UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua

Sidang Akademik 2007/2008

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EEE 223 – TEORI ELEKTROMAGNET

Masa : 3 Jam

Sila pastikan kertas peperiksaan ini mengandungi **SEPULUH** muka surat beserta **EMPAT** muka surat LAMPIRAN bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi **ENAM** soalan.

Jawab **LIMA** soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Agihan markah diberikan di sudut sebelah kanan soalan berkenaan.

Jawab semua soalan dalam Bahasa Malaysia atau Bahasa Inggeris.

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1. (a) Diberikan $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$ dan $\vec{B} = \vec{a}_x - 3\vec{a}_y + 2\vec{a}_z$, cari $\vec{C}$ apabila ia adalah $\vec{C} = 2\vec{A} - 3\vec{B}$.

Given $\vec{A} = 3\vec{a}_x + 2\vec{a}_y - \vec{a}_z$ *and* $\vec{B} = \vec{a}_x - 3\vec{a}_y + 2\vec{a}_z$ *, find* $\vec{C}$ *such that* $\vec{C} = 2\vec{A} - 3\vec{B}$*.*

(20 marks)

(b) Dua vektor $\vec{A}$ and $\vec{B}$ diberikan pada titik P(r,θ, ϕ) didalam ruang sebagai $\vec{A} = 10\vec{a}_r + 30\vec{a}_\theta - 10\vec{a}_\phi$ dan $\vec{B} = -3\vec{a}_r - 10\vec{a}_\theta + 20\vec{a}_\phi$. Dapatkan

Two vectors $\vec{A}$ *and* $\vec{B}$ *are given at a point P(r,θ, ϕ) in space as* $\vec{A} = 10\vec{a}_r + 30\vec{a}_\theta - 10\vec{a}_\phi$ *and* $\vec{B} = -3\vec{a}_r - 10\vec{a}_\theta + 20\vec{a}_\phi$*. Determine*

(i) $2\vec{A} - 5\vec{B}$

(ii) $\vec{A} \bullet \vec{B}$

(iii) komponen skala bagi $\vec{A}$ di dalam arah $\vec{B}$
the scalar component of $\vec{A}$ *in the direction of* $\vec{B}$

(iv) komponan vektor $\vec{A}$ diunjurkan di dalam arah $\vec{B}$
the vector projection of $\vec{A}$ *in the direction of* $\vec{B}$

(v) satu unit vektor normal ke kedua-dua $\vec{A}$ dan $\vec{B}$
a unit vector perpendicular to both $\vec{A}$ *and* $\vec{B}$

(50 marks)

(c) Nilaikan $\oint \vec{r} \bullet ds$ meliputi permukaan tertutup pada kiub yang bersempadan dengan $0 \le x \le 1$, $0 \le y \le 1$, dan $0 \le z \le 1$ di mana $\vec{r}$ adalah vektor kedudukan pada mana-mana titik pada permukaan kiub tersebut.

Evaluate $\oint \vec{r} \bullet ds$ *over the closed surface of the cube bounded by* $0 \le x \le 1$ *,* $0 \le y \le 1$*, and* $0 \le z \le 1$ *where* $\vec{r}$ *is the position vector of any point on the surface of the cube.*

(30 marks)

2. (a) Bermula dari hukum Coulomb, kaitkan kuasa diantara dua titik bercas dengan manitud cas dan jarak diantara mereka, bina persamaan-persamaan untuk ketumpatan medan elektrik, D dan kekuatan medan elektrik, E, pada jarak r meter dari satu cas yang bermagnitud $+q_1$ Coulomb.

Starting from Coulomb's Law, relating the force between two point charges to the magnitude of the charges and the distance between them, develop expressions for the electric field density, D and electric field strength, E, at a distance of r meter from a charge of magnitude $+q_1$ *Coulomb.*

(20 marks)

Kemudian, tunjukkan bahawa keupayaan mutlak pada jarak r meter dari satu titik bercas $+q_1$ Coulomb dalam ruang udara adalah diberikan oleh

Hence show that the absolute potential at a distance of r meter from a point charge of $+ q_1$ *Coulomb, in air is given by*

$$V = \frac{q_1}{4\pi\varepsilon_0 r}$$

disini ε_0 adalah ketelapan untuk ruang bebas.

here ε_0 is the permittivity of free space.

(30 marks)

(b) Tiga cas yang sama bernilai 200 nC diletakkan dalam ruang bebas pada (0, 0, 0), (2, 0, 0), dan (0, 2, 0). Dapatkan jumlah kuasa yang bertindak ke atas cas 500 nC pada (2, 2, 0).

