UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 1996/97

April 1997

MSG 363/MSG 464 - Rekabentuk dan Analisis Ujikaji

Masa: [3 jam]

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA soalan di dalam LAPAN halaman yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **SEMUA** soalan.

1. (a) Kertas "*Towards Improving the Properties of Plaster Moulds and Castings*" (J. Engr. Manuf., 1991, pp.265-269) menghuraikan beberapa ANOVA dijalankan untuk mengkaji bagaimana amaun fiber karbon dan tambahan pasir mempengaruhi berbagai cirian proses "*moulding*".

Output diperolehi melalui SPSS ditunjukkan di bawah:

```
* * * A N A L Y S I S   O F   V A R I A N C E * * *

        KEKUATAN kekuatan wet-mold
   by   KARBON    tamb. fiber karbon
        PASIR     tambahan pasir

        UNIQUE sums of squares
        All effects entered simultaneously
```

Source of Variation	Sum of Squares	DF	Mean Square	F	Sig of F
Main Effects	1983.556	4	495.889	5.297	.018
KARBON	1278.111	2	639.056	6.827	.016
PASIR	705.444	2	352.722	3.768	.065
2-Way Interactions	278.889	4	69.722	.745	.585
KARBON PASIR	278.889	4	69.722	.745	.585
Explained	2262.444	8	282.806	3.021	.060
Residual	842.500	9	93.611		
Total	3104.944	17	182.644		

18 cases were processed.
0 cases (.0 pct) were missing.

...2/-

* * * A N A L Y S I S   O F   V A R I A N C E * * *

	KERAS	kekerasan casting
by	KARBON	tamb. fiber karbon
	PASIR	tambahan pasir

UNIQUE sums of squares
All effects entered simultaneously

Source of Variation	Sum of Squares	DF	Mean Square	F	Sig of F
Main Effects	193.889	4	48.472	5.935	.013
KARBON	87.111	2	43.556	5.333	.030
PASIR	106.778	2	53.389	6.537	.018
2-Way Interactions	8.889	4	2.222	.272	.889
KARBON PASIR	8.889	4	2.222	.272	.889
Explained	202.778	8	25.347	3.104	.056
Residual	73.500	9	8.167		
Total	276.278	17	16.252		

18 cases were processed.
0 cases (.0 pct) were missing.

Huraikan kesimpulan-kesimpulan yang anda boleh perolehi daripada kajian ini. Gunakan $\alpha = .05$. Nyatakan anggapan-anggapan yang telah anda gunakan.

(50/100)

(b) Apakah kegunaan polinomial ortogon? **Andaikan** penyelidik dalam (a) selepas analisis ANOVA di atas juga menggunakan polinomial ortogon dalam analisisnya dan memperolehi output ditunjukkan di bawah.

\- - - - - O N E W A Y - - - - -

Variable	KEKUATAN	kekuatan wet-mold
By Variable	KARBON	tamb. fiber karbon

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	1278.1111	639.0556	5.2472	.0187
Linear Term	1	1260.7500	1260.7500	10.3519	.0058
Deviation from Linear	1	17.3611	17.3611	.1426	.7110
Quad. Term	1	17.3611	17.3611	.1426	.7110
Within Groups	15	1826.8333	121.7889		
Total	17	3104.9444			

...3/-

----- O N E W A Y -----

Variable KERAS kekerasan casting
By Variable KARBON tamb. fiber karbon

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	87.1111	43.5556	3.4537	.0584
Linear Term	1	65.3333	65.3333	5.1806	.0379
Deviation from Linear	1	21.7778	21.7778	1.7269	.2086
Quad. Term	1	21.7778	21.7778	1.7269	.2086
Within Groups	15	189.1667	12.6111		
Total	17	276.2778			

----- O N E W A Y -----

Variable KEKUATAN kekuatan wet-mold
By Variable PASIR tambahan pasir

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	705.4444	352.7222	2.2050	.1447
Linear Term	1	705.3333	705.3333	4.4093	.0531
Deviation from Linear	1	.1111	.1111	.0007	.9793
Quad. Term	1	.1111	.1111	.0007	.9793
Within Groups	15	2399.5000	159.9667		
Total	17	3104.9444			

----- O N E W A Y -----

Variable KERAS kekerasan casting
By Variable PASIR tambahan pasir

Analysis of Variance

Source	D.F.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	106.7778	53.3889	4.7247	.0256
Linear Term	1	102.0833	102.0833	9.0339	.0089
Deviation from Linear	1	4.6944	4.6944	.4154	.5290
Quad. Term	1	4.6944	4.6944	.4154	.5290
Within Groups	15	169.5000	11.3000		
Total	17	276.2778			

Huraikan kesimpulan-kesimpulan yang beliau perolehi. Gunakan $\alpha = .05$. Adakah terdapat kelemahan dalam kesimpulan-kesimpulannya?

