UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 1997/98

September 1997

MSG 423 - Kaedah Unsur Terhingga

Masa: [3 jam]

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA soalan di dalam TIGA halaman dan EMPAT halaman lampiran yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **SEMUA** soalan.

1.(a) Cari penyelesaian hampiran bagi persamaan

$$\frac{d^2y}{dx^2} + y = x^2, \quad 0 < x < 1$$

$$y(0) = 0, \quad y(1) = 0$$

dengan menggunakan kaedah unsur terhingga dengan tiga unsur linear.

(b) Dapatkan penyelesaian tepat bagi masalah bahagian (a).

(c) Bandingkan penyelesaian di bahagian (a) dan (b).

(100/100)

2.(a) Pertimbangkan masalah yang bersandarkan masa berikut:

$$\frac{\partial^2\phi}{\partial x^2} = 6\frac{\partial\phi}{\partial t}, \quad 0 < x < 3, \quad t > 0$$

$$\phi(x, 0) = 80, \quad 0 \le x \le 3$$

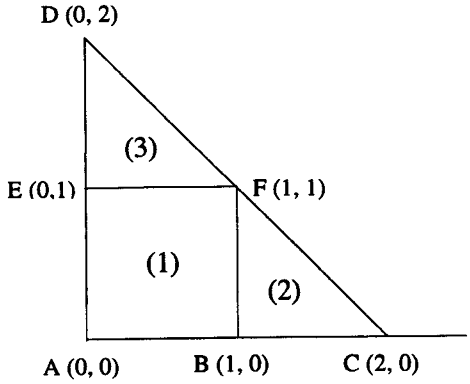
$$\phi(0, t) = 10, \quad t > 0$$

$$\phi(3, t) = 20, \quad t > 0$$

Dengan menggunakan tiga unsur linear dengan perumusan konsisten dan $\theta = \frac{1}{2}$ dapatkan $\phi(1, 1)$, $\phi(2, 1)$ dengan satu langkah masa $\Delta t = 1$.

(b) Selesaikan masalah berikut:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+2=0$$ di dalam segitiga ACD dengan syarat,



$\phi(x, y)=0$ pada sisi CD

$\frac{\partial\phi}{\partial n}=0$ pada sisi AC dan AD

Gunakan tiga unsur seperti ditunjukkan dalam rajah.

(100/100)

3.(a) Pertimbangkan penyelesaian bagi persamaan

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=\frac{\partial\phi}{\partial t}$$

melalui unsur segitiga linear dengan perumusan konsisten. Untuk segitiga $A(0, 0)$, $B(1, 0)$, $C(0, 1)$, cari syarat atas Δt supaya ayunan berangka dapat dielakkan.

(b) Pertimbangkan penyelesaian persamaan haba

$$D\frac{\partial^2\phi}{\partial x^2}=\lambda\frac{\partial\phi}{\partial t}$$

melalui kaedah unsur terhingga dengan skema beza ke depan dan perumusan bergumpah. Dapatkan syarat atas Δt supaya skema pengiraan berangka ini adalah stabil.

(100/100)

...3/-

(c) Pertimbangkan masalah:

$$D\frac{d^2y}{dx^2} + Q = 0, \quad 0 < x < \ell$$

$$y(0) = A, \quad y(\ell) = B$$

di mana D, Q, ℓ, A dan B ialah pemalar.

Terangkan bagaimana masalah ini dapat diselesaikan dengan menggunakan kaedah unsur terhingga dengan unsur kuadratik dan kamiran yang terlibat dinilaikan dengan kuadratur Gauss.

(100/100)

-oooOooo-

Lampiran 1

Unsur Segiempat Kuadratik 8-Nod

$$N_1 = -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta), \qquad N_2 = \frac{1}{2}(1-\xi^2)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1), \qquad N_4 = \frac{1}{2}(1-\eta^2)(1+\xi)$$

$$N_5 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1), \qquad N_6 = \frac{1}{2}(1-\xi^2)(1+\eta)$$

$$N_7 = -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1), \qquad N_8 = \frac{1}{2}(1-\eta^2)(1-\xi)$$

Kuadratur Gauss-Legendre

$n=1$	$\xi_i = 0.0$	$W_i = 2.0$
$n=2$	$\xi_i = \pm 0.577350$	$W_i = 1.0$
$n=3$	$\xi_i = 0.0$	$W_i = 8/9$
	$\xi_i = \pm 0.774597$	$W_i = 5/9$
$n=4$	$\xi_i = \pm 0.861136$	$W_i = 0.347855$
	$\xi_i = \pm 0.339981$	$W_i = 0.652145$

