UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1995/96

Oktober/November 1995

MSG 442 - Kaedah Unsur Terhingga

Masa : [3 jam]

Jawab semua **TIGA** soalan.

1. (a) Cari penyelesaian kaedah unsur terhingga bagi masalah berikut:

$$y'' + y + 1 = 0, \quad 0 < x < 3$$
$$y(0) = 2, \quad y(3) = 3$$

dengan menggunakan tiga unsur linear.

(40/100)

(b) Katakan Ω ialah segiempat $i(0, 0)$, $j(1, 0)$, $k(1, 1)$, $m(0, 1)$ dan N_i, N_j, N_k, N_m ialah fungsi bentuk segiempat tepat bilinear.

Cari (i) $\int_\Omega N_i N_j N_k \, d\Omega$

(ii) $\int_\Omega x N_i \, d\Omega$

(iii) $\int_\Omega \frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} \, d\Omega$

(iv) $\int_\Gamma N_i N_j \, d\Gamma$

di mana Γ ialah sisi ij.

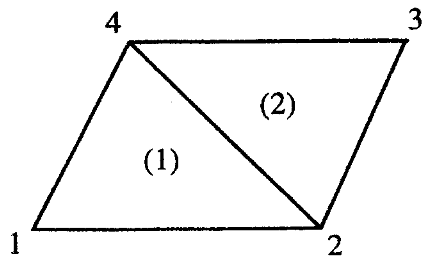
(20/100)

...2/-

(c) Apakah hubungan di antara lebar jalur dan sistem penomboran nod-nod? Terangkan mengapa lebar jalur seharusnya diminimumkan.

(20/100)

(d) Katakan $\phi^{(1)}$ dan $\phi^{(2)}$ ialah fungsi interpolasi dengan menggunakan unsur segitiga linear untuk unsur (1) dan (2) masing-masing seperti ditunjukkan di Rajah 1. Tunjukkan bahawa $\phi^{(1)} = \phi^{(2)}$ di sepanjang sempadan sepunya dua unsur itu.



Rajah 1

(20/100)

2. (a) Pertimbangkan masalah berikut:

$$D_x \frac{\partial^2 \phi}{\partial x^2} + D_y \frac{\partial^2 \phi}{\partial y^2} + Q = \lambda \frac{\partial \phi}{\partial t}$$

dengan syarat sempadan dan syarat awal yang sesuai.

(i) Dengan menggunakan kaedah Galerkin, tunjukkan bahawa masalah tersebut boleh dijadikan sistem persamaan pembezaan berikut:

$$[C]\left\{\frac{d\Phi}{dt}\right\} + [K]\{\Phi\} = \{F\}$$

(ii) Tunjukkan bahawa sistem persamaan di bahagian (i) boleh dijadikan sistem persamaan berikut:

$$\left([C] + \theta\Delta t\,[K] \right) \{\Phi\}_b$$
$$= \left([C] - (1-\theta)\,\Delta t\,[K] \right) \{\Phi\}_a + \Delta t\left((1-\theta)\,\{F\}_a + \theta\{F\}_b\right)$$

di mana $a = t,\ b = t + \Delta t$

(50/100)

.../3-

2. (b) Selesaikan:

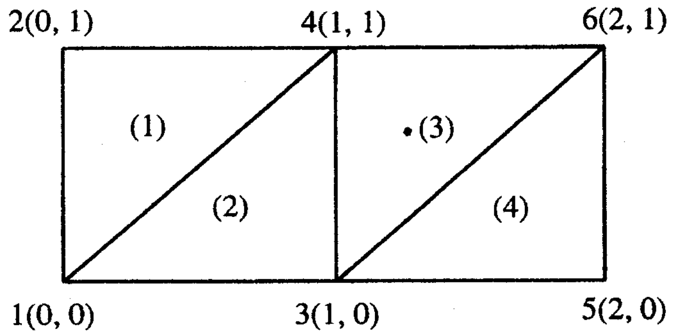
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2 = 0 \quad \text{dalam } \Omega$$

di mana Ω ialah segiempat (0, 0), (2, 0), (2, 1), (0, 1) dengan syarat seperti berikut:

$$\Phi_2 = \Phi_4 = \Phi_5 = \Phi_6 = 0$$

$$\frac{\partial \phi}{\partial x} = 0 \quad \text{pada sisi 1-2}$$

$$\frac{\partial \phi}{\partial y} = -1 \quad \text{pada sisi 1-3-5}$$



Rajah 2

Gunakan empat unsur segitiga seperti ditunjukkan di Rajah 2.

(50/100)

3. (a) Pertimbangkan masalah berikut:

$$\frac{\partial^2 \phi}{\partial x^2} = 8\frac{\partial \phi}{\partial t}, \quad 0 < x < 4, \quad t > 0$$

dengan syarat

$$\phi(x, 0) = 50, \quad 0 < x \le 4$$

$$\phi(0, t) = 10, \quad t > 0$$

$$\frac{\partial \phi}{\partial x}(4, t) = 0, \quad t > 0$$

.../4-

Dengan menggunakan kaedah bergumpal, $\theta = 0$ dan dengan empat unsur linear, cari penyelesaian bagi masalah itu pada masa 2 saat dengan mengambil $\Delta t = 1$ saat.