(50/100)

...4/-

2.(a) Terangkan prinsip kuasa dua terkecil. Pertimbangkan model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \epsilon.$$

Andaikan n cerapan dikutip. Gunakan prinsip kuasa dua terkecil untuk mendapat sistem persamaan normal bagi model di atas.

(50/100)

(b) Di dalam proses "*wave soldering*" PCB, sebuah PCB dimasukkan ke dalam mesin "*wave soldering*", dan semua "*joint*" solder dibuat. Andaikan 5 pembolehubah major yang terlibat dalam setup mesin diukur bagi setiap larian. Sejumlah 25 larian berasingan dengan 5 PCB per larian dijalankan. (Setiap PCB mengandungi 460 "*joint*" solder.) Selepas operasi, PCB tersebut diinspeksi secara visual dan elektrikal, dan bilangan "*joint*" solder yang cacat per 100 "*joints*" yang diinspeksi direkodkan.

Data yang dikutip dianalisiskan dengan menggunakan SPSS. Output ditunjukkan di bawah.

```
* * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y    faults per 100 solder joint

 Descriptive Statistics are printed on Page    6

Block Number  1.  Method:  Enter
   X1        X2        X3        X4        X5

Variable(s) Entered on Step Number
   1..    X5        suhu pra-haba
   2..    X3        pemekatan flux
   3..    X1        sudut konveyor
   4..    X4        laju konveyor
   5..    X2        suhu solder

Multiple R            .85779
R Square              .73580
Adjusted R Square     .66627
Standard Error        .05806

Analysis of Variance
                     DF      Sum of Squares      Mean Square
Regression            5              .17837           .03567
Residual             19              .06405           .00337

F =      10.58293       Signif F =  .0001

--------------------- Variables in the Equation ----------------------

Variable              B          SE B     95% Confdnce Intrvl B        Beta

X1              .213574       .036299      .137599      .289548     .838262
X2         -9.58502E-04       .001873     -.004879      .002962    -.075855
X3              .897832      1.046987    -1.293537     3.089200     .105832
X4              .121588       .216682     -.331933      .575108     .071743
X5          1.69476E-04    9.4574E-04     -.001810      .002149     .026044
(Constant)    -1.788473       .965517    -3.809323      .232377

----------- in ------------

Variable            T   Sig T

X1              5.884   .0000
X2              -.512   .6148
X3               .858   .4018
X4               .561   .5813
X5               .179   .8597
(Constant)     -1.852   .0796
```

...5/-

* * * * M U L T I P L E   R E G R E S S I O N * * * *

Listwise Deletion of Missing Data

	Mean	Std Dev	Label
Y	.177	.101	faults per 100 solder joint
X1	6.068	.394	sudut konveyor
X2	253.520	7.954	suhu solder
X3	.872	.012	pemekatan flux
X4	.724	.059	laju konveyor
X5	248.760	15.444	suhu pra-haba

N of Cases = 25

Correlation, 1-tailed Sig:

	Y	X1	X2	X3	X4	X5
Y	1.000	.845	-.320	.078	-.149	.204
	.	.000	.060	.355	.239	.165
X1	.845	1.000	-.328	-.039	-.281	.251
	.000	.	.055	.426	.087	.113
X2	-.320	-.328	1.000	.174	.030	.402
	.060	.055	.	.203	.444	.023
X3	.078	-.039	.174	1.000	.215	.117
	.355	.426	.203	.	.151	.289
X4	-.149	-.281	.030	.215	1.000	-.207
	.239	.087	.444	.151	.	.161
X5	.204	.251	.402	.117	-.207	1.000
	.165	.113	.023	.289	.161	.

* * * * M U L T I P L E   R E G R E S S I O N * * * *

Equation Number 1   Dependent Variable..   Y   faults per 100 solder joint

End Block Number   1   All requested variables entered.

