

**UNIVERSITI SAINS MALAYSIA**

**Peperiksaan Semester Pertama**

**Sidang Akademik 1998/99**

**Ogos/September 1998**

**IUK 106 - STATISTIK UNTUK TEKNOLOGI**

**Masa: [3 jam]**

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Sila pastikan bahawa kertas soalan ini mengandungi SEPULUH (10) mukasurat (termasuk 4 salinan Lampiran) yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab **SEMUA** soalan. Semua soalan mesti dijawab di dalam **Bahasa Malaysia**.

Bagi soalan objektif, jawapan hendaklah ditulis di dalam kotak yang disediakan. Bagi soalan di mana jalan kerja diperlukan, jawapan hendaklah ditulis di dalam ruangan yang disediakan.

1. (a) Cari varians untuk set data ini: 17.5, 10.5, 13.5, 30.7, 23.1, 12.0, 16.2, 29.1, 28.8, 10.7.

A. 64.54    B. 58.18    C. 64.4    D. 229.32    E. 60.87

(2 markah)

- (b) Mean untuk satu set data ialah 0.26, manakala sisisian piawainya ialah 2.68. Cari nilai-Z untuk data yang bernilai 4.45.

A. 1.72    B. 1.56    C. 1.41    D. 1.86    E. 1.09

(2 markah)

2. Sebuah pengilang hendak memeriksa bilangan produk cacat yang dikilanginya. Dari 169 keping produk yang dikeluarkan oleh mesin A, diketahui 50 adalah cacat, manakala daripada 137 keping produk yang dikeluarkan oleh mesin B, 37 adalah cacat. Untuk mesin C pula, 30 keping diketahui cacat dari 100 keping yang dibina. Sekiranya sekeping produk diambil secara rawak, cari kebarangkalian untuk mendapatkan produk yang dikeluarkan oleh mesin A ATAU produk yang tidak cacat.

A. 0.654    B. 1.128    C. 0.293    D. 0.704    E. 0.835

(5 markah)

3. Sebuah beg mengandungi 62 biji lampu. 10 daripada adalah rosak. Sekiranya 6 lampu diambil secara rawak (bagi setiap lampu yang diambil, ia diganti semula dengan lampu lain), hitung kebarangkalian yang semua lampu yang dipilih TIDAK rosak.

A. 0    B. 0.35    C. 0.839    D. 0.41    E. 0.577

(5 markah)

4. Sebuah kilang mengeluarkan mesin pengira. Setiap kelompok keluaran mengandungi 64 keping mesin pengira. Diketahui bahawa kadar kecacatan ialah 4%. Cari kebarangkalian terdapat 3 mesin pengira yang cacat dalam satu kelompok keluaran.

A. 0.091    B. 0.375    C. 0.139    D. 0.221    E. 0.769

(10 markah)

5. Bilangan rompakan dalam sebuah bandar ialah 324 dalam satu tahun yang tertentu. Gunakan taburan Poisson untuk mencari kebarangkalian yang pada satu hari yang dipilih secara rawak, bilangan rompakan ialah 2.

A. 0.2058    B. 0.1811    C. 0.1482    D. 0.1647    E. 0.4531

(10 markah)

6. (a) Sebuah populasi TIDAK bertaburan normal. Sekiranya meannya ialah 60.0 dan sisisian piawainya ialah 4.0, cari  $P(X < 53.0)$ .

(5 markah)

- A. 0.0802   B. 0.9599   C. 0.0401   D. 0.5589   E. 0.7896

- (b) Sejenis gred telur mempunyai berat sekurang-kurangnya 2 oz. Sekiranya berat telur-telur ini mempunyai taburan normal dengan mean 1.5 oz dan sisisian piawai 0.4 oz, berapa biji telurkah dari sampel rawak yang terdiri dari 22 dozen telur akan mempunyai berat lebih dari 2 oz?

