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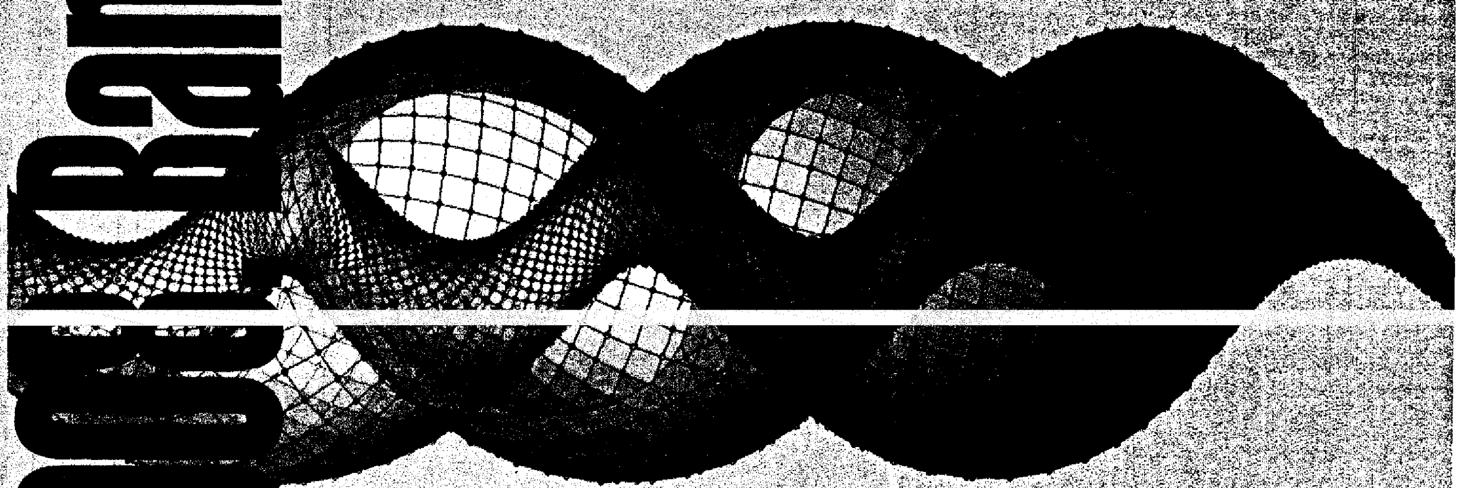

15 – 19 December, 2008
Suan Sunandha Rajabhat University
Bangkok, Thailand

problem in teaching and learning Mathematics will be discussed. It's also good for Cooperative learning approach.

All participants will get hand-outs and TI-Flash Debuggers software.

Abstract for 15211

EXAMPLES OF MATH ASSIGNMENTS USING MATLAB AND INTERNET

Authors: Sarwar J. Abbasi, Kahkashan Iqbal
Affiliations: Karachi Institute of Economics & Technology, Department of Mathematics, University of Karachi, Karachi, Pakistan

In 2007, we had concluded in [1] after studying a sample of 75 students' responses at under /and post graduate levels in Karachi, Pakistan that math teaching can be made interesting by dedication of the math teacher, by giving the students logical and concrete examples and by equipping students with know-how of math software and technology. In this paper, we extend our work by combining the last two effective [1] ways. That is, we examine some math assignments for creating good examples for students' learning of Mathematics while with the assistance of technology. These assignments were given to the engineering students of PAF-KIET in the course of "Vector Analysis and Multivariable Calculus" at undergraduate level in supervision of the second author. These examples include construction of polar flower, geometric designs by polar equations and graphing space curves traced by the position vector r using Matlab and plotting z = f (x, y) surfaces using live math software and internet.

Abstract for 15214

THE FOURIER SPECTRAL METHOD FOR THE SIVASHINSKY EQUATION

Authors: Abdur Rashid, Ahmad Izani Md. Ismail
Affiliations: University Sains Malaysia, Penang, Malaysia

Keywords: Sivashinsky equation, Fourier Spectral Method

In this paper, a Fourier spectral method for solving the Sivashinsky equation with periodic boundary conditions is developed. We establish semi-discrete and fully discrete schemes of the Fourier spectral method. A fully discrete scheme is constructed in such a way that the linear part is treated implicitly and the nonlinear part explicitly. We use an energy estimation method to obtain error estimates for the approximate solutions. We also perform some numerical experiments.