* *

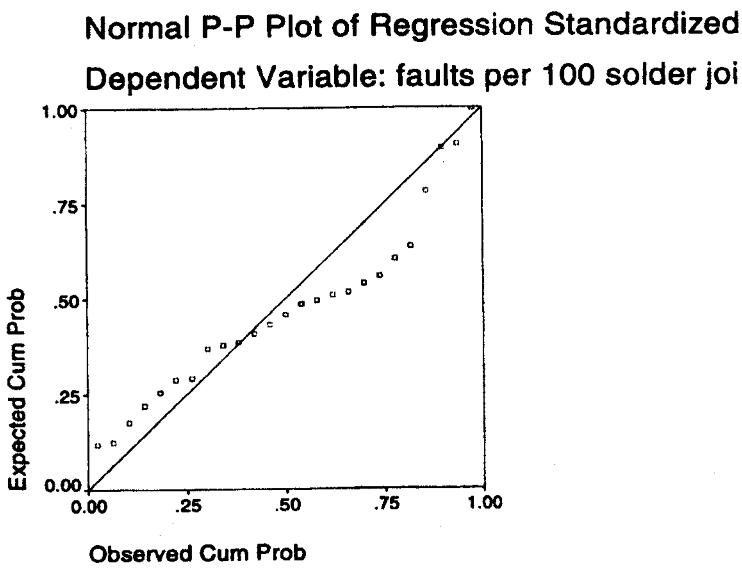
Residuals Statistics:

	Min	Max	Mean	Std Dev	N
*PRED	.0216	.3269	.1772	.0862	25
*RESID	-.0685	.1766	.0000	.0517	25
*ZPRED	-1.8041	1.7374	.0000	1.0000	25
*ZRESID	-1.1803	3.0413	.0000	.8898	25

Total Cases = 25

Hi-Res Chart # 3:Normal p-p plot of *zresid

...6/-



Huraikan kesimpulan-kesimpulan anda. Gunakan $\alpha = .05$.

(50/100)

3.(a) Bincangkan perbezaan di antara

(i) rekabentuk faktorial
(ii) rekabentuk tersarang
(iii) rekabentuk belahan plot

(20/100)

(b) Suatu rekabentuk tersarang tiga peringkat telah dijalankan. Data dikutip ditunjukkan di bawah:

Aloi	1						2					
Haba	1		2		3		1		2		3	
Ketulan	1	2	1	2	1	2	1	2	1	2	1	2
	-10	-23	45	19	15	28	-28	-27	33	25	11	-15
	13	-20	17	-3	4	-5	-40	-11	12	14	27	-8

...7/-

Andaikan bahawa aloi dan haba ialah faktor tetap manakala ketulan ialah faktor rawak. Sebahagian daripada jadual ANOVA ditunjukkan di bawah:

Punca Variasi	Hasil Tambah Kuasa dua (SS)	Darjah Kebebasan	Min Kuasa dua	F
Aloi	315.375	1		
Haba (dalam Aloi)	6,453.833	4		
Ketulan (dalam Haba)	2,226.250	6		
Ralat				
Jumlah	11,136,958	23		

Lengkapkan jadual ANOVA di atas. Dapatkan jangkaan min kuasa dua bagi rekabentuk ini juga. Huraikan kesimpulan-kesimpulan yang anda memperolehi. Gunakan $\alpha = .05$.

(40/100)

(c) Pertimbangkan rekabentuk plot belahan berikut. Empat jenis baja B_1, B_2, B_3 dan B_4, dan dua persediaan tanah T_1 dan T_2 telah dijalankan bagi setiap plot tanah. Ujikaji telah dijalankan dalam dua blok dan data diperolehi diberikan di bawah:

Blok	Blok (*B*) (Plot Utama)	Persediaan Tanah (*T*) (Subplot)	
		T_1	T_2
1	B_1	6	3
	B_2	9	6
	B_3	5	8
	B_4	4	6
2	B_1	3	7
	B_2	6	6
	B_3	2	4
	B_4	5	8

$$\sum_i \sum_j \sum_k y_{ijk}^2 = 542.$$

Huraikan kesimpulan-kesimpulan yang anda boleh perolehi daripada analisis data ini. Gunakan $\alpha = .05$.

(40/100)

...8/-

4.(a) Suatu ujikaji telah dijalankan untuk meningkatkan hasil daripada satu proses kimia. Tiga faktor telah dipilih, dan dua replika bagi suatu ujikaji rawak lengkap telah dijalankan. Keputusannya dipamerkan dalam jadual berikut.

Analisiskan data ini dan dapatkan kesimpulan-kesimpulan yang sesuai. Gunakan $\alpha = .05$.

