05)$

v_2 \ v_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	249.3	250.1	251.1	252.2
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49

12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.38
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.80
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.79
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.75
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43
∞	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.51	1.46	1.39	1.32

TABLE 6 *F*-DISTRIBUTION CRITICAL POINTS ($\alpha = .01$)



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	4052.	5000.	5403.	5625.	5764.	5859.	5928.	5981.	6023.	6056.	6106.	6157.	6209.	6240.	6261.	6287.	6313.
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.32
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.65
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.20
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.06
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.82
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.03
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.48
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.08
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.78

Lampiran 15

12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.54
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.34
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.18
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.05
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.93
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.83
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.75
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.67
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.61
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.55
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.50
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.45
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.40
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.60	2.54	2.45	2.36
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.57	2.50	2.42	2.33
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.54	2.47	2.38	2.29
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.51	2.44	2.35	2.26
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.48	2.41	2.33	2.23
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.45	2.39	2.30	2.21
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.27	2.20	2.11	2.02
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.10	2.03	1.94	1.84
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.93	1.86	1.76	1.66
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.78	1.70	1.59	1.47