(5 markah)

- A. 104   B. 28   C. 2   D. 1   E. 7

7. 63 orang pekerja lelaki dipilih secara rawak dari sebuah syarikat gergasi. Didapati bahawa mean pendapatan mereka ialah RM520 dengan sisisian piawai RM181. Seramai 65 orang pekerja wanita dipilih secara rawak dari syarikat yang sama dan didapati bahawa mean pendapatan mereka ialah RM480 dengan sisisian piawainya RM189. Anda hendak menguji tuntutan bahawa pekerja lelaki mempunyai pendapatan yang lebih tinggi dari pekerja wanita. Apakah angka statistik z yang berkaitan di sini?

(10 markah)

- A. 1.22   B. 2.39   C. 0.04   D. 16.64   E. 10.71

8. Sebuah kajian tentang hubungkait antara penggunaan *aras teknologi* dengan *gred* bagi sesebuah produk yang tertentu telah dijalankan. Data adalah seperti berikut (dalam unit bilangan produk).

<u>Aras Teknologi</u>	<u>Gred A</u>	<u>Gred B</u>	<u>Gred C</u>	<u>Gred D</u>	<u>Jumlah</u>
Sangat Tinggi	10	40	75	40	_____
Tinggi	5	30	80	60	_____
Sederhana	21	45	63	49	_____
Rendah	18	44	70	50	_____
<b>JUMLAH</b>	_____	_____	_____	_____	_____

- (a) Isikan tempat kosong dalam jadual di atas ini.

(5 markah)

- (b) Lakukan perhitungan yang sewajarnya, dan nyatakan nilai  $\chi^2$ .  
 (Tuliskan jawapan anda dalam kotak yang disediakan)

(5 markah)

- (c) Adakah data ini menunjukkan bahawa terdapat hubungkait antara gred produk dengan aras teknologi yang digunakan? Huraikan. (Gunakan alfa = 0.05).

(5 markah)

9. Sebuah syarikat yang mengeluarkan bahan binaan untuk tujuan lanskap menjual granit hancur dengan harga RM50 untuk setiap ton, dengan harga pengangkutan RM20 setiap ton. Sekiranya  $x$  mewakili jumlah granit hancur yang dijual (dalam unit ton), dan  $y$  mewakili jumlah kos (dalam unit ringgit).

- (a) Cari persamaan yang terbabit.

Jawapan =

(3 markah)

- (b) Cari  $b_1$  dan  $b_2$ .

$b_1 =$	$b_2 =$	(1 markah)
<input type="text"/>		

- (c) Tanpa pertolongan lakaran graf, tentukan sama ada garisan mencondong ke atas, ke bawah, atau melintang. Terangkan mengapa anda fikirkan sedemikian.  
(2 markah)

- (d) Bina jadual yang di dalamnya termasuk nilai- $x = 5$  dan nilai- $x = 10$ , serta nilai  $y$  yang terbabit.

X	Y

(2 markah)

- (e) Bina graf menggunakan data dari jadual yang anda bina ini. (2 markah)

- (f) Gunakan graf anda untuk membuat anggaran mata kasar untuk kos 7 ton granit hancur.

(2 markah)

Jawapan =

- (g) Gunakan persamaan yang anda dapati dari (a) di atas untuk membuat perhitungan tentang kos 7 ton granit.

(2 markah)

Jawapan =

- (h) Adakah terdapat perbezaan? Sekiranya YA atau TIDAK, terangkan mengapa.

(2 markah)

10. Sebuah kajian tentang jangka hayat 3 jenis pen hendak dilakukan oleh sebuah pengilang pen. Untuk setiap jenis pen, 4 batang pen diambil secara rawak. Jangka hayat pen itu, dalam unit 'jam penggunaan', diberikan di bawah ini:

<u>Jenis A</u>	<u>Jenis B</u>	<u>Jenis C</u>
231	254	176
254	225	255
179	164	196
213	214	260

Bina jadual ANOVA yang berkaitan (pada ruang yang disediakan), dan tentukan sama ada terdapat perbezaan dalam jangka hayat pen antara ketiga-tiga jenis pen ini. Gunakan aras ralat 0.05.