Gabungan Rawatan	Replika	
	I	II
(1)	90	93
a	74	78
b	81	85
ab	83	80
c	77	78
ac	81	80
bc	88	82
abc	73	70

(50/100)

(b) Bina rekabentuk 2^{7-3} dengan memilih 3 saling tindak empat faktor sebagai penjana tak bersandar. Tuliskan struktur alias yang lengkap untuk rekabentuk ini. Kemukakan jadual analisis varians. Apakah peleraian rekabentuk ini?

(50/100)

5. Tuliskan nota-nota pendek mengenai topik-topik yang berikut:

(a) Analisis residual
(b) Pembauran
(c) Pemblokan
(d) Perawakan

(100/100)

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BERBAGAI RUMUS (Tatatanda seperti di dalam nota kuliah)

1. **Dua sampel tak bersandar** ($n_1 < 25$ atau $n_2 < 25$)

$$s_p^2 = \frac{\sum_i (x_i - \bar{x})^2 + \sum_j (y_j - \bar{y})^2}{n_1 + n_2 - 2}$$

$$= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

2. **Sampel berpasangan**

$$s_d^2 = \frac{\sum_i (d_i - \bar{d})^2}{n - 1}$$

$$= \frac{\sum_i d_i^2 - \frac{(\sum d_i)^2}{n}}{n - 1}$$

3. **Analisis varian satu hala**

$$SST = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SSA = \sum_i \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N}$$

$$SSE = SST - SSA$$

Bagi sebarang kontras $L = \sum_i c_i y_{i.}$,

$$SSL = (\sum_i c_i y_{i.})^2 / (n \sum_i c_i^2)$$

4. <u>Rekabentuk blok rawakan</u>

$$SST = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SSA = \sum_i \frac{y_{i.}^2}{b} - \frac{y_{..}^2}{N}$$

$$SSB = \sum_j \frac{y_{.j}^2}{a} - \frac{y_{..}^2}{N}$$

$$SSE = SST - SSA - SSB$$

5. <u>Rekabentuk segiempat sama Latin</u>

$$SST = \sum_i \sum_j y_{ijk}^2 - \frac{y_{...}^2}{N}$$

$$SSR = \sum_i \frac{y_{i..}^2}{p} - \frac{y_{...}^2}{N}$$

$$SSC = \sum_k \frac{y_{..k}^2}{p} - \frac{y_{...}^2}{N}$$

$$SSA = \sum_j \frac{y_{.j.}^2}{p} - \frac{y_{...}^2}{N}$$

$$SSE = SST - SSR - SSC - SSA$$

6. <u>Rekabentuk faktorial (dua faktor)</u>

$$SST = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$$

$$SSA = \sum_i \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{N}$$

$$SSB = \sum_j \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{N}$$

$$SS_{sub-jumlah} = \sum_i \sum_j \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{N}$$

$$SSAB = SS_{sub-jumlah} - SSA - SSB$$

$$SSE = SST - SS_{sub-jumlah}$$

7. <u>Korelasi dan regresi linear mudah</u>

$$r^2 = \frac{\left(\sum_i x_i y_i - (\sum_i x_i)(\sum_i y_i)/n\right)^2}{[\sum_i x_i^2 - (\sum_i x_i)^2/n][\sum_i y_i^2 - (\sum y_i)^2/n]}$$

$$\hat{\beta}_1 = \frac{\sum_i x_i y_i - (\sum_i x_i)(\sum_i y_i)/n}{\sum_i x_i^2 - (\sum x_i)^2/n}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$SSE = S_{yy} - \hat{\beta}_1^2 S_{xx}$$

$$= S_{yy} - \hat{\beta}_1 S_{xy}$$

$$S_{Y \cdot X}^2 = \frac{SSE}{n-2}$$

Anggaran ralat piawai bagi $\hat{\beta}_0$ ialah

$$\sqrt{MSE\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right)}$$

Anggaran ralat piawai bagi $\hat{\beta}_1$ ialah

$$\sqrt{MSE/S_{xx}}$$

Selang peramalan 100(1 - α)% pada $x = x_0$:

$$\hat{y} \pm t_{\alpha/2}\sqrt{MSE\left(1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$

Selang keyakinan 100(1 - α)% pada $x = x_0$ bagi $\mu_{Y \cdot x_0}$ ialah

$$\hat{y} \pm t_{\alpha/2}\sqrt{MSE\left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}\right)}$$