Three equal charges of 200 nC are placed in free space at (0,0,0), (2,0,0), and (0,2,0). Determine the total force acting on a charge of 500 nC at (2,2,0).

(30 marks)

(c) Dua titik cas bernilai 20 nC dan -20 nC adalah diletakkan pada (1, 0, 0) dan (0, 1, 0) dalam ruang bebas. Dapatkan kekuatan medan elektrik pada (0,0,1).

Two point charges of 20 nC and -20 nC are situated at (1,0,0) and (0,1,0) in free space. Determine the electric field strength at (0,0,1).

(20 marks)

3. Satu panjang kabel berongga terdiri daripada satu konduktor dalaman yang berjejari ***a***, dan konduktor luarnya adalah berjejari ***b***, dengan konduktor dielektrik yang bukan magnetik. Tunjukkan dengan mengambil kira kuasa tersimpan di dalam tiub nipis berperingkat bahawa induktor pada konduktor dalaman adalah diberikan oleh

A length of coaxial cable consists of an inner conductor of radius ***a****, and an outer conductor of radius* ***b****, with a non-magnetic inner conductor dielectric. Show that by considering the stored energy in a thin incremental tube that the inductance of the inner conductor is given by*

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$$L^{'} = \frac{\mu_0}{8\pi} \text{ H/m}$$

(40 marks)

Tunjukkan juga bahawa induktor luaran hasil daripada medan magnetik di dalam dielektrik adalah diberikan oleh

Also, show that the external inductance, due to magnetic field inside the dielectric is given by

$$L^{'} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \text{ H/m}$$

(30 marks)

Satu panjang kabel berongga mempunyai konduktor dalaman yang berjejari 3 mm, konduktor luaran yang berjejari 5 cm dan satu dielektrik bukan magnetik dengan “permittivity” relatif bernilai 7. Kapasitan per unit panjang boleh diperolehi daripada:

A length of coaxial cable has an inner conductor of radius 3 mm, an outer conductor of radius 5 cm and a non-magnetic dielectric with a relative permittivity of 7. Given the capacitance per unit length can be found from:

$$C^{'} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \text{ F/m}$$

Dapatkan frekuensi resonan sendiri bagi satu kabel sepanjang 3 m.

Determine the self-resonant frequency of a 3 m length of cable.

(30 marks)

4. Satu gelombang mengembara boleh diwakili oleh voltan dan arus seperti berikut:

A travelling wave can be represented by the following voltage and current expressions:

$$V(x,t) = V_0 \exp(j\omega t)\exp(\gamma x)$$
$$I(x,t) = I_0 \exp(j\omega t)\exp(\gamma x)$$

Dapatkan satu persamaan untuk galangan bagi satu panjang kabel berongga, yang membawa gelombang mengembara dengan menggunakan persamaan

Find an expression for the impedance of a length of coaxial cable, carrying a travelling wave, by using relationship

$$\frac{dV}{dx} = RI + L\frac{dl}{dt}$$

(30 marks)

Satu panjang kabel berongga mempunyai nilai parameter seperti berikut:

A length of coaxial cable has the following lumped parameter values:

$R = 98\,m\Omega / m$ $\quad$ $L = 320\,nH / m$

$G = 1.5\,\mu S / m$ $\quad$ $C = 34.5\,nF / m$

Diberikan pemalar perambatan, γ sebagai

Given the propagation coefficient, γ is given by

$$\gamma^2 = (R + j\varpi L)(G + j\varpi C)$$

dapatkan pada frekuensi 27 MHz, yang berikut:

determine at a frequency of 27 MHz, the following:

(i) pemalar perambatan
the propagation coefficient

(ii) pemalar pelemahan
the attenuation coefficient

(iii) pemalar fasa
the phase coefficient

(iv) galangan kecirian
the characteristic impedance

(40 marks)

Kabel ini adalah sepanjang 30 m dan ditamatkan dengan betul supaya tiada pantulan. Dapatkan magnitud voltan pada hujung talian sekiranya kabel tersebut disambungkan kepada sumber 10V, 27 MHz. Sekiranya magnitud pada beban digandakan dua kali, dapatkan voltan baru pada beban.

This cable is 30 m long and correctly terminated so that there are no reflections. Determine the magnitude of the voltage at the end of the line if the cable is connected to a 10V, 27 MHz source. If the magnitude of the load is now doubled, determine the new voltage across the load.