(15 markah)

## Test Statistics

$$= \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \quad \text{Mean—one population} \\ (\sigma \text{ known or } n > 30)$$

$$= \frac{\bar{x} - \mu}{s / \sqrt{n}} \quad \text{Mean—one population} \\ (\sigma \text{ unknown and } n \leq 30)$$

$$= \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \quad \text{Proportion—one population}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{Standard deviation or variance—one population}$$

$$= \frac{\bar{d} - \mu_d}{s_d / \sqrt{n}} \quad \text{Two means—dependent} \\ (\text{df} = n - 1)$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \quad \text{Two means—-independent} \\ (\sigma_1, \sigma_2 \text{ known or} \\ n_1 > 30 \text{ and } n_2 > 30)$$

$$F = \frac{s_1^2}{s_2^2} \quad \text{Standard deviation or variance—} \\ \text{two populations (where } s_1^2 \geq s_2^2)$$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad (\text{df} = \text{smaller of} \\ n_1 - 1, n_2 - 1)$$

Two means—Independent  
(reject  $\sigma_1^2 = \sigma_2^2$  and  $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad (\text{df} = n_1 + n_2 - 2)$$

where  $s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Two means—Independent (fail to reject  $\sigma_1^2 = \sigma_2^2$  and  
 $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}} \quad \text{Two proportions}$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} \quad \text{Multinomial} \\ (\text{df} = k - 1)$$

$$\chi^2 = \sum \frac{(O - E)^2}{E} \quad \text{Contingency table} \\ (\text{df} = (r - 1)(c - 1))$$

## Linear Correlation/Regression

$$\text{Correlation } r = \frac{n \Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{n(\Sigma x^2) - (\Sigma x)^2} \sqrt{n(\Sigma y^2) - (\Sigma y)^2}}$$

$$\hat{\beta}_1 = \frac{n \Sigma xy - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} \quad \text{or } \hat{\beta}_1 = r \frac{s_y}{s_x} \quad \text{Slope}$$

$$\hat{\beta}_0 = \frac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2} \quad \text{or } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x \quad \text{Estimated eq. of regression line}$$

$$r^2 = \frac{\text{explained variation}}{\text{total variation}}$$

$$s_e = \sqrt{\frac{\sum (y - \hat{y})^2}{n-2}} \quad \text{or } \sqrt{\frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 \Sigma xy}{n-2}}$$

$$\hat{y} = E < y < \hat{y} + E$$

$$\text{where } E = t_{\alpha/2} s_e \sqrt{1 + \frac{1}{n} + \frac{n(x_0 - \bar{x})^2}{n(\Sigma x^2) - (\Sigma x)^2}}$$

## One-Way Analysis of Variance

$$F = \frac{ns_p^2}{s_p^2} \quad k \text{ samples each of size } n \\ (\text{num. df} = k - 1; \text{den. df} = k(n - 1))$$

$$F = \frac{\text{MS(treatment)}}{\text{MS(error)}} \quad \leftarrow \text{df} = k - 1 \\ \leftarrow \text{df} = N - k$$

$$\text{MS(treatment)} = \frac{\text{SS(treatment)}}{k - 1}$$

$$\text{MS(error)} = \frac{\text{SS(error)}}{N - k} \quad \text{MS(total)} = \frac{\text{SS(total)}}{N - 1}$$

$$\text{SS(treatment)} = n_1(\bar{x}_1 - \bar{x})^2 + \dots + n_k(\bar{x}_k - \bar{x})^2$$