8. __Regresi linear berganda__

$$\underset{\sim}{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \underset{\sim}{X} = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}$$

$$\underset{\sim}{\beta} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \underset{\sim}{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$$\hat{\underset{\sim}{\beta}} = (\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{y}$$

$$SSE = \underset{\sim}{y}'\underset{\sim}{y} - \hat{\underset{\sim}{\beta}}'\underset{\sim}{X}'\underset{\sim}{y}$$

$$SSR = \hat{\underset{\sim}{\beta}}'\underset{\sim}{X}'\underset{\sim}{y} - (\Sigma\, y_i)^2/n$$

$$r^2_{12.3} = \frac{(r_{12} - r_{13}\, r_{23})^2}{(1 - r^2_{13})(1 - r^2_{23})}$$

$$MSE = \frac{SSE}{n-p}, \quad p = k+1 .$$

9. **Polinomial ortogon**

$$P_0(x) = 1$$

$$P_1(x) = \lambda_1 \left[\frac{x-\bar{x}}{d}\right]$$

$$P_2(x) = \lambda_2 \left[\left(\frac{x-\bar{x}}{d}\right)^2 - \left(\frac{n^2-1}{12}\right)\right]$$

$$P_3(x) = \lambda_3 \left[\left(\frac{x-\bar{x}}{d}\right)^3 - \left(\frac{x-\bar{x}}{d}\right)\left(\frac{3n^2-7}{20}\right)\right]$$

$$P_4(x) = \lambda_4 \left[\left(\frac{x-\bar{x}}{d}\right)^4 - \left(\frac{x-\bar{x}}{d}\right)^2\left(\frac{3n^2-13}{14}\right) + \frac{3(n^2-1)(n^2-9)}{560}\right]$$

$$\hat{\alpha}_j = \frac{\sum_{i=1}^{n} P_j(x_i)y_i}{\sum_{i=1}^{n} P_j^2(x_i)} \quad , \quad j = 0, 1, \ldots, k$$

$$SSR(\alpha_j) = \hat{\alpha}_j \sum_{i=1}^{n} P_j(x_i)y_i$$

$$SSE(k) = S_{yy} - \sum_{j=1}^{k} \hat{\alpha}_j \left[\sum_{i=1}^{n} P_j(x_i)y_i\right]$$

10. **Pemilihan pembolehubah dan pembangunan model dalam regresi**

$$R_p^2 = \frac{SSR(p)}{S_{yy}} = 1 - \frac{SSE(p)}{S_{yy}}$$

$$\overline{R_p^2} = 1 - \left(\frac{n-1}{n-p}\right)(1 - R_p^2)$$

$$MSE(p) = \frac{SSE(p)}{n-p}$$

$$C_p = \frac{SSE(p)}{\hat{\sigma}^2} - (n-2p),$$ di mana $\hat{\sigma}^2$ adalah suatu anggaran σ^2 .

Rekabentuk Blok Tak Lengkap

$N = ar = bk$

$$\lambda = \frac{r(k-1)}{a-1}$$

$$SST = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_{Blok} = \sum_j \frac{y_{.j}^2}{k} - \frac{y_{..}^2}{N}$$

$$Q_i = y_{i.} - \frac{1}{k} \sum_j n_{ij}\, y_{.j}\ ,\ i = 1, 2, \ldots, a$$

$$SS_{Rawatan\ (terlaras)} = \frac{k \sum_i Q_i^2}{\lambda a}$$

$$SSE = SST - SS_{Rawatan\ (terlaras)} - SS_{Blok}$$

Bagi Kontras $L = \sum_i c_i\, Q_i$,

$$SSL = k \frac{\left(\sum_i c_i\, Q_i\right)^2}{\lambda a \sum_i c_i^2}.$$

Ralat piawai bagi suatu rawatan terlaras = $\sqrt{\dfrac{k\, MSE}{\lambda a}}$

$$Q_j' = y_{.j} - \frac{1}{r} \sum_i n_{ij}\, y_{i.}\ ,\ j = 1, 2, \ldots, b$$

$$SS_{Blok\ (terlaras)} = r \frac{\sum_j (Q_j')^2}{\lambda b}$$

Rekabentuk Faktorial (Tiga faktor)

$$SST = \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SSA = \sum_i \frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SSB = \sum_j \frac{y_{.j..}^2}{acn} - \frac{y_{....}^2}{abcn}$$

$$SSC = \sum_k \frac{y_{..k.}^2}{abn} - \frac{y_{....}^2}{abcn}$$

$$SS_{subjumlah\ (AB)} = \sum_i \sum_j \frac{y_{ij..}^2}{cn} - \frac{y_{....}^2}{abcn}$$

