

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1995/96

Oktober/November 1995

MSG 466 - Analisis Multivariat

Masa : [3 jam]

Jawab **SEMUA** soalan. Terdapat **EMPAT** soalan.

1. (a) Diberikan lima sukatan pada pembolehubah-pembolehubah x_1, x_2 dan x_3 :-

$$\begin{array}{rcccccc} x_1: & 9 & 2 & 6 & 5 & 8 \\ x_2: & 12 & 8 & 6 & 4 & 10 \\ x_3: & 3 & 4 & 0 & 2 & 1 \end{array}$$

Dapatkan \bar{x} , S_n dan R .

[10/100]

- (b) Katakan

$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

- (i) Adakah A bersimetri?
- (ii) Tunjukkan A adalah tentu positif?
- (iii) Tentukan nilai-nilai eigen dan vektor-vektor eigen bagi A .
- (iv) Tuliskan penghuraian spektrum bagi A .
- (v) Cari A^{-1} .
- (vi) Cari nilai-nilai eigen dan vektor-vektor eigen bagi A^{-1} .

[30/100]

- (c) Pertimbangkan matriks data

$$X = \begin{bmatrix} 1 & 6 & 8 \\ 4 & 2 & 3 \\ 3 & 6 & 3 \end{bmatrix}$$

...2/-

dan gabungan-gabungan linear

$$b'X = [1 \quad 1 \quad 1] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

dan

$$c'X = [1 \quad 2 \quad -3] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix},$$

nilaikan min, varians dan kovarians sampel bagi $b'X$ dan $c'X$.

[30/100]

- (d) Katakan X tertabur $N_3(\mu, \Sigma)$ dengan $\mu' = [2, -3, 1]$ dan

$$\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

- (i) Cari taburan bagi $3X_1 - 2X_2 + X_3$.
- (ii) Labelkan semula pembolehubah-pembolehubah, jika perlu, dan cari satu 2×1 vektor q supaya X_2 dan

$$X_2 - q' \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

adalah tak bersandar.

[30/100]

[100/100]

2. (a) Fisher (1936) mengkaji data daripada Anderson yang berkaitan dengan sampel-sampel rawak bunga iris daripada turunan iris setosa (Spesis 1), versicolor (Spesis 2) dan virginica (Spesis 3). Balasan-balasan adalah empat sukatan:

$x_1 =$ panjang "sepal"
 $x_2 =$ lebar "sepal"
 $x_3 =$ panjang "petal"
 $x_4 =$ lebar "petal"

...3/-

Bagi setiap jenis iris, cerapan-cerapan yang terdiri daripada empat sukatan itu diperhatikan untuk 50 pokok iris. Vektor-vektor minnya adalah

$$\bar{y}_1 = \begin{bmatrix} 5.006 \\ 3.428 \\ 1.462 \\ 0.246 \end{bmatrix}, \quad \bar{y}_2 = \begin{bmatrix} 5.936 \\ 2.770 \\ 4.260 \\ 1.326 \end{bmatrix}, \quad \bar{y}_3 = \begin{bmatrix} 6.588 \\ 2.974 \\ 5.552 \\ 2.026 \end{bmatrix}$$

Matriks SSP di antara spesies ialah

$$B = \begin{bmatrix} 63.21 & -19.95 & 165.25 & 71.28 \\ & 11.35 & -57.24 & -22.93 \\ & & 437.11 & 186.78 \\ & & & 80.41 \end{bmatrix}$$

Matriks SSP di dalam spesies ialah

$$W = \begin{bmatrix} 38.60 & 13.63 & 24.62 & 5.64 \\ & 16.96 & 8.12 & 4.81 \\ & & 27.22 & 6.27 \\ & & & 6.16 \end{bmatrix}$$

- (i) Ujikan hipotesis bahawa ketiga-tiga vektor min adalah sama pada paras keertian 0.01.
- (ii) Dapatkan selang keyakinan serentak 99% untuk perbezaan-perbezaan lebar "sepal".
- (iii) Nyatakan anggapan-anggapan yang telah anda gunakan.

[50/100]

- (b) Kita ingin membanding membanding kehilangan berat tikus-tikus jantan dan betina ($r = 2$ jantina) di bawah $c = 3$ dadah di mana $n = 4$ tikus dari setiap jantina diberi dadah itu secara rawak. Kehilangan berat diperhatikan untuk minggu pertama dan minggu kedua ($p = 2$) dan data adalah seperti berikut:

Kehilangan berat (dalam gram) untuk minggu pertama dan minggu kedua bagi tikus-tikus setiap jantina di bawah dadah A, B dan C.

...4/-

Jantina	Dadah			Jumlah baris
	A	B	C	
Jantan	(5, 6)	(7, 6)	(21, 15)	(33, 27)
	(5, 4)	(7, 7)	(14, 11)	(26, 22)
	(9, 9)	(9, 12)	(17, 12)	(35, 33)
	(7, 6)	(6, 8)	(12, 10)	(25, 24)
Jumlah lajur	(26, 25)	(29, 33)	(64, 48)	(119, 106)
Betina	(7, 10)	(10, 13)	(16, 12)	(33, 35)
	(6, 6)	(8, 7)	(14, 9)	(28, 22)
	(9, 7)	(7, 6)	(14, 8)	(30, 21)
	(8, 10)	(6, 9)	(10, 5)	(24, 24)
Jumlah lajur	(30, 33)	(31, 35)	(54, 34)	(115, 102)