Abstract for 15225

THE IMPACT OF VIRTUAL MANIPULATIVES ON THIRD GRADERS – LEARNING OF PERIMETER AND AREA

Authors: Yuan Yuan
Affiliations: Chung Yuan Christian University

Keywords: virtual manipulatives, third grader, perimeter, area

The researcher developed a web-based virtual manipulative (Magic Board) and used it to design an instructional material for third grade teachers¡¦ teaching of perimeter and area unit. A pretest-posttest quasi-experimental design was used to examine the effectiveness of using this instructional material to introduce concepts of perimeter and area. The study involved 63 students in two different classes of an elementary school at Hsinchu city in Taiwan. The classes were randomly assigned to two methods of instruction; a virtual manipulative group and a traditional group. Equivalency of treatment groups was determined by independent t-test on pre-test scores. A posttest was conducted to measure immediate treatment effect. To measure retention, the posttest was used again two weeks after the posttest was first administered to students. For an in-depth comparison between the two groups, the classroom climate and interactions among students and teachers were also investigated.

The Fourier Spectral Method for the Sivashinsky Equation

Abdur Rashid* and Ahmad Izani Bin Md. Ismail†

Abstract

In this paper, a Fourier spectral method for solving the Sivashinsky equation with periodic boundary conditions is developed. We establish semi-discrete and fully discrete schemes of the Fourier spectral method. A fully discrete scheme is constructed in such a way that the linear part is treated implicitly and the nonlinear part explicitly. We use an energy estimation method to obtain error estimates for the approximate solutions. We also perform some numerical experiments.

Key words: Sivashinsky equation, Fourier spectral method.
2000 MR Subject Classification: 35Q35, 35B05, 58F39 76M22.

1 Introduction

Spectral methods provide a computational approach which has achieved substantial popularity over the last three decades. They have gained recognition for highly accurate computations of a wide class of physical problems in the field of computational fluid dynamics. Fourier spectral methods, in particular, have become increasingly popular for solving partial differential equations and they are also very useful in obtaining highly accurate solutions to partial differential equations [6, 7, 8].

The purpose of this paper is to develop a Fourier spectral spectral method for numerically solving the Sivashinsky equation with periodic boundary conditions. We consider the following nonlinear evolution equation in one space dimension,(see [1, 2, 3]).

$$\frac{\partial u}{\partial t} + \frac{\partial^4 u}{\partial x^4} + \frac{\partial}{\partial x}\left[(2-u)\frac{\partial u}{\partial x}\right] + \alpha u = 0, \tag{1.1}$$

where $\alpha > 0$ is constant.

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In particular we consider the mathematical modal of the Sivashinsky equation for the Fourier spectral approximation of the following initial-boundary value problem as follows:

$$\frac{\partial u}{\partial t}+\frac{\partial^4 u}{\partial x^4}+\alpha u = \frac{\partial^2 f(u)}{\partial x^2}, \qquad x\in\Omega, \qquad t\in(0,T], \tag{1.2}$$

$$\frac{\partial u}{\partial x}(-L,t) = \frac{\partial u}{\partial x}(L,t), \qquad t>0, \tag{1.3}$$

$$\frac{\partial^3 u}{\partial x^3}(-L,t) = \frac{\partial^3 u}{\partial x^3}(L,t), \qquad t>0, \tag{1.4}$$

$$u(x,0) = u_0(x), \qquad \in\Omega. \tag{1.5}$$

where $f(u)=\frac{1}{2}u^2-2u$, u_0 is a given function, $T>0$, and $\Omega=(-L,L)$.

The Sivashinsky equation arises in the modelling of directional solidification of a dilute binary alloy [1]. The dependent variable $u(x,t)$ is the location of the solid-liquid interface, and the equation may be formally derived asymptotically from the one-sided model for directional solidification in which diffusion of the solute in the solid is neglected and thermal conductivities, densities and specific heats of the liquid and solid are assumed to be equal.

Numerical methods for Sivashinsiky equation can be found in many references (see [4, 5] and the references listed therein). Omrani [5] studied the error estimates of semi discrete finite element method and fully discrete scheme based on the backward Galerkin scheme, a linearized backward Euler's method and Crank Nickoloson Galerkin scheme for the Sivashinsky equation. In [4] Omrani used the finite difference method for the approximate solution of the Sivashinsky equation. Cohen and Peletier [10] calculated the number of steady states for Sivashinsky equation . Daniel [9] reduced the Sivashinsky equation into ordinary differential equation and found the solution in spherical coordinates.

There have been some studies of the solution of the Sivashinsiky equation using numerical and approximate methods. However as far as we are aware, there are no studies of the solution of the Sivashinsky equation using the Fourier spectral method. In this paper, we develop a Fourier spectral method for the Sivashinsky equation (1.1). We establish their semi discrete and fully discrete schemes.