$$\text{SS(error)} = (n_1 - 1)s_1^2 + \dots + (n_k - 1)s_k^2$$

$$\text{SS(total)} = \sum (x - \bar{x})^2$$

$$\text{SS(total)} = \text{SS(treatment)} + \text{SS(error)}$$

## Two-Way Analysis of Variance

$$\text{Interaction: } F = \frac{\text{MS(interaction)}}{\text{MS(error)}}$$

$$\text{Row Factor: } F = \frac{\text{MS(row factor)}}{\text{MS(error)}}$$

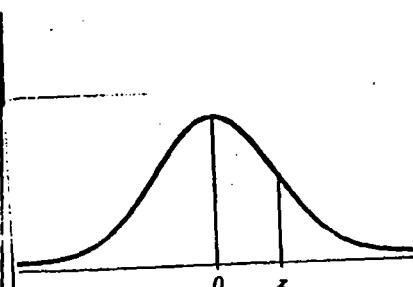


TABLE A-2 Standard Normal (z) Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429
1.6	.4452	.4463	.4474	.4484	.4495 *	.4505	.4515	.4525	.4535
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949 *	.4951
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990

z score Area

1.645 0.500

NOTE: For values of z above 3.09, use 0.4999 for the area.

\*Use these common values that result from interpolation.



Critical point. For example:  
 $F_{0.05}$  leaves 5% probability in the tail.

TABLE VI  $F$  Critical Points

DEGREES OF FREEDOM FOR NUMERATOR										
1	2	3	4	5	6	8	10	20	40	*
5.83	7.50	8.20	8.58	8.82	8.98	9.19	9.32	9.58	9.71	9.85
39.9	49.5	53.6	55.8	57.2	58.2	59.4	60.2	61.7	62.5	63.3
181	200	216	225	230	234	239	242	248	251	254
2.57	3.00	3.15	3.23	3.28	3.31	3.35	3.38	3.43	3.45	3.48
8.53	9.00	9.16	9.24	9.29	9.33	9.37	9.39	9.44	9.47	9.49
18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.5	19.5
98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.5	99.5
998	999	999	999	999	999	999	999	999	999	999
2.02	2.28	2.36	2.39	2.41	2.42	2.44	2.44	2.46	2.47	2.47
5.54	5.46	5.39	5.34	5.31	5.28	5.25	5.22	5.18	5.10	5.13
10.1	9.55	9.28	9.12	9.10	8.94	8.85	8.79	8.66	8.59	8.53
34.1	30.8	29.5	28.7	28.2	27.9	27.5	27.2	26.7	26.4	26.1
167	149	141	137	135	133	131	129	126	125	124
1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08
4.54	4.32	4.19	4.11	4.05	4.01	3.95	3.92	3.84	3.80	3.76
7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.96	5.80	5.72	5.63
21.2	18.0	16.7	16.0	15.5	15.2	14.8	14.5	14.0	13.7	13.5
74.1	61.3	56.2	53.4	51.7	50.5	49.0	48.1	46.1	45.1	44.1
1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.88	1.88	1.87	1.87
4.06	3.78	3.62	3.52	3.45	3.40	3.34	3.30	3.21	3.16	3.10
6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.74	4.56	4.48	4.36
16.3	13.3	12.1	11.4	11.0	10.7	10.3	10.1	9.85	9.29	9.02
47.2	37.1	33.2	31.1	29.8	28.8	27.6	26.9	25.4	24.6	23.8
1.62	1.76	1.78	1.79	1.79	1.78	1.77	1.77	1.76	1.75	1.74
3.78	3.46	3.29	3.18	3.11	3.05	2.96	2.94	2.84	2.78	2.72
5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.06	3.87	3.77	3.67
13.7	10.9	9.78	9.15	8.75	8.47	8.10	7.87	7.40	7.14	6.88
35.5	27.0	23.7	21.9	20.8	20.0	19.0	18.4	17.1	16.4	15.8
1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.69	1.67	1.66	1.65
3.59	3.26	3.07	2.96	2.88	2.03	2.75	2.70	2.59	2.54	2.47
5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.64	3.44	3.34	3.23
12.2	9.55	8.45	7.85	7.46	7.19	6.84	6.62	6.16	5.91	5.65
29.3	21.7	18.8	17.2	16.2	15.5	14.6	14.1	12.9	12.3	11.7
1.54	1.60	1.67	1.66	1.66	1.65	1.64	1.63	1.61	1.59	1.58
3.46	3.11	2.92	2.81	2.73	2.67	2.59	2.54	2.42	2.36	2.29
5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.35	3.15	3.04	2.93
11.3	8.65	7.59	7.01	6.63	6.37	6.03	5.81	5.36	5.12	4.86
25.4	18.5	15.0	14.4	13.5	12.9	12.0	11.5	10.5	9.92	9.33
1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.59	1.56	1.55	1.53
3.36	3.01	2.81	2.69	2.61	2.55	2.47	2.42	2.30	2.23	2.16
5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.14	2.94	2.83	2.71
10.6	8.02	6.99	6.42	6.06	5.80	5.47	5.26	4.81	4.57	4.31
22.9	16.4	13.9	12.6	11.7	11.1	10.4	9.89	9.00	8.37	7.61