- (i) Binakan jadual MANOVA bagi data ini.
(ii) Ujikan bagi kesan-kesan dadah, kesan jantina dan kesan tindakan bersaling. Gunakan $\alpha = 0.01$.
(iii) Nyatakan anggapan-anggapan yang telah anda menggunakan.
- [50/100]
[100/100]

3. (a) Tuliskan nota pendek tentang tajuk-tajuk di bawah:

- (i) Kaedah kelompok "hierarchical"
(ii) Kaedah k -min MacQueen

[50/100]

(b) Diberikan sepuluh objek mempunyai matriks jarak berikut

$$D = \{d_{ik}\} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} & \begin{bmatrix} 0 & & & & & & & & & \\ 1.61 & 0 & & & & & & & & \\ 1.97 & 2.02 & 0 & & & & & & & \\ 1.97 & 2.51 & 2.88 & 0 & & & & & & \\ 1.40 & 1.70 & 1.35 & 2.21 & 0 & & & & & \\ 2.45 & 3.49 & 3.34 & 3.83 & 3.19 & 0 & & & & \\ 2.83 & 3.94 & 3.64 & 2.89 & 3.01 & 3.00 & 0 & & & \\ 9.58 & 9.59 & 10.05 & 8.78 & 9.30 & 9.74 & 9.23 & 0 & & \\ 7.79 & 7.82 & 8.43 & 7.08 & 7.76 & 7.86 & 7.76 & 2.64 & 0 & \\ 7.86 & 7.92 & 8.36 & 7.44 & 7.79 & 7.90 & 8.26 & 3.38 & 2.56 & 0 \end{bmatrix} \end{matrix}$$

Gunakan (i) kaedah pautan tunggal, (ii) kaedah pautan lengkap daripada prosedur "agglomerative hierarchical" untuk memperolehi gambarajah-gambarajah dendogram untuk sepuluh objek itu.

[50/100]

[100/100]

...5/-

4. Bagi setiap bahagian yang berikut, tuliskan suatu perenggan yang menghuraikan kesimpulan-kesimpulan anda.

- (a) Markah-markah peperiksaan untuk berbagai-bagai topik dikutip. Lima topik dipilih dan peperiksaan dikendalikan secara buku-buka (*B*) dan buku-tutup (*T*):

Mekanik (*T*)
 Vektor (*T*)
 Aljabar (*B*)
 Analisis (*B*)
 Statistik (*B*)

Analisis komponen prinsipal dijalankan dengan pakej SAS kepada data peperiksaan buku buka/tutup, dan hasilnya adalah di dalam Lampiran 1.

[20/100]

- (b) Analisis pembezaan dijalankan pada rekod-rekod masa kebangsaan dalam pelbagai acara sukan larian bagi bahagian lelaki dan bahagian wanita dari 55 buah negara. Pembolehubah-pembolehubah adalah seperti berikut:

Simbol	Bahagian Lelaki	Bahagian Wanita	Unit
sex	1	2	.
<i>x</i> ₁	100 <i>m</i>	100 <i>m</i>	saat
<i>x</i> ₂	200 <i>m</i>	200 <i>m</i>	saat
<i>x</i> ₃	400 <i>m</i>	400 <i>m</i>	saat
<i>x</i> ₄	800 <i>m</i>	800 <i>m</i>	minit
<i>x</i> ₅	1500 <i>m</i>	1500 <i>m</i>	minit
<i>x</i> ₆	*	3000 <i>m</i>	minit
<i>x</i> ₇	5000 <i>m</i>	*	minit
<i>x</i> ₈	10,000 <i>m</i>	*	minit
<i>x</i> ₉	Maraton	Maraton	minit

Pakej SPSS digunakan, dan hasil-hasilnya adalah di dalam Lampiran 2 - 5.

[30/100]

...6/-

(c) Dalam sebuah temuduga pekerjaan, 48 calon dinilai pada 15 pembolehubah. Pembolehubah-pembolehubahnya adalah

1. "Form of letter of application"
2. "Appearance"
3. "Academic ability"
4. "Likeability"
5. "Self-confidence"
6. "Lucidity"
7. "Honesty"
8. "Salesmanship"
9. "Experience"
10. "Drive"
11. "Ambition"
12. "Grasp"
13. "Potential"
14. "Keeness to join"
15. "Suitability"

Analisis faktor dengan kaedah komponen prinsipal dan kaedah kebolehdjian maksimum dijalankan dengan pakej SPSS dan hasil-hasilnya adalah di dalam Lampiran 6 - 12.

[50/100]

- ooo0ooo -

Principal Component Analysis

88 Observations
5 Variables

Simple Statistics

	X1	X2	X3	X4	X5
Mean	38.95454545	50.59090909	50.60227273	46.68181818	42.30681818
Std	17.48622387	13.14694699	10.62478102	14.84521321	17.25558910

Correlation Matrix

	X1	X2	X3	X4	X5	
X1	1.0000	0.5534	0.5468	0.4094	0.3891	MECHANICS (C)
X2	0.5534	1.0000	0.6096	0.4851	0.4364	VECTORS (C)
X3	0.5468	0.6096	1.0000	0.7108	0.6647	ALGEBRA (O)
X4	0.4094	0.4851	0.7108	1.0000	0.6072	ANALYSIS (O)
X5	0.3891	0.4364	0.6647	0.6072	1.0000	STATISTICS (O)