The layout of the paper is as follows: We introduce notation and lemmas in section 2, In section 3, we consider a semi discrete Fourier spectral approximation. In section 4, we consider a fully discrete implicit scheme and prove convergence to the solution of the associated continuous problem. In section 5, we perform some numerical experiments.

2 Notations and Lemmas

We denote by $(\cdot,\cdot)$ and $\|\cdot\|_0$ the inner product and the norm of $L^2(\Omega)$ defined by

$$(u,v)=\int_\Omega u(x)v(x)dx, \qquad \|u\|_0^2=(u,u),$$

where $\Omega = (0,1)$. Let N be a positive integer and V_N be the set of all trigonometric polynomials of degree at most N, that is

$$V_N = \text{span}\left\{ \frac{1}{\sqrt{2L}} e^{i\pi\ell.x/L} : -N/2 \le \ell \le N/2 \right\}.$$

The L^2 orthogonal projection operator $P_N : L^2(\Omega) \to V_N$ is a mapping such that for any $u \in L^2(\Omega)$,

$$(u - P_N u, v) = 0, \qquad \forall v \in V_N.$$

Let $L^\infty(\Omega)$ denote the Lebesgue space with norm $\|u\|_\infty = \text{ess sup}_{x\in\Omega}|u(x)|$ and $H_p^m(\Omega)$ denote the periodic Sobolev space with the norm $\|u\|_m = \left(\sum_{|\ell|\le m} \|D^\ell u\|_0^2\right)^{1/2}$. Denote

$$L^2\left(0,T; H_p^m(\Omega)\right) = \left\{ u(x,t) \in H_p^m(\Omega); \int_0^T \|u\|_m^2 dt < +\infty \right\},$$

$$L^\infty\left(0,T; H_p^m(\Omega)\right) = \left\{ u(x,t) \in H_p^m(\Omega); \sup_{0\le t\le T} \|u\|_m^2 < +\infty \right\}.$$

Lemma 1. *[6] For any real $0 \le \mu \le \sigma$, and $u \in H_p^\sigma(\Omega)$, then*

$$\|u - P_N u\|_\mu \le cN^{\mu-\sigma}|u|_\sigma,$$

For the discretization in time t, let τ be the mesh spacing of the variable t and we set

$$S_\tau = \left\{ t = k\tau : 1 \le k \le \left[\frac{T}{t}\right] \right\},$$

For simplicity $u(x,t)$ is denoted by $u(t)$ or u usually. We define the following difference quotient:

$$u_{\hat{t}}(t) = \frac{1}{2\tau}[u(t+\tau) - u(t-\tau)], \qquad \widehat{u}(t) = \frac{1}{2}[u(t+\tau) + u(t-\tau)].$$

3 Semi-discrete approximation

Now we construct an approximate solution for the periodic initial value problem (1.2)-(1.5) as follows: Find $u_N(t) \in V_N$, such that

$$\begin{cases} \left(\dfrac{\partial u_N}{\partial t}, w_N\right) + \alpha(u_N, w_N) + \left(\dfrac{\partial^2 u_N}{\partial x^2}, \dfrac{\partial^2 w_N}{\partial x^2}\right) = \left(f(u_N), \dfrac{\partial^2 w_N}{\partial x^2}\right), \quad \forall w_N \in V_N \\ u_N(x,0) = P_N u(0) \end{cases} \tag{3.1}$$

Now we estimate the error $\|u(t) - u_N(t)\|_0$. Denote $e_N = P_N u(t) - u_N(t)$, we have from (1.2) and (3.1),

$$\begin{cases} \left(\dfrac{\partial e_N}{\partial t}, w_N\right) + \alpha(e_N, w_N) + \left(\dfrac{\partial^2 e_N}{\partial x^2}, \dfrac{\partial^2 w_N}{\partial x^2}\right) = \left(f(u) - f(u_N), \dfrac{\partial^2 w_N}{\partial x^2}\right), \quad \forall w_N \in V_N \\ u_N(x,0) = P_N u(0) \end{cases} \tag{3.2}$$

Choosing $w_N = e_N$ in (3.2) and by applying the Cauchy-Schwartz inequality as well as the algebraic inequality, we obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\|e_N\|_0^2 + \left\|\frac{\partial^2 e_N}{\partial x^2}\right\|_0^2 + \alpha\|e_N\|_0^2 &= \left(f(u) - f(u_N), \frac{\partial^2 e_N}{\partial x^2}\right) \\ &\leq \|(f(u) - f(u_N)\|_0 \left\|\frac{\partial^2 e_N}{\partial x^2}\right\|_0 \\ &\leq \|(f(u) - f(u_N)\|_0^2 + \left\|\frac{\partial^2 e_N}{\partial x^2}\right\|_0^2 .\end{aligned} \tag{3.3}$$