$$SS_{subjumlah\ (AC)} = \sum_i \sum_k \frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_{subjumlah\ (BC)} = \sum_j \sum_k \frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{subjumlah\ (ABC)} = \sum_i \sum_j \sum_k \frac{y_{ijk.}^2}{n} - \frac{y_{....}^2}{abcn}$$

$$SS_{A \times B} = SS_{\text{subjumlah (AB)}} - SS_A - SS_B$$

$$SS_{A \times C} = SS_{\text{subjumlah (AC)}} - SS_A - SS_C$$

$$SS_{B \times C} = SS_{\text{subjumlah (BC)}} - SS_B - SS_C$$

$$SS_{A \times B \times C} = SS_{\text{subjumlah (ABC)}} - SS_A - SS_B - SS_C$$

$$- SS_{A \times B} - SS_{A \times C} - SS_{B \times C}$$

$$SS_E = SS_T - SS_{\text{subjumlah (ABC)}}$$

Rekabentuk 2^k

$$\text{Kontras}_{AB..K} = (a \pm 1)\ (b \pm 1)\ \ldots\ (k \pm 1)$$

$$AB \ldots K = \frac{2}{n2^k} (\text{Kontras}_{AB..K})$$

$$SS_{AB...K} = \frac{1}{n2^k} (\text{Kontras}_{AB...K})^2$$

$$SS_{\text{curvature}} = \frac{n_F\, n_c\, (\overline{y_F} - \overline{y_C})^2}{n_F + n_C}$$

$$L = \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_K x_K$$

Rekabentuk Tersarang

1. Dua Tahap

$$SST = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_i \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \sum_i \sum_j \frac{y_{ij.}^2}{n} - \sum_i \frac{y_{i..}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

2. Tiga Tahap

$$SS_T = \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SS_A = \sum_i \frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SS_{B(A)} = \sum_i \sum_j \frac{y_{ij..}^2}{cn} - \sum_i \frac{y_{i...}^2}{bcn}$$

$$SS_{C(B)} = \sum_i \sum_j \sum_k \frac{y_{ijk.}^2}{n} - \sum_i \sum_j \frac{y_{ij..}^2}{cn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)} - SS_{C(B)}$$

Analisis Kovarians

$$S_{yy} = \sum_i \sum_j y_{ij}^2 - \frac{y_{..}^2}{an}$$

$$S_{xx} = \sum_i \sum_j x_{ij}^2 - \frac{x_{..}^2}{an}$$

$$S_{xy} = \sum_i \sum_j x_{ij}\, y_{ij} - \frac{(x_{..})\,(y_{..})}{an}$$

$$T_{xy} = \sum_i \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{an}$$

$$T_{xx} = \sum_i \frac{x_{i.}^2}{n} - \frac{x_{..}^2}{an}$$

$$T_{xy} = \sum_i \frac{(x_{i.})\,(y_{i.})}{n} - \frac{(x_{..})\,(y_{..})}{an}$$

$$E_{yy} = S_{yy} - T_{yy}$$

$$E_{xx} = S_{xx} - T_{xx}$$

$$E_{xy} = S_{xy} - T_{xy}$$

$$\hat{\beta} = \frac{E_{xy}}{E_{xx}}$$

$$SS_E = E_{yy} - (E_{xy})^2 / E_{xx}$$

$$SS_E' = S_{yy} - (S_{xy})^2 / S_{xx}$$

$$MSE = \frac{SSE}{a(n - 1) - 1}$$

$$\bar{y}_{i.(\text{terlaras})} = \bar{y}_{i.} - \hat{\beta} (\overline{x_{i.}} - \overline{x_{..}}), i = 1, 2, ..., a$$

$$S_{\bar{y}_{i.(\text{terlaras})}} = \left[MSE \left(\frac{1}{n} + \frac{(\overline{x_{i.}} - \overline{x_{..}})^2}{E_{xx}}\right) \right]^{1/2}$$