Eigenvalues of the Correlation Matrix

	Eigenvalue	Difference	Proportion	Cumulative
PRIN1	3.18098	2.44141	0.636196	0.63620
PRIN2	0.73957	0.29461	0.147914	0.78411
PRIN3	0.44497	0.05707	0.088993	0.87310
PRIN4	0.38789	0.14130	0.077578	0.95068
PRIN5	0.24659	.	0.049318	1.00000

Eigenvectors

	PRIN1	PRIN2	PRIN3	PRIN4	PRIN5	
X1	0.399605	0.645458	0.620782	-.145787	0.130672	MECHANICS (C)
X2	0.431419	0.441505	-.705006	0.298135	0.181748	VECTORS (C)
X3	0.503282	-.129068	-.037049	-.108599	-.846689	ALGEBRA (O)
X4	0.456994	-.387906	-.136182	-.666256	0.422189	ANALYSIS (O)
X5	0.438244	-.470454	0.312533	0.658916	0.234022	STATISTICS (O)

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- - - - - D I S C R I M I N A N T A N A L Y S I S - - - - -

On groups defined by SEX SEX

110 (unweighted) cases were processed.
 0 of these were excluded from the analysis.
 110 (unweighted) cases will be used in the analysis.

Number of Cases by Group

SEX	Number of Cases			Label
	Unweighted	Weighted		
1	55	55.0		LELAKI
2	55	55.0		WANITA
Total	110	110.0		

Group Means

SEX	X1	X2	X3	X4
1	10.47109	20.94036	46.43873	1.79327
2	11.61855	23.64164	53.40582	2.07636
Total	11.04482	22.29100	49.92227	1.93482

SEX	X5	X9
1	3.69818	136.62727
2	4.32545	173.25327
Total	4.01182	154.94027

Group Standard Deviations

SEX	X1	X2	X3	X4
1	.35143	.64465	1.45702	.06368
2	.45221	1.11106	2.67834	.10822
Total	.70333	1.63046	4.10512	.16742

SEX	X5	X9
1	.15591	9.22436
2	.33243	30.42954
Total	.40750	28.97116

Pooled Within-Groups Covariance Matrix with 108 degrees of freedom

	X1	X2	X3	X4
X1	.1639983			
X2	.3438677	.8250125		
X3	.7208272	1.674599	4.648194	
X4	.2626678E-01	.6008946E-01	.1705777	.7884108E-02
X5	.7393005E-01	.1679125	.4455975	.2077609E-01
X9	5.564584	13.36120	33.48265	1.519983

	X5	X9
X5	.6740724E-01	
X9	5.062661	505.5230

Pooled Within-Groups Correlation Matrix

	X1	X2	X3	X4	X5	X9
X1	1.00000					
X2	.93485	1.00000				
X3	.82560	.85514	1.00000			
X4	.73048	.74506	.89105	1.00000		
X5	.70315	.71203	.79606	.90123	1.00000	
X9	.61114	.65425	.69073	.76136	.86727	1.00000

Correlations which cannot be computed are printed as '.'

Wilks' Lambda (U-statistic) and univariate F-ratio with 1 and 108 degrees of freedom

Variable	Wilks' Lambda	F	Significance
X1	.32848	220.8	.0000
X2	.30749	243.2	.0000
X3	.27329	287.2	.0000
X4	.27869	279.5	.0000
X5	.40220	160.5	.0000
X9	.59677	72.97	.0000

Covariance Matrix for Group 1, LELAKI

	X1	X2	X3	X4
X1	.1235025			
X2	.2090218	.4155702		
X3	.4306996	.7990560	2.122900	
X4	.1692044E-01	.3311545E-01	.8074313E-01	.4055758E-02
X5	.3836684E-01	.7788771E-01	.1897421	.9115320E-02
X9	1.684809	3.543775	9.472828	.4735924
	X5	X9		
X5	.2430774E-01			
X9	1.244536	85.08878		

Covariance Matrix for Group 2, WANITA

	X1	X2	X3	X4
X1	.2044941			
X2	.4787135	1.234455		
X3	1.010955	2.550142	7.173488	
X4	.3561313E-01	.8706347E-01	.2604123	.1171246E-01
X5	.1094933	.2579372	.7014529	.3243687E-01
X9	9.444360	23.17863	57.49246	2.566373

	X5	X9
X5	.1105067	
X9	8.880786	925.9572

Total Covariance Matrix with 109 degrees of freedom

	X1	X2	X3	X4
X1	.4946766			
X2	1.122719	2.658398		
X3	2.731155	6.407404	16.85197	
X4	.1079793	.2524685	.6666165	.2803070E-01
X5	.2548444	.5938670	1.544099	.6538657E-01
X9	16.11659	38.19974	97.55490	4.121941

	X5	X9
X5	.1660590	
X9	10.81253	839.3279

DISCRIMINANT ANALYSIS

On groups defined by SEX SEX

Analysis number 1

Direct method: All variables passing the tolerance test are entered.

Minimum Tolerance Level..... .00100

Canonical Discriminant Functions

Maximum number of functions..... 1
 Minimum cumulative percent of variance... 100.00
 Maximum significance of Wilks' Lambda.... 1.0000

Prior Probabilities

Group	Prior	Label
1	.50000	LELAKI
2	.50000	WANITA
Total	1.00000	

Canonical Discriminant Functions

Fcn	Eigenvalue	Pct of Variance	Cum Pct	Canonical Corr	After Fcn	Wilks' Lambda	Chisquare	DF	Sig
1*	3.3693	100.00	100.00	.8781	0	.2289	154.834	6	.0000

* marks the 1 canonical discriminant functions remaining in the analysis.