But

$$(f(u) - f(u_N) = \phi\prime(u_N - \theta(u - u_N))(u - u_N), \qquad 0 < \theta < 1.$$

Since

$$\|u\|_\infty \leq c, \qquad \|u_N\|_\infty \leq c,$$

we have

$$\phi\prime(u_N - \theta(u - u_N)) \leq c,$$

Thus

$$\begin{aligned}\|f(u) - f(u_N)\|_0 &\leq \|(u - u_N)\|_0 \\ &\leq c\left(\|(u - u_N)\|_0 + \|e_N\|_0\right).\end{aligned} \tag{3.4}$$

Substituting the value of (3.3) and (3.4) in (3.2), we obtain

$$\frac{d}{dt}\|e_N\|_0^2 \leq c_1\|e_N\|_0^2 + c_2\|(u - P_N u)\|_0^2 \tag{3.5}$$

where c_1 and c_2 are constants independent of N. By Gronwall's lemma, we have

$$\|e_N\|_0^2 \leq \|e_N(0)\|_0^2 + c\int_0^t \|(u(s) - P_N u(s))\|_0^2 ds \tag{3.6}$$

Thus we obtain the following theorem

Theorem 1. *Let $u_0 \in H_0^1(\Omega)$ and $u(t)$ be the solution for periodic initial value problem (1.2-1.5). If $u_N(t)$ is the solution of the semi-discrete approximation (3.1), then there exists a constant c independent of N, such that*

$$\|u(t) - u_N(t)\|_0 \leq cN^{-m}\left(\|u(t)\|_m + \left(\int_0^t \|u(s)\|_m^2 ds\right)^{1/2}\right).$$

4 A fully discrete scheme

In this section we give a fully discrete scheme of the Fourier spectral method for Sivashinsky equation. Using the modified leap-frog scheme such that the linear part is treated implicitly and the nonlinear part explicitly, we obtain the fully discrete spectral scheme for solving (1.1): find $u_N(t) \in S_N$, such that

$$\begin{cases} (u_{N\hat{t}}, w_N) + \alpha(\widehat{u}_N, w_N) + \left(\dfrac{\partial^2 \widehat{u}_N}{\partial x^2}, \dfrac{\partial^2 w_N}{\partial x^2}\right) = \left(f(u_N), \dfrac{\partial^2 w_N}{\partial x^2}\right), \quad \forall w_N \in V_N \\ u_N(x,0) = P_N u(0), \qquad u_N(x,\tau) = P_N\left(u(0) + \tau \dfrac{\partial u}{\partial t}(0)\right). \end{cases} \tag{4.1}$$

Let

$$e = u - u_N = (u - P_N u) + (P_N u - u_N) = \xi + \psi,$$

where

$$\xi = u - P_N u, \qquad \psi = P_N u - u_N,$$

Subtracting (1.2) from (4.1), we obtain

$$\begin{cases} (\psi_{\hat{t}}, w_N) + \alpha\left(\widehat{\psi}, w_N\right) + \left(\dfrac{\partial^2 \widehat{\psi}}{\partial x^2}, \dfrac{\partial^2 w_N}{\partial x^2}\right) = \left(f(u_N) - f(u), \dfrac{\partial^2 w_N}{\partial x^2}\right) + \left(\dfrac{\partial u}{\partial t} - P_N u_{\hat{t}}, w\right) \\ \qquad + \alpha(u - P_N\widehat{u}, w_N) + \left(\dfrac{\partial^2}{\partial x^2}(u - P_N \widehat{u}), \dfrac{\partial^2}{\partial x^2} w\right), \quad \forall w_N \in V_N \\ \psi(0) = P_N u(0) - u_N(0), \qquad \psi(\tau) = P_N\left(u(\tau) - \left(u(o) + \tau \dfrac{\partial u}{\partial t}(0)\right)\right). \end{cases} \tag{4.2}$$

Taking $w = 2\widehat{\psi}(t)$ in (4.2), we obtain

$$\begin{aligned} (\|\psi(t)\|_0^2)_{\hat{t}} + 2\alpha \left\|\widehat{\psi}(t)\right\|_0^2 \le c\Bigg(&\|f(\widehat{u}_N) - f(u)\|_0^2 + \left\|\frac{\partial u}{\partial t} - P_N u_{\hat{t}}(t)