X. Coefficients of Orthogonal Polynomials[a]

	n = 3		n = 4			n = 5				n = 6					n = 7					
X_j	P_1	P_2	P_1	P_2	P_3	P_1	P_2	P_3	P_4	P_1	P_2	P_3	P_4	P_5	P_1	P_2	P_3	P_4	P_5	P_6
1	−1	1	−3	1	−1	−2	2	−1	1	−5	5	−5	1	−1	−3	5	−1	3	−1	1
2	0	−2	−1	−1	3	−1	−1	2	−4	−3	−1	7	−3	5	−2	0	1	−7	4	−6
3	1	1	1	−1	−3	0	−2	0	6	−1	−4	4	2	−10	−1	−3	1	1	−5	15
4			3	1	1	1	−1	−2	−4	1	−4	−4	2	10	0	−4	0	6	0	−20
5						2	2	1	1	3	−1	−7	−3	−5	1	−3	−1	1	5	15
6										5	5	5	1	1	2	0	−1	−7	−4	−6
7															3	5	1	3	1	1
$\sum_{i=1}^{n} \{P_i(X_j)\}^2$	2	6	20	4	20	10	14	10	70	70	84	180	28	252	28	84	6	154	84	924
λ	1	3	8	1	$\frac{10}{3}$	1	1	$\frac{5}{6}$	$\frac{35}{12}$	2	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{7}{12}$	$\frac{21}{10}$	1	1	$\frac{1}{6}$	$\frac{7}{12}$	$\frac{7}{20}$	$\frac{77}{60}$

	n = 8						n = 9						n = 10					
X_j	P_1	P_2	P_3	P_4	P_5	P_6	P_1	P_2	P_3	P_4	P_5	P_6	P_1	P_2	P_3	P_4	P_5	P_6
1	−7	7	−7	7	−7	1	−4	28	−14	14	−4	4	−9	6	−42	18	−6	3
2	−5	1	5	−13	23	−5	−3	7	7	−21	11	−17	−7	2	14	−22	14	−11
3	−3	−3	7	−3	−17	9	−2	−8	13	−11	−4	22	−5	−1	35	−17	−1	10
4	−1	−5	3	9	−15	−5	−1	−17	9	9	−9	1	−3	−3	31	3	−11	6
5	1	−5	−3	9	15	−5	0	−20	0	18	0	−20	−1	−4	12	18	−6	−8
6	3	−3	−7	−3	17	9	1	−17	−9	9	9	1	1	−4	−12	18	6	−8
7	5	1	−5	−13	−23	−5	2	−8	−13	−11	4	22	3	−3	−31	3	11	6
8	7	7	7	7	7	1	3	7	−7	−21	−11	−17	5	−1	−35	−17	1	10
9							4	28	14	14	4	4	7	2	−14	−22	−14	−11
10													9	6	42	18	6	3
$\sum_{i=1}^{n} \{P_i(X_j)\}^2$	168	168	264	616	2184	264	60	2772	990	2002	468	1980	330	132	8580	2860	780	660
λ	2	1	$\frac{2}{3}$	$\frac{7}{12}$	$\frac{7}{10}$	$\frac{11}{60}$	1	3	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{3}{20}$	$\frac{11}{60}$	2	$\frac{1}{2}$	$\frac{5}{3}$	$\frac{5}{12}$	$\frac{1}{10}$	$\frac{11}{240}$

[a] Adapted with permission from *Biometrika Tables For Statisticians*, Vol. 1, 3rd edition by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

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VII. Significant Ranges for Duncan's Multiple Range Test[a]

$r_{01}(p, f)$

f	p = 2	3	4	5	6	7	8	9	10	20	50	100
1	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
2	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
3	8.26	8.5	8.6	8.7	8.8	8.9	8.9	9.0	9.0	9.3	9.3	9.3
4	6.51	6.8	6.9	7.0	7.1	7.1	7.2	7.2	7.3	7.5	7.5	7.5
5	5.70	5.96	6.11	6.18	6.26	6.33	6.40	6.44	6.5	6.8	6.8	6.8
6	5.24	5.51	5.65	5.73	5.81	5.88	5.95	6.00	6.0	6.3	6.3	6.3
7	4.95	5.22	5.37	5.45	5.53	5.61	5.69	5.73	5.8	6.0	6.0	6.0
8	4.74	5.00	5.14	5.23	5.32	5.40	5.47	5.51	5.5	5.8	5.8	5.8
9	4.60	4.86	4.99	5.08	5.17	5.25	5.32	5.36	5.4	5.7	5.7	5.7
10	4.48	4.73	4.88	4.96	5.06	5.13	5.20	5.24	5.28	5.55	5.55	5.55
11	4.39	4.63	4.77	4.86	4.94	5.01	5.06	5.12	5.15	5.39	5.39	5.39
12	4.32	4.55	4.68	4.76	4.84	4.92	4.96	5.02	5.07	5.26	5.26	5.26
13	4.26	4.48	4.62	4.69	4.74	4.84	4.88	4.94	4.98	5.15	5.15	5.15
14	4.21	4.42	4.55	4.63	4.70	4.78	4.83	4.87	4.91	5.07	5.07	5.07
15	4.17	4.37	4.50	4.58	4.64	4.72	4.77	4.81	4.84	5.00	5.00	5.00
16	4.13	4.34	4.45	4.54	4.60	4.67	4.72	4.76	4.79	4.94	4.94	4.94
17	4.10	4.30	4.41	4.50	4.56	4.63	4.68	4.73	4.75	4.89	4.89	4.89
18	4.07	4.27	4.38	4.46	4.53	4.59	4.64	4.68	4.71	4.85	4.85	4.85
19	4.05	4.24	4.35	4.43	4.50	4.56	4.61	4.64	4.67	4.82	4.82	4.82
20	4.02	4.22	4.33	4.40	4.47	4.53	4.58	4.61	4.65	4.79	4.79	4.79
30	3.89	4.06	4.16	4.22	4.32	4.36	4.41	4.45	4.48	4.65	4.71	4.71
40	3.82	3.99	4.10	4.17	4.24	4.30	4.34	4.37	4.41	4.59	4.69	4.69
60	3.76	3.92	4.03	4.12	4.17	4.23	4.27	4.31	4.34	4.53	4.66	4.66
100	3.71	3.86	3.98	4.06	4.11	4.17	4.21	4.25	4.29	4.48	4.64	4.65
∞	3.64	3.80	3.90	3.98	4.04	4.09	4.14	4.17	4.20	4.41	4.60	4.68

