

UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 1994 / 95

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IUK 291 - MATEMATIK II

Masa : 3 jam

Kertas ini mengandungi LIMA soalan. Jawab **SEMUA** soalan.

1. (a) Nilaikan

$$\iint_D (x^2 + xy) \, dx \, dy$$

$$D = \{ (x, y) \mid 0 \leq y \leq x, 0 \leq x \leq 1 \}$$

- (b) Dengan menggunakan koordinat kutub, nilaikan

$$\int_0^\infty \int_0^\infty e^{-x^2 - y^2} \, dx \, dy$$

(100/100)

2. (a) Jika $u = u(r)$ dan $r = (x^2 + y^2 + z^2)^{1/2}$.

tunjukkan

$$\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial u}{\partial z} \right)^2 = \left(\frac{du}{dr} \right)^2$$

- (b) Jika $u = F(x, t) = G(\xi, \eta)$, $\xi = x + ct$, $\eta = x - ct$

dan $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 1$, cari $\frac{\partial^2 u}{\partial \xi \partial \eta}$.

- (c) Cari semua titik pegun bagi fungsi

$$f(x, y) = 2x^3 - 6xy - 3y^2$$

dan tentukan cirinya.

(100/100)

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3. (a) Kembangkan fungsi

$$f(x) = x, \quad 0 < x < \pi$$

dalam (i) siri sinus, (ii) siri kosinus.

- (b) Selesaikan

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < 1, \quad 0 < y < 1$$

$$u(0, y) = u(1, y) = 0, \quad 0 < y < 1$$

$$u(x, 0) = 0, \quad 0 < x < 1$$

$$u(x, 1) = x(1 - x), \quad 0 < x < 1$$

(100/100)

4. Jika $0 < \alpha < 1$, cari siri Taylor dalam kuasa-kuasa x bagi fungsi

$$f(x) = (1 + x)^\alpha, \quad |x| < 1.$$

Jika baki diberi sebagai

$$R_{n+1}(x) = \frac{f^{(n+1)}(\theta x)}{n!} x^{n+1} (1 - \theta)^n, \quad 0 < \theta < 1$$

Cari $R_{n+1}(x)$ dan tunjukkan

$$\lim_{n \rightarrow \infty} R_{n+1}(x) = 0 \text{ jika } |x| < 1$$

(100/100)

5. Tunjukkan bahawa penyelesaian siri bagi persamaan $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0, x > 0$

boleh ditulis sebagai $y(x) = y_1(x) + y_2(x)$ dengan

$$y_1 = a_0 x^{1/2} \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{(2m+1)!}$$

$$y_2 = a_1 x^{-1/2} \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{(2m)!}$$

(100/100)

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