Untuk Domain Segitiga

n	Titik	L_1	L_2	W_i
2	a	1/3	1/3	1/2
3	a	1/2	0	1/6
	b	1/2	1/2	1/6
	c	0	1/2	1/6

Masalah Bersandarkan Masa

$$\left([C] + \theta\Delta t[K]\right)\{\Phi\}_b = \left([C] - (1-\theta)\Delta t[K]\right)\{\Phi\}_a + \Delta t\left((1-\theta)\{F\}_a + \theta\{F\}_b\right)$$

Perumusan Konsisten

$$\left[c^{(e)}\right] = \frac{\lambda L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \left[c^{(e)}\right] = \frac{\lambda A}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}$$

$$\Delta t > \frac{\lambda L^2}{6D\theta}, \qquad \Delta t < \frac{\lambda L^2}{12D(1-\theta)}$$

Perumusan Tergumpal

$$\left[c^{(e)}\right] = \frac{\lambda L}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \left[c^{(e)}\right] = \frac{\lambda A}{3}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[c^{(e)}\right] = \frac{\lambda A}{4}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Delta t < \frac{\lambda L^2}{4D(1-\theta)}$$

Unsur Linear 1-D

$$\left[k^{(e)}\right] = \frac{D}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QL}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Unsur Segitiga Linear

$$N_i = [a_i + b_i x + c_i y]/(2A), \quad N_j = [a_j + b_j x + c_j y]/(2A)$$
$$N_k = [a_k + b_k x + c_k y]/(2A)$$

dengan

$$2A = \begin{vmatrix} 1 & X_i & Y_i \\ 1 & X_j & Y_j \\ 1 & X_k & Y_k \end{vmatrix}$$

dan

$$a_i = X_j Y_k - X_k Y_j, \quad b_i = Y_j - Y_k, \quad c_i = X_k - X_j$$
$$a_j = X_k Y_i - X_i Y_k, \quad b_j = Y_k - Y_i, \quad c_j = X_i - X_k$$
$$a_k = X_i Y_j - X_j Y_i, \quad b_k = Y_i - Y_j, \quad c_k = X_j - X_i$$

$$\left[k_D^{(e)}\right] = \frac{D_x}{4A}\begin{bmatrix} b_i^2 & b_i b_j & b_i b_k \\ b_i b_j & b_j^2 & b_j b_k \\ b_i b_k & b_j b_k & b_k^2 \end{bmatrix} + \frac{D_y}{4A}\begin{bmatrix} c_i^2 & c_i c_j & c_i c_k \\ c_i c_j & c_j^2 & c_j c_k \\ c_i c_k & c_j c_k & c_k^2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad \left\{f^{(e)}\right\} = \frac{QA}{3}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

$$\int_A L_1^a L_2^b L_3^c \, dA = \frac{a!\,b!\,c!}{(a+b+c+2)!} 2A$$

Unsur Segiempat Tepat Bilinear

$$N_i = \frac{1}{4}(1-\xi)(1-\eta), \quad N_j = \frac{1}{4}(1+\xi)(1-\eta)$$
$$N_k = \frac{1}{4}(1+\xi)(1+\eta), \quad N_m = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_i = \left(1-\frac{s}{2b}\right)\left(1-\frac{t}{2a}\right), \quad N_j = \frac{s}{2b}\left(1-\frac{t}{2a}\right)$$
$$N_k = \frac{st}{4ab}, \quad N_m = \frac{t}{2a}\left(1-\frac{s}{2b}\right)$$

$$\left[k_D^{(e)}\right] = \frac{D_x a}{6b}\begin{bmatrix} 2 & -2 & -1 & 1 \\ -2 & 2 & 1 & -1 \\ -1 & 1 & 2 & -2 \\ 1 & -1 & -2 & 2 \end{bmatrix} + \frac{D_y b}{6a}\begin{bmatrix} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 \\ -1 & -2 & 2 & 1 \\ -2 & -1 & 1 & 2 \end{bmatrix}$$

$$\left[k_G^{(e)}\right] = \frac{GA}{36}\begin{bmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{bmatrix}; \qquad \left\{f^{(e)}\right\} = \frac{QA}{4}\begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\left[k_M^{(e)}\right] = \frac{ML_{ij}}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \text{dll.}$$

Unsur Kuadratik 1-D

$$N_1 = \frac{1}{2}\xi(\xi-1), \quad N_2 = -(\xi+1)(\xi-1), \quad N_3 = \frac{1}{2}\xi(\xi+1)$$

Unsur Segitiga Kuadratik 6-Nod

$$N_1 = L_1(2L_1-1), \quad N_2 = 4L_1L_2,$$
$$N_3 = L_2(2L_2-1), \quad N_4 = 4L_2(1-L_1-L_2)$$
$$N_5 = 1 - 3(L_1+L_2) + 2(L_1+L_2)^2, \quad N_6 = 4L_1(1-L_1-L_2)$$