f = degrees of freedom.
[a] Reproduced with permission from "Multiple Range and Multiple F Tests," by D. B. Duncan, *Biometrics*, Vol 1, No. 1, pp. 1–42, 1955

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VII. Significant Ranges for Duncan's Multiple Range Test (*continued*)

$r_{.05}(p, f)$

f	p = 2	3	4	5	6	7	8	9	10	20	50	100
1	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0
2	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09	6.09
3	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50
4	3.93	4.01	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02	4.02
5	3.64	3.74	3.79	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83	3.83
6	3.46	3.58	3.64	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68	3.68
7	3.35	3.47	3.54	3.58	3.60	3.61	3.61	3.61	3.61	3.61	3.61	3.61
8	3.26	3.39	3.47	3.52	3.55	3.56	3.56	3.56	3.56	3.56	3.56	3.56
9	3.20	3.34	3.41	3.47	3.50	3.52	3.52	3.52	3.52	3.52	3.52	3.52
10	3.15	3.30	3.37	3.43	3.46	3.47	3.47	3.47	3.47	3.48	3.48	3.48
11	3.11	3.27	3.35	3.39	3.43	3.44	3.45	3.46	3.46	3.48	3.48	3.48
12	3.08	3.23	3.33	3.36	3.40	3.42	3.44	3.44	3.46	3.48	3.48	3.48
13	3.06	3.21	3.30	3.35	3.38	3.41	3.42	3.44	3.45	3.47	3.47	3.47
14	3.03	3.18	3.27	3.33	3.37	3.39	3.41	3.42	3.44	3.47	3.47	3.47
15	3.01	3.16	3.25	3.31	3.36	3.38	3.40	3.42	3.43	3.47	3.47	3.47
16	3.00	3.15	3.23	3.30	3.34	3.37	3.39	3.41	3.43	3.47	3.47	3.47
17	2.98	3.13	3.22	3.28	3.33	3.36	3.38	3.40	3.42	3.47	3.47	3.47
18	2.97	3.12	3.21	3.27	3.32	3.35	3.37	3.39	3.41	3.47	3.47	3.47
19	2.96	3.11	3.19	3.26	3.31	3.35	3.37	3.39	3.41	3.47	3.47	3.47
20	2.95	3.10	3.18	3.25	3.30	3.34	3.36	3.38	3.40	3.47	3.47	3.47
30	2.89	3.04	3.12	3.20	3.25	3.29	3.32	3.35	3.37	3.47	3.47	3.47
40	2.86	3.01	3.10	3.17	3.22	3.27	3.30	3.33	3.35	3.47	3.47	3.47
60	2.83	2.98	3.08	3.14	3.20	3.24	3.28	3.31	3.33	3.47	3.48	3.48
100	2.80	2.95	3.05	3.12	3.18	3.22	3.26	3.29	3.32	3.47	3.53	3.53
∞	2.77	2.92	3.02	3.09	3.15	3.19	3.23	3.26	3.29	3.47	3.61	3.67

f = degrees of freedom.

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