DEGREES OF FREEDOM FOR NUMERATOR											
1	2	3	4	5	6	8	10	20	40	*	
$F_{0.05}$	1.49	1.60	1.60	1.59	1.59	1.58	1.56	1.55	1.52	1.51	1.48
$F_{0.01}$	3.28	2.92	2.73	2.61	2.52	2.46	2.38	2.32	2.20	2.13	2.06
$F_{0.001}$	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.98	2.77	2.66	2.54
$F_{0.0001}$	10.0	7.56	6.55	5.99	5.64	5.39	5.06	4.85	4.41	4.17	3.91
$F_{0.00001}$	21.0	14.9	12.6	11.3	10.5	9.92	9.20	8.75	7.80	7.30	6.76
$F_{0.05}$	1.56	1.56	1.55	1.54	1.53	1.51	1.50	1.47	1.45	1.42	1.40
$F_{0.01}$	3.18	2.81	2.61	2.48	2.39	2.33	2.24	2.19	2.06	1.99	1.90
$F_{0.001}$	4.75	3.89	3.49	3.26	3.11	3.00	2.85	2.75	2.54	2.43	2.30
$F_{0.0001}$	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.30	3.86	3.62	3.36
$F_{0.00001}$	18.6	13.0	10.8	9.63	8.89	8.38	7.71	7.29	6.40	5.93	5.42
$F_{0.05}$	1.44	1.53	1.53	1.52	1.51	1.50	1.48	1.46	1.45	1.41	1.38
$F_{0.01}$	3.10	2.73	2.52	2.39	2.31	2.24	2.15	2.10	1.96	1.89	1.80
$F_{0.001}$	4.60	3.74	3.34	3.11	2.96	2.65	2.70	2.60	2.39	2.27	2.13
$F_{0.0001}$	8.86	5.51	5.56	5.04	4.69	4.46	4.14	3.94	3.51	3.27	3.00
$F_{0.00001}$	17.1	11.8	9.73	8.62	7.92	7.43	6.80	6.40	5.56	5.10	4.60
$F_{0.05}$	1.42	1.51	1.51	1.50	1.48	1.48	1.46	1.45	1.40	1.37	1.34
$F_{0.01}$	3.05	2.67	2.46	2.33	2.24	2.18	2.09	2.03	1.89	1.81	1.72
$F_{0.001}$	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.49	2.28	2.15	2.01
$F_{0.0001}$	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.69	3.26	3.02	2.75
$F_{0.00001}$	16.1	11.0	9.00	7.94	7.27	6.81	6.19	5.81	4.99	4.54	4.06
$F_{0.05}$	1.40	1.49	1.48	1.46	1.45	1.44	1.42	1.40	1.36	1.33	1.29
$F_{0.01}$	2.97	2.59	2.38	2.25	2.16	2.09	2.00	1.94	1.79	1.71	1.61
$F_{0.001}$	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.35	2.12	1.99	1.84
$F_{0.0001}$	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.37	2.94	2.69	2.42