(30 marks)

5. Satu satah gelombang elektromagnet mengembara di udara telah menuju secara normal di atas satu blok bahan. Terbitkan persamaan-persamaan untuk medan elektrik terpantul dan terhantar di dalam bentuk medan elektrik tertuju, E_i, galangan untuk ruang bebas, Z_0, dan galangan bahan, η.

A plane electromagnetic wave, travelling in air is normally incident on a block of material. Derive the expressions for the reflected and transmitted electric field in terms of the incident electric field, E_i, the impedance of free-space, Z_0, and the impedance of the material, η.

(40 marks)

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Gelombang ini mempunyai kekuatan medan elektrik sebanyak 100V/m dan frekuensinya 100 MHz. Ia adalah menuju secara normal di atas satu blok bahan penebat yang mempunyai kekonduksian $4 \times 10^{-4} S/m$ dan 'permittivity' relatif sebanyak 10. Dapatkan:

This wave has electric field strength of 100V/m and a frequency of 100 MHz. It is normally incident on a block of insulating material, which has a conductivity of $4 \times 10^{-4} S/m$ *and a relative permittivity of 10. Determine:*

(i) purata ketumpatan kuasa bagi gelombang terlanggar.
the average power density of the incident wave.

(ii) medan E yang terpantul dan terhantar.
the reflected and transmitted E fields.

(iii) purata ketumpatan kuasa gelombang tersebut di dalam bahan.
the average power density of the wave just inside the material.

(60 marks)

Nota: Galangan gelombang bagi bahan tersebut adalah diberikan oleh:

Note: The wave impedance of the material is given by:

$$\eta = \sqrt{\frac{j\varpi\mu}{\sigma + j\varpi\varepsilon}}$$

6. (a) Terangkan makna istilah “skin effect” di dalam konduktor membawa arus dan berikan komen secara ringkas mekanisma yang menghasilkannya.

Explain the meaning of the term “skin effect” in a current-carrying conductor and briefly comment on the mechanism that produces it.

(10 marks)

(b) Diberikan bahawa parameter perambatan γ, untuk bahan berkehilangan boleh didapati daripada

Given that the propagation parameter γ, for a lossy material can be found from

$$\gamma^2 = -\varpi^2 \mu\varepsilon + j\varpi\mu\sigma$$

Tunjukkan bahawa “skin-depth” di dalam konduktor yang baik ialah

Show that the skin-depth in a good conductor is given by

$$\delta = \sqrt{\frac{2}{\varpi\mu\sigma}}$$

Di sini simbol-simbolnya mempunyai maknanya seperti biasa

Here the symbols have their usual meaning.

(40 marks)

(c) Satu tempat pemprosesan makanan menggunakan pemanasan RF pada frekuensi 270 MHz untuk membakar biskut. Apabila basah,biskut-biskut ini mempunyai ‘permittiviti’ relative sebanyak 10 dan kekonduksian sebanyak $1x10^{-3}$ S/m, untuk membakar biskut-biskut ini dengan berkesan, ketebalannya sepatutnya adalah 0.1 kali “skin depth. Dapatkan ketebalan maksima untuk biskut tersebut.

A food processing plant uses RF heating, at a frequency of 270 MHz, to bake biscuits. When wet, these biscuits have a relative permittivity of 10 and a conductivity of $1x10^{-3}$ S/m, to efficiently bake the biscuits, their thickness should be at least 0.1 times the skin depth. Determine the maximum thickness of the biscuits.

(30 marks)

Apabila biskut-biskut tersebut mengering, “permittivity” relatif dan kekonduksian menurun masing-masing kepada 3 dan $1x10^{-5}$ S/m. Dapatkan “skin depth” yang baru apabila ianya kering.

As the biscuits dry out, their relative permittivity and conductivity reduce to 3 and $1x10^{-5}$ S/m respectively. Determine the new skin depth when they are dry.

(20 marks)

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Table 2-1: Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation, A =	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A, $\lvert A \rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product, A · B =	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product, A × B =	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume, $d\mathcal{V} =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Table 2-2: Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

CARTESIAN (RECTANGULAR) COORDINATES (x, y, z)

$$\nabla V = \hat{\mathbf{x}}\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla V = \hat{\mathbf{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\phi}}\frac{1}{r}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\frac{\partial V}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}}r & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Constants:

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m $\qquad \mu_0 = 4\pi \times 10^{-7}$ H/m

Equations:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla . \mathbf{D} = \rho$$

$$\nabla . \mathbf{B} = 0$$