(30/100)

(b) Pertimbangkan masalah berikut:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \lambda \frac{\partial \phi}{\partial t}, \quad (x, y) \in \Omega, \ t > 0$$

dengan syarat sempadan dan syarat awal yang sesuai. Jika perumusan bergumpal digunakan, cari syarat atas Δt bagi segitiga $A(0, 0)$, $B(1, 0)$, $C(0, 1)$ supaya ayunan berangka dapat dielakkan.

(30/100)

(c) Nilaikan

$$\int_A (x + y)\, dA$$

dengan kuadratur Gauss dengan menggunakan tiga titik pensampelan jika A ialah segitiga (1, 2), (6, 3), (4, 5).

(10/100)

(d) Diberi segiempat (1, 1), (7, 1), (6, 5), (1, 4). Cari transformasi dari koordinat (x, y) kepada koordinat asli (ξ, η) dengan menggunakan fungsi bentuk bilinear.

Jika $\phi = N_1\Phi_1 + N_2\Phi_2 + N_3\Phi_3 + N_4\Phi_4$, cari $\frac{\partial \phi}{\partial x}$ dan $\frac{\partial \phi}{\partial y}$ pada $\xi = \eta = 0.5$.

(30/100)

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Unsur Linear 1-D

$$\left[k^{(e)}\right] = \frac{D}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Unsur Segitiga Linear

$$N_i = [a_i + b_i x + c_i y]/(2A), \quad N_j = [a_j + b_j x + c_j y]/(2A)$$
$$N_k = [a_k + b_k x + c_k y]/(2A)$$

dengan

$$2A = \begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}$$

dan

$$a_i = X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k, \quad c_i = X_k - X_j$$
$$a_j = X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i, \quad c_j = X_i - X_k$$
$$a_k = X_i Y_j - X_j Y_i, \quad b_k = Y_i - Y_j, \quad c_k = X_j - X_i$$

$$\left[k_D^{(e)}\right] = \frac{D_x}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_i b_j & b_j^2 & b_j b_k \\ b_i b_k & b_j b_k & b_k^2 \end{bmatrix} + \frac{D_y}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_i c_j & c_j^2 & c_j c_k \\ c_i c_k & c_j c_k & c_k^2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}, \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

$$\int_A L_1^a \, L_2^b \, L_3^c \, dA = \frac{a!\,b!\,c!}{(a+b+c+2)!} 2A$$

Unsur Segiempat Tepat Bilinear

$$N_i = \frac{1}{4}(1-\xi)(1-\eta), \quad N_j = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_k = \frac{1}{4}(1+\xi)(1+\eta), \quad N_m = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_i = \left(1-\frac{s}{2b}\right)\left(1-\frac{t}{2a}\right), \quad N_j = \frac{s}{2b}\left(1-\frac{t}{2a}\right)$$

$$N_k = \frac{st}{4ab}, \quad N_m = \frac{t}{2a}\left(1-\frac{s}{2b}\right)$$

$$\left[k_D^{(e)}\right] = \frac{D_x a}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{D_y b}{6a}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{4}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

Unsur Kuadratik 1-D

$$N_1 = \frac{1}{2}\xi(\xi-1), \quad N_2 = -(\xi+1)(\xi-1), \quad N_3 = \frac{1}{2}\xi(\xi+1)$$

Unsur Segitiga Kuadratik 6-Nod

$$N_1 = L_1(2L_1-1), \quad N_2 = 4L_1L_2,$$

$$N_3 = L_2(2L_2-1), \quad N_4 = 4L_2(1-L_1-L_2)$$

$$N_5 = 1 - 3(L_1+L_2) + 2(L_1+L_2)^2, \quad N_6 = 4L_1(1-L_1-L_2)$$

Unsur Segiempat Kuadratik 8-Nod

$$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \qquad N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), \qquad N_4 = \frac{1}{2}(1-\eta^2)(1+\xi)$$

$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1), \qquad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_7 = -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1), \qquad N_8 = \frac{1}{2}(1-\eta^2)(1-\xi)$$

Kuadratur Gauss-Legendre

$n=1$	$\xi_i = 0.0$	$W_i = 2.0$
$n=2$	$\xi_i = \pm 0.577350$	$W_i = 1.0$
$n=3$	$\xi_i = 0.0$	$W_i = 8/9$
	$\xi_i = \pm 0.774597$	$W_i = 5/9$
$n=4$	$\xi_i = \pm 0.861136$	$W_i = 0.347855$
	$\xi_i = \pm 0.339981$	$W_i = 0.652145$

Untuk Domain Segitiga

n	Titik	L_1	L_2	W_i
2	a	1/3	1/3	1/2
3	a	1/2	0	1/6
	b	1/2	1/2	1/6
	c	0	1/2	1/6

Masalah Bersandarkan Masa

$$\Big([C] + \theta\Delta t[K]\Big)\{\Phi\}_b = \Big([C] - (1-\theta)\Delta t[K]\Big)\{\Phi\}_a + \Delta t\Big((1-\theta)\{F\}_a + \theta\{F\}_b\Big)$$

Perumusan Konsisten

$$\left[c^{(e)}\right] = \frac{\lambda L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

$$\Delta t > \frac{\lambda L^2}{6D\theta}, \qquad \Delta t < \frac{\lambda L^2}{12D(1-\theta)}$$

Perumusan Tergumpal

$$\left[c^{(e)}\right] = \frac{\lambda L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \left[c^{(e)}\right] = \frac{\lambda A}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta t < \frac{\lambda L^2}{4D(1-\theta)}$$