$F_{0.00001}$	14.8	9.95	8.10	7.10	6.46	6.02	5.44	5.08	4.29	3.86	3.38
$F_{0.05}$	1.38	1.45	1.44	1.42	1.41	1.41	1.39	1.37	1.35	1.30	1.27
$F_{0.01}$	2.88	2.49	2.28	2.14	2.05	1.98	1.88	1.82	1.67	1.57	1.46
$F_{0.001}$	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.16	1.93	1.79	1.62
$F_{0.0001}$	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.98	2.55	2.30	2.01
$F_{0.00001}$	13.3	8.77	7.05	6.12	5.53	5.12	4.58	4.24	3.49	3.07	2.59
$F_{0.05}$	1.36	1.44	1.42	1.40	1.39	1.37	1.35	1.32	1.30	1.25	1.21
$F_{0.01}$	2.84	2.44	2.23	2.09	2.00	1.93	1.83	1.76	1.61	1.51	1.38
$F_{0.001}$	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.08	1.84	1.69	1.51
$F_{0.0001}$	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.80	2.37	2.11	1.80
$F_{0.00001}$	12.6	8.25	6.60	5.70	5.13	4.73	4.21	3.87	3.15	2.73	2.23
$F_{0.05}$	1.35	1.42	1.41	1.38	1.37	1.35	1.32	1.30	1.25	1.21	1.15
$F_{0.01}$	2.79	2.39	2.16	2.04	1.95	1.87	1.77	1.71	1.54	1.44	1.29
$F_{0.001}$	4.00	3.15	2.76	2.53	2.37	2.25	2.10	1.99	1.75	1.59	1.39
$F_{0.0001}$	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.63	2.20	1.91	1.60
$F_{0.00001}$	12.0	7.76	6.17	5.31	4.76	4.37	3.87	3.54	2.83	2.41	1.89
$F_{0.05}$	1.34	1.40	1.39	1.37	1.35	1.33	1.30	1.28	1.22	1.18	1.10
$F_{0.01}$	2.75	2.35	2.13	1.99	1.90	1.82	1.72	1.65	1.48	1.37	1.19
$F_{0.001}$	3.92	3.07	2.68	2.45	2.29	2.17	2.02	1.91	1.66	1.50	1.25
$F_{0.0001}$	6.85	4.79	3.95	3.48	3.17	2.96	2.66	2.47	2.03	1.76	1.38
$F_{0.00001}$	11.4	7.32	5.79	4.95	4.42	4.04	3.55	3.24	2.53	2.11	1.54
$F_{0.05}$	1.32	1.39	1.37	1.35	1.33	1.31	1.28	1.25	1.19	1.14	1.00
$F_{0.01}$	2.71	2.30	2.08	1.94	1.85	1.77	1.67	1.60	1.42	1.30	1.00
$F_{0.001}$	3.84	3.00	2.60	2.37	2.21	2.10	1.94	1.83	1.57	1.39	1.00
$F_{0.0001}$	6.63	4.61	3.78	3.32	3.02	2.80	2.51	2.32	1.88	1.59	1.00
$F_{0.00001}$	10.8	6.91	5.42	4.62	4.10</td						