Standardized Canonical Discriminant Function Coefficients

	FUNC 1
X1	-.12414
X2	.53836
X3	.14852

X4 1.05509
 X5 -.25830
 X9 -.51042

Structure Matrix:

Pooled-within-groups correlations between discriminating variables
 and canonical discriminant functions
 (Variables ordered by size of correlation within function)

 FUNC 1
 X3 .88837
 X4 .87646
 X2 .81756
 X1 .77893
 X5 .66418
 X9 .44782

Unstandardized Canonical Discriminant Function Coefficients

 FUNC 1
 X1 -.3065450
 X2 .5927079
 X3 .6888937E-01
 X4 11.88262
 X5 -.9948650
 X9 -.2270153E-01
 (constant) -28.74755

Canonical Discriminant Functions evaluated at Group Means (Group Centroids)

Group FUNC 1
 1 -1.81881
 2 1.81881

Classification Results -

Actual Group	No. of Cases	Predicted Group Membership	
		1	2
Group 1 LELAKI	55	53 96.4%	2 3.6%
Group 2 WANITA	55	4 7.3%	51 92.7%

Percent of "grouped" cases correctly classified: 94.55%

- - - - F A C T O R A N A L Y S I S - - - -

Analysis Number 1 Matrix input

Correlation Matrix:

	X1	X2	X3	X4	X5	X6	X7
X1	1.00000						
X2	.24000	1.00000					
X3	.04000	.12000	1.00000				
X4	.31000	.38000	.00000	1.00000			
X5	.09000	.43000	.00000	.30000	1.00000		
X6	.23000	.37000	.08000	.48000	.81000	1.00000	
X7	-.11000	.35000	-.03000	.65000	.41000	.36000	1.00000
X8	.27000	.48000	.05000	.35000	.82000	.83000	.23000
X9	.55000	.14000	.27000	.14000	.02000	.15000	-.16000
X10	.35000	.34000	.09000	.39000	.70000	.70000	.28000
X11	.28000	.55000	.04000	.35000	.84000	.76000	.21000
X12	.34000	.51000	.20000	.50000	.72000	.88000	.39000
X13	.37000	.51000	.29000	.61000	.67000	.78000	.42000
X14	.47000	.28000	-.32000	.69000	.48000	.53000	.45000
X15	.59000	.38000	.14000	.33000	.25000	.42000	.00000

	X8	X9	X10	X11	X12	X13	X14
X8	1.00000						
X9	.23000	1.00000					
X10	.81000	.34000	1.00000				
X11	.86000	.20000	.78000	1.00000			
X12	.77000	.30000	.71000	.78000	1.00000		
X13	.73000	.35000	.79000	.77000	.88000	1.00000	
X14	.55000	.21000	.61000	.55000	.55000	.54000	1.00000
X15	.55000	.69000	.62000	.43000	.53000	.57000	.40000

	X15
X15	1.00000

Kaiser-Meyer-Olkin Measure of Sampling Adequacy = .77802

Bartlett Test of Sphericity = 656.13037, significance = .00000

There are 16 (7.6%) off-diagonal elements of AIC Matrix > 0.09

Anti-Image Covariance Matrix:

	X1	X2	X3	X4	X5
X1	.41619				
X2	-.07663	.39900			

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X3	-.05002	.00230	.48820		
X4	.00076	-.02714	-.05359	.17227	
X5	-.01470	.00538	.00085	.05199	.12021
X6	-.00473	.05374	.03559	-.05422	-.04633
X7	.10651	-.10129	.01189	-.09507	-.08612
X8	.03790	-.04270	-.03435	.01280	-.02199
X9	-.05951	.05528	-.10990	.02209	-.00565
X10	.01426	.07422	-.03737	.05840	.00101
X11	.01847	-.07671	.03081	-.01678	-.05356
X12	.00107	-.02592	-.03827	.05546	.02352
X13	-.01935	.00493	-.03702	-.06072	-.00375
X14	-.10845	.05921	.14549	-.09632	-.00210
X15	-.06327	-.07796	.07646	-.03512	.02198

	X6	X7	X8	X9	X10
X6	.09401				
X7	.03723	.23510			
X8	-.04129	.01806	.11861		
X9	.01676	.01646	.02157	.39908	
X10	-.00570	-.04429	-.02542	.02276	.14451
X11	.02353	.08100	-.02592	-.00875	-.02850
X12	-.05994	-.04024	.01252	-.00673	.03359
X13	.00777	-.00214	.01108	-.01234	-.05325
X14	.02956	-.02346	-.02548	-.03294	-.05397
X15	.01226	.03719	-.05270	-.14978	-.07148

	X11	X12	X13	X14	X15
X11	.09682				
X12	-.02188	.08626			
X13	-.01894	-.04399	.09251		
X14	-.01383	-.03779	.03901	.16516	

	X11	X12	X13	X14	X15
X15	.04074	-.02460	.00241	.03719	.21586

Anti-Image Correlation Matrix:

	X1	X2	X3	X4	X5	X6	X7
X1	.78092						
X2	-.18804	.75219					
X3	-.11097	.00520	.37906				
X4	.00282	-.10351	-.18479	.62765			
X5	-.06574	.02456	.00351	.36132	.81800		
X6	-.02393	.27748	.16612	-.42603	-.43579	.79099	
X7	.34051	-.33070	.03509	-.47239	-.51230	.25040	.55814
X8	.17060	-.19631	-.14274	.08958	-.18415	-.39106	.10814
X9	-.14602	.13853	-.24898	.08424	-.02579	.08650	.05375
X10	.05815	.30909	-.14069	.37012	.00768	-.04890	-.24031
X11	.09198	-.39027	.14172	-.12992	-.49648	.24667	.53689

X1	.92982*	-.00994	.00424	-.05136	.04303
X2	.24994	.98836*	.00394	-.00066	-.00853
X3	.03576	.11606	.93975*	-.00453	.01326
X4	.36136	.38066	.00453	.89307*	-.01975
X5	.04697	.43853	-.01326	.31975	.88550*
X6	.24345	.36091	.12440	.46389	.82580
X7	-.18295	.36178	-.03394	.73006	.36441
X8	.26997	.48227	.02166	.32163	.83929
X9	.56119	.13321	.29091	.15009	-.03323
X10	.31833	.32828	.05152	.40595	.72289
X11	.30397	.55966	.02595	.32516	.84525
X12	.36125	.50948	.25259	.53659	.75840
X13	.38310	.51314	.32921	.60703	.70005
X14	.49975	.26478	-.37741	.71327	.48311
X15	.58783	.39382	.14281	.31121	.27254

	X6	X7	X8	X9	X10
X1	-.01345	.07295	.00003	-.01119	.03167
X2	.00909	-.01178	-.00227	.00679	.01172
X3	-.04440	.00394	.02834	-.02091	.03848
X4	.01611	-.08006	.02837	-.01009	-.01595
X5	-.01580	.04559	-.01929	.05323	-.02289
X6	.87009*	-.01175	.00571	.01712	-.07555
X7	.37175	.90518*	-.00596	.01593	.00811
X8	.82429	.23596	.90021*	-.02135	-.01583
X9	.13288	-.17593	.25135	.85420*	-.06989
X10	.77555	.27189	.82583	.40989	.84817*
X11	.82011	.22492	.88541	.16927	.76836
X12	.83386	.38764	.80571	.28179	.77148
X13	.80806	.44396	.76632	.36632	.77109
X14	.57545	.46428	.54167	.19636	.59547
X15	.38439	.01353	.53613	.79369	.63312

	X11	X12	X13	X14	X15
X1	-.02397	-.02125	-.01310	-.02975	.00217
X2	-.00966	.00052	-.00314	.01522	-.01382
X3	.01405	-.05259	-.03921	.05741	-.00281
X4	.02484	-.03659	.00297	-.02327	.01879
X5	-.00525	-.03840	-.03005	-.00311	-.02254
X6	-.06011	.04614	-.02806	-.04545	.03561
X7	-.01492	.00236	-.02396	-.01428	-.01353
X8	-.02541	-.03571	-.03632	.00833	.01387
X9	.03073	.01821	-.01632	.01364	-.10369
X10	.01164	-.06148	.01891	.01453	-.01312
X11	.90354*	-.02743	.01496	.02214	-.03899
X12	.80743	.86693*	.00958	-.01279	.01313
X13	.75504	.87042	.90107*	-.02600	-.00479
X14	.52786	.56279	.56600	.89795*	-.03210
X15	.46899	.51687	.57479	.43210	.88386*

The lower left triangle contains the reproduced correlation matrix; The diagonal, communalities; and the upper right triangle, residuals between

the observed correlations and the reproduced correlations.

There are 11 (10.0%) residuals (above diagonal) that are > 0.05

Varimax Rotation 1, Extraction 1, Analysis 1 - Kaiser Normalization.

Varimax converged in 6 iterations.

Analysis Number 1 Matrix input

Extraction 1 for Analysis 1, Maximum Likelihood (ML)

Initial Statistics:

Variable	Communality	* Factor	Eigenvalue	Pct of Var	Cum Pct
X1	.58381	* 1	7.50399	50.0	50.0
X2	.60100	* 2	2.06150	13.7	63.8
X3	.51180	* 3	1.46769	9.8	73.6
X4	.82773	* 4	1.20910	8.1	81.6
X5	.87979	* 5	.74142	4.9	86.6
X6	.90599	* 6	.48402	3.2	89.8
X7	.76490	* 7	.34408	2.3	92.1
X8	.88139	* 8	.31027	2.1	94.1
X9	.60092	* 9	.25965	1.7	95.9
X10	.85549	* 10	.20575	1.4	97.2
X11	.90318	* 11	.15093	1.0	98.3
X12	.91374	* 12	.09327	.6	98.9
X13	.90749	* 13	.07628	.5	99.4
X14	.83484	* 14	.05766	.4	99.8
X15	.78414	* 15	.03441	.2	100.0

ML Attempted to extract 6 factors.

More than 25 iterations required. Convergence = .00100

Chi-square Statistic: 46.9788, D.F.: 30, Significance: .0250

Factor Matrix:

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
X1	.32250	.16197	-.00287	.31942	.25208
X2	.41728	-.03549	.38857	.05507	.39917
X3	.08142	.00332	-.07587	-.46755	.50964
X4	.47492	-.11248	-.11396	.56363	.41336
X5	.83652	-.14489	.23424	-.04561	-.13587
X6	.91717	-.39229	-.06174	-.00882	-.00419
X7	.34256	-.10640	-.16668	.36830	.28343
X8	.90228	-.02806	.15092	-.03214	-.07107
X9	.26441	.24680	-.07721	.01777	.37530
X10	.92097	.38081	-.07297	-.02118	-.00323
X11	.87315	.02838	.48286	-.01285	-.00059
X12	.86965	-.22262	.05763	.04083	.27784

X13	.85913	.00866	.04101	.01648	.42294
X14	.63353	.11269	.00683	.75283	-.04036
X15	.55940	.25687	-.13293	.02825	.32115

FACTOR 6

X1	.56852
X2	-.10223
X3	.03490
X4	-.18426
X5	-.23885
X6	.00246
X7	-.67331
X8	.05520
X9	.56570
X10	-.00236
X11	-.00018
X12	.03897
X13	-.03508
X14	.02835
X15	.46861

Final Statistics:

Variable	Communality	*	Factor	Eigenvalue	Pct of Var	Cum Pct
X1	.61904	*	1	6.87192	45.8	45.8
X2	.49919	*	2	.56220	3.7	49.6
X3	.49196	*	3	.54614	3.6	53.2
X4	.77369	*	4	1.35060	9.0	62.2
X5	.85322	*	5	1.25905	8.4	70.6
X6	.99900	*	6	1.42545	9.5	80.1
X7	.82577	*				
X8	.84680	*				
X9	.59796	*				
X10	.99900	*				
X11	.99652	*				
X12	.88956	*				
X13	.92025	*				
X14	.98329	*				
X15	.72011	*				

Varimax Rotation 1, Extraction 1, Analysis 1 - Kaiser Normalization.

Varimax converged in 8 iterations.

Rotated Factor Matrix:

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR 5
X1	.10782	.75512	.07024	-.13471	.09979
X2	.34124	.13260	.25214	.15246	.52731
X3	.04728	.14024	-.00402	.68102	.07804
X4	.23647	.26709	.78110	-.09941	.13927

X5	.87658	-.08969	.18118	-.07241	.17548
X6	.92098	.14738	.28297	.03823	-.08378
X7	.19847	-.23607	.84509	.01414	.06526
X8	.86939	.23324	.09746	-.05989	.11207
X9	.06267	.74671	-.04376	.17367	.05556
X10	.74755	.38214	.19800	-.00499	-.01160
X11	.85539	.18013	.04834	-.09123	.45181
X12	.77824	.31427	.35509	.14613	.15418
X13	.68855	.37623	.42466	.25114	.22560
X14	.39851	.40384	.54242	-.59046	.09639
X15	.33582	.74911	.07761	.14821	.00134

FACTOR 6

X1	-.06457
X2	-.01702
X3	.01213
X4	-.08388
X5	.08895
X6	-.20130
X7	.10966
X8	.10444
X9	.03603
X10	.50476
X11	.13260
X12	-.11801
X13	.10144
X14	.09617
X15	.13482

Factor Transformation Matrix:

	FACTOR 1	FACTOR 2	FACTOR 3	FACTOR 4	FACTOR
FACTOR 1	.91146	.28431	.24833	-.00578	.01818
FACTOR 2	-.23033	.30830	-.11048	-.07170	.10705
FACTOR 3	.13086	-.15196	-.32677	-.19290	.89998
FACTOR 4	-.21501	.24333	.57565	-.72886	.11342
FACTOR 5	-.22637	.37528	.46813	.64655	.39436
FACTOR 6	-.03994	.77523	-.51849	-.09138	-.09933

FACTOR 6

FACTOR 1	.16242
FACTOR 2	.90724
FACTOR 3	-.07637
FACTOR 4	-.13816
FACTOR 5	-.12343
FACTOR 6	-.33222

MSG 466 - ANALISIS MULTIVARIAT

LAMPIRAN

Tatatanda adalah seperti di dalam kuliah.

1. Penguraian spektrum bagi suatu matriks simetrik $k \times k$, A diberikan oleh

$$A = \lambda_1 e_1 e_1' + \lambda_2 e_2 e_2' + \dots + \lambda_k e_k e_k'$$

di mana $\lambda_1, \lambda_2, \dots, \lambda_k$ adalah nilai-nilai eigen A dan e_1, e_2, \dots, e_k adalah vektor-vektor eigen terpiawai yang berkaitan.

2. Katakan X mempunyai $E(X) = \mu$ dan $\text{Kov}(X) = \Sigma$. Maka $c'X$ mempunyai min, $c'\mu$ dan varians, $c'\Sigma c$.

3. f.k.k. normal bivariat:

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{\sigma_{11}\sigma_{22}(1-\rho_{12}^2)}} \times \exp \left\{ -\frac{1}{2(1-\rho_{12}^2)} \left[\left[\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right]^2 + \left[\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right]^2 - 2\rho_{12} \left[\frac{x_1 - \mu_1}{\sqrt{\sigma_{11}}} \right] \left[\frac{x_2 - \mu_2}{\sqrt{\sigma_{22}}} \right] \right] \right\}$$

4. f.k.k. normal multivariat:

$$f(x) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-(1/2)(x - \mu)' \Sigma^{-1} (x - \mu)}$$

5. Jika $X \sim N_p(\mu, \Sigma)$, maka $AX \sim N_q(A\mu, A\Sigma A')$.

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[MSG466]

6. Satu sampel:

$$(a) \quad T^2 = n (\bar{X} - \mu)' S^{-1} (\bar{X} - \mu)$$

$$\bar{X} = \frac{1}{n} \sum_{j=1}^n X_j, \quad S = \frac{1}{n-1} \sum_{j=1}^n (X_j - \bar{X})(X_j - \bar{X})'$$

$$T^2 \sim \frac{(n-1)p}{n-p} F_{p, n-p}$$

$$(b) \quad \text{Lambda Wilks } \Lambda^{2/n} = \frac{|\hat{\Sigma}|}{|\hat{\Sigma}_0|} = \left[1 + \frac{T^2}{(n-1)} \right]^{-1}$$

(c) Selang keyakinan serentak 100(1- α)% bagi $\ell' \mu$:

$$\ell' \bar{X} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p, n-p}(\alpha) \ell' S \ell}$$

(d) Selang keyakinan serentak Bonferroni 100(1- α)% bagi

$$\mu_i, \quad i = 1, \dots, p:$$

$$\bar{X}_i \pm t_{n-1} \left[\frac{\alpha}{2p} \right] \sqrt{\frac{S_{ii}}{n}}$$

7. Dua sampel tak bersandar:

$$(a) \quad T^2 = \left[\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right]' \left[\left(\frac{1}{n_1} + \frac{1}{n_2} \right) S_p \right]^{-1} \left[\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2) \right]$$

$$T^2 \sim \frac{\left[\frac{n_1 + n_2 - 2}{n_1 + n_2 - p - 1} \right] p}{F_{p, n_1 + n_2 - p - 1}}$$

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[MSG 466]

(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi

$$\ell' (\mu_1 - \mu_2)$$

$$\ell' \left[\bar{X}_1 - \bar{X}_2 \right] \pm c \sqrt{\ell' \left[\frac{1}{n_1} + \frac{1}{n_2} \right] S_p^2}$$

$$\text{di mana } c^2 = \frac{\left[n_1 + n_2 - 2 \right] p}{n_1 + n_2 - p - 1} F_{p, n_1 + n_2 - p - 1}$$

8. MANOVA satu-hala:

$$(a) \quad B = \sum_{\ell=1}^g n_{\ell} (\bar{x}_{\ell} - \bar{x}) (\bar{x}_{\ell} - \bar{x})'$$

$$W = \sum_{\ell=1}^g \sum_{j=1}^{n_{\ell}} \left[x_{\ell j} - \bar{x}_{\ell} \right] \left[x_{\ell j} - \bar{x}_{\ell} \right]'$$

$$\Lambda^* = \frac{|W|}{|B + W|}$$

(b) Selang keyakinan serentak $100(1-\alpha)\%$ bagi $\tau_{k1} - \tau_{\ell 1}$:

$$\bar{X}_{k1} - \bar{X}_{\ell 1} \pm t_{n-g} \left(\frac{\alpha}{pg(g-1)} \right) \sqrt{\frac{W_{11}}{n-g} \left[\frac{1}{n_k} + \frac{1}{n_{\ell}} \right]}$$

$$i = 1, 2, \dots, p, \quad \ell < k = 1, 2, \dots, g$$

9. Andaikan E mempunyai d.k. m_E dan H mempunyai d.k. m_H .

$$\text{Katakan } \Lambda = \frac{|E|}{|E + H|}$$

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[MSG466]

Maka (1) Untuk $p = 1$,

$$\left(\frac{1 - \Lambda}{\Lambda} \right) \frac{m_E}{m_H} \sim F_{m_H, m_E} \text{ bagi sebarang } m_H$$

(2) Untuk $m_H = 1$,

$$\left(\frac{1 - \Lambda}{\Lambda} \right) \frac{m_E + 1 - p}{p} \sim F_{p, m_E + 1 - p} \text{ bagi sebarang } p.$$

(3) Untuk $p = 2$,

$$\left(\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}} \right) \left(\frac{m_E - 1}{m_H} \right) \sim F_{2m_H, 2(m_E - 1)}$$

untuk $m_H \geq 2$.(4) Untuk $m_H = 2$,

$$\left(\frac{1 - \Lambda^{1/2}}{\Lambda^{1/2}} \right) \left(\frac{m_E + 1 - p}{p} \right) \sim F_{2p, 2(m_E + 1 - p)}$$

untuk $p \geq 2$.Pembetulan Bartlett: Katakan $n_o = m_E + m_H$.Bagi m_E besar,

$$-f \log \Lambda \sim \chi_{pm_H}^2$$

$$\begin{aligned} \text{di mana } f &= m_E - \frac{1}{2} (p - m_H + 1) \\ &= n_o - \frac{1}{2} (p + m_H + 1) \end{aligned}$$

10. MANOVA dua-hala:

$$SSP_{\text{faktor 1}} = \sum_{\ell=1}^g b_n \left[\bar{x}_{\ell} - \bar{x} \right] \left[\bar{x}_{\ell} - \bar{x} \right]'$$

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$$SSP_{\text{faktor 2}} = \sum_{k=1}^b gn \left[\bar{x}_{.k} - \bar{\bar{x}} \right] \left[\bar{x}_{.k} - \bar{\bar{x}} \right]'$$

$$SSP_{\text{tindakan bersaling}} = \sum_{\ell=1}^g \sum_{k=1}^b n \left[\bar{x}_{\ell k} - \bar{\bar{x}}_{\ell.} - \bar{\bar{x}}_{.k} + \bar{\bar{x}} \right]$$

$$\left[\bar{x}_{\ell k} - \bar{\bar{x}}_{\ell.} - \bar{\bar{x}}_{.k} + \bar{\bar{x}} \right]'$$

$$SSP_{\text{residual}} = \sum_{\ell=1}^g \sum_{k=1}^b \sum_{r=1}^n \left[x_{\ell kr} - \bar{\bar{x}}_{\ell k} \right] \left[x_{\ell kr} - \bar{\bar{x}}_{\ell k} \right]'$$

11. Komponen Prinsipal

$$(a) \quad \underset{\sim}{Y}_i = \underset{\sim}{e}'_i \underset{\sim}{X}_k, \quad i = 1, 2, \dots, p.$$

$$\rho_{Y_i, X_k} = \frac{e_{ki} \sqrt{\lambda_i}}{\sqrt{\sigma_{kk}}}, \quad i, k = 1, 2, \dots, p.$$

$$(b) \quad \underset{\sim}{Y}_i = \underset{\sim}{e}'_i \underset{\sim}{Z}$$

$$\rho_{Y_i, Z_k} = e_{ki} \sqrt{\lambda_i}, \quad i, k = 1, 2, \dots, p.$$

12. Analisis Faktor

$$(a) \quad \underset{\sim}{X} - \underset{\sim}{\mu} = \underset{\sim}{L} \underset{\sim}{F} + \underset{\sim}{\epsilon}$$

$$(b) \quad \underset{\sim}{Kov}(X) = \underset{\sim}{L} \underset{\sim}{L}' + \underset{\sim}{\Psi}$$

$$\underset{\sim}{Kov}(X, F) = \underset{\sim}{L}$$

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[MSG466]

$$(c) \quad h_i^2 = \ell_{i1}^2 + \ell_{i2}^2 + \dots + \ell_{im}^2, \quad i = 1, 2, \dots, p.$$

$$\sigma_{ii} = h_i^2 + \psi_i, \quad i = 1, 2, \dots, p.$$

(d) Kriteria varimax: Pilih transformasi ortogon T yang menjadikan

$$V = \frac{1}{p} \sum_{j=1}^m \left[\sum_{i=1}^p \tilde{\ell}_{ij}^{*4} - \left(\sum_{i=1}^p \tilde{\ell}_{ij}^{*2} \right)^2 / p \right]$$

sebesar yang mungkin.

13. Analisis Pembezaian

$$(a) \quad Y = \tilde{\ell}' X = (\tilde{\mu}_1 - \tilde{\mu}_2)' \tilde{\Sigma}^{-1} X$$

$$\hat{m} = \frac{1}{2} (\tilde{\mu}_1 - \tilde{\mu}_2)' \tilde{\Sigma}^{-1} (\tilde{\mu}_1 + \tilde{\mu}_2)$$

$$(b) \quad y = \hat{\ell}' x = (\bar{x}_1 - \bar{x}_2)' S_p^{-1} x$$

$$\hat{m} = \frac{1}{2} (\bar{x}_1 - \bar{x}_2)' S_p^{-1} (\bar{x}_1 + \bar{x}_2)$$

(c) Petua peruntukan:

$$\text{Untukkan } x_o \text{ kepada } \begin{cases} \pi_1 & \text{jika } y_o \geq \hat{m} \\ \pi_2 & \text{jika } y_o < \hat{m} \end{cases}$$

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$$(d) \quad \hat{B}_o = \sum_{i=1}^g \left[\begin{array}{c} \mu_i - \bar{\mu} \\ \sim \end{array} \right] \left[\begin{array}{c} \mu_i - \bar{\mu} \\ \sim \end{array} \right]'$$

$\lambda_1, \dots, \lambda_s$ nilai eigen dan

e_1, \dots, e_s vektor eigen $\sum_{i=1}^g \hat{B}_o$.

$\hat{\ell}_i X = e_i X$ pembezalayan ke- i , $i = 1, 2, \dots, s$.

$$(e) \quad \hat{B}_o = \sum_{i=1}^g \left[\begin{array}{c} \bar{x}_i - \bar{x} \\ \sim \end{array} \right] \left[\begin{array}{c} \bar{x}_i - \bar{x} \\ \sim \end{array} \right]'$$

$$W = \sum_{i=1}^g \sum_{j=1}^{n_i} \left[\begin{array}{c} x_{ij} - \bar{x}_i \\ \sim \end{array} \right] \left[\begin{array}{c} x_{ij} - \bar{x}_i \\ \sim \end{array} \right]'$$

$\hat{\ell}_i x = e_i x$ pembezalayan sampel ke- i , $i = 1, \dots, s$.

(f) Petua peruntukan:

Untukkan x kepada π_k jika

$$\begin{aligned} \sum_{j=1}^r \left[\hat{y}_j - \bar{y}_{kj} \right]^2 &= \sum_{j=1}^r \left[\hat{\ell}'_j \left[\begin{array}{c} x - \bar{x}_k \\ \sim \end{array} \right] \right]^2 \\ &\leq \sum_{j=1}^r \left[\hat{\ell}'_j \left[\begin{array}{c} x - \bar{x}_i \\ \sim \end{array} \right] \right]^2 \end{aligned}$$

bagi semua $i \neq k$, $r \leq s$.

- ooo0ooo -