### Confidence Intervals (one population)

$$\bar{x} - E < \mu < \bar{x} + E \text{ Mean}$$

where  $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$  ( $\sigma$  known or  $n \geq 30$ )

$$\text{or } E = t_{\alpha/2} \frac{s}{\sqrt{n}} \text{ ( $\sigma$  unknown and } n \leq 30 \text{)}$$

$$\hat{p} - E < p < \hat{p} + E \text{ Proportion}$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\frac{(n-1)s^2}{(n-1)} < \sigma^2 < \frac{(n-1)s^2}{X_1^2} \text{ Variance}$$

### Sample Size

$$n = \left[ \frac{z_{\alpha/2} \sigma}{E} \right]^2 \text{ Mean}$$

$$n = \frac{[z_{\alpha/2}]^2 \cdot 0.25}{E^2} \text{ Proportion}$$

$$n = \frac{[z_{\alpha/2}]^2 \hat{p} \hat{q}}{E^2} \text{ Proportion}$$

( $\hat{p}$  and  $\hat{q}$  are known)

### Confidence Intervals (two populations)

$$\bar{d} - E < \mu_d < \bar{d} + E \text{ (Dependent)}$$

$$\text{where } E = t_{\alpha/2} \frac{s_d}{\sqrt{n}} \quad (\text{df} = n - 1)$$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E \text{ (Independent)}$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

( $\sigma_1, \sigma_2$  known or  $n_1 \geq 30$  and  $n_2 \geq 30$ )

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (\text{df} = \text{smaller of } n_1 - 1, n_2 - 1)$$

(reject  $\sigma_1^2 = \sigma_2^2$  and  $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (\text{df} = n_1 + n_2 - 2)$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

(fail to reject  $\sigma_1^2 = \sigma_2^2$  and  $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

mean

Mean (frequency table)

$$\bar{x} = \frac{\sum x_i}{n}$$

Standard deviation

$$\text{Standard deviation (shortcut)}$$

$$\text{Standard deviation (frequency table)}$$

$s^2$

$= P(A) + P(B)$  if  $A, B$  are mutually exclusive

$= P(A) + P(B) - P(A \text{ and } B)$

are not mutually exclusive

$= P(A) \cdot P(B)$  if  $A, B$  are independent

$= P(A) \cdot P(B|A)$  if  $A, B$  are dependent

$- P(A)$  Rule of complements

Permutations

Combinations

$$\begin{aligned} P(x) &\text{ Mean (prob. dist.)} \\ [P(x)] - \mu^2 &\text{ Standard deviation (prob. dist.)} \end{aligned}$$

$$\frac{n!}{x_1! x_2!} \cdot p^{x_1} \cdot q^{x_2} \text{ Binomial probability}$$

$x! x!$

Mean (binomial)

$\cdot q$  Variance (binomial)

$p \cdot q$  Standard deviation (binomial)

$\text{or } \frac{x - \mu}{\sigma}$  Standard score

Central limit theorem

Standard error

$$\sqrt{\frac{N-n}{N-1}} \text{ Standard error}$$

if  $n > 0.05N$

### Confidence Intervals (one population)

$$\bar{x} - E < \mu < \bar{x} + E \text{ Mean}$$

where  $E = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$  ( $\sigma$  known or  $n \geq 30$ )

$$\text{or } E = t_{\alpha/2} \frac{s}{\sqrt{n}} \text{ ( $\sigma$  unknown and } n \leq 30 \text{)}$$

$$\hat{p} - E < p < \hat{p} + E \text{ Proportion}$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\frac{(n-1)s^2}{(n-1)} < \sigma^2 < \frac{(n-1)s^2}{X_1^2} \text{ Variance}$$

### Sample Size

$$n = \left[ \frac{z_{\alpha/2} \sigma}{E} \right]^2 \text{ Mean}$$

$$n = \frac{[z_{\alpha/2}]^2 \cdot 0.25}{E^2} \text{ Proportion}$$

$$n = \frac{[z_{\alpha/2}]^2 \hat{p} \hat{q}}{E^2} \text{ Proportion}$$

( $\hat{p}$  and  $\hat{q}$  are known)

### Confidence Intervals (two populations)

$$\bar{d} - E < \mu_d < \bar{d} + E \text{ (Dependent)}$$

$$\text{where } E = t_{\alpha/2} \frac{s_d}{\sqrt{n}} \quad (\text{df} = n - 1)$$

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E \text{ (Independent)}$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

( $\sigma_1, \sigma_2$  known or  $n_1 \geq 30$  and  $n_2 \geq 30$ )

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (\text{df} = \text{smaller of } n_1 - 1, n_2 - 1)$$

(reject  $\sigma_1^2 = \sigma_2^2$  and  $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (\text{df} = n_1 + n_2 - 2)$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}$$

(fail to reject  $\sigma_1^2 = \sigma_2^2$  and  $n_1 \leq 30$  or  $n_2 \leq 30$ )

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

$$\text{where } E = z_{\alpha/2} \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$

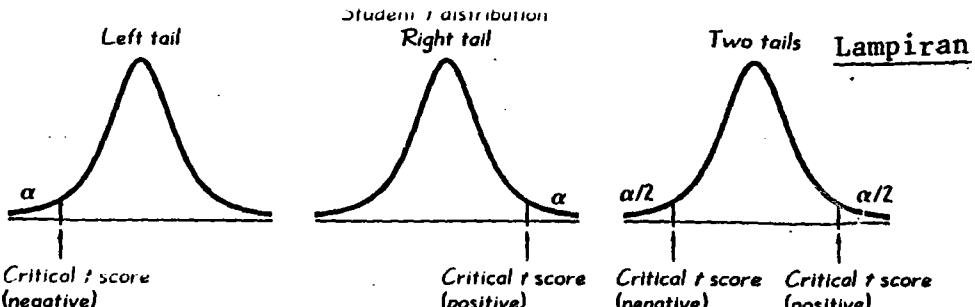


TABLE A-3  $t$  Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.2 (one tail) .5 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.878
3	5.841	4.541	3.182	2.353	1.638	.761
4	4.604	3.747	2.776	2.132	1.533	.707
5	4.032	3.365	2.571	2.015	1.476	.671
6	3.707	3.143	2.447	1.943	1.440	.631
7	3.500	2.998	2.365	1.895	1.415	.597
8	3.355	2.896	2.306	1.860	1.397	.561
9	3.250	2.821	2.262	1.833	1.383	.525
10	3.169	2.764	2.228	1.812	1.372	.489
11	3.106	2.718	2.201	1.796	1.363	.453
12	3.054	2.681	2.179	1.782	1.356	.417
13	3.012	2.650	2.160	1.771	1.350	.381
14	2.977	2.625	2.145	1.761	1.345	.345
15	2.947	2.602	2.132	1.753	1.341	.309
16	2.921	2.584	2.120	1.746	1.337	.273
17	2.898	2.567	2.110	1.740	1.333	.237
18	2.878	2.552	2.101	1.734	1.330	.201
19	2.861	2.540	2.093	1.729	1.328	.165
20	2.845	2.528	2.086	1.725	1.325	.129
21	2.831	2.518	2.080	1.721	1.323	.093
22	2.819	2.508	2.074	1.717	1.321	.057
23	2.807	2.500	2.069	1.714	1.320	.021
24	2.797	2.492	2.064	1.711	1.318	.016
25	2.787	2.485	2.060	1.708	1.316	.011
26	2.779	2.479	2.056	1.706	1.315	.006
27	2.771	2.473	2.052	1.703	1.314	.001
28	2.763	2.467	2.048	1.701	1.313	.000
29	2.756	2.462	2.045	1.699	1.311	.000
Large (z)	2.575	2.327	1.960	1.645	1.305	.000

TABLE A-4 Chi-Square ( $\chi^2$ ) Distribution

Area to the Right of the Critical Value

Degrees of freedom	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.001
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	10.828
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	11.345
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.833
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.833
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.919
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.510
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.090
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.666
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.209
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.090
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	27.000
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.090
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.090
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.090
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	33.090
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	33.090
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	33.090
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	33.090
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	33.090
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	33.090
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.090
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.090
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.090
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.090
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.090
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.090
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.090
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.090
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.090
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.090
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.090
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.090
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.090
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.090
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.090
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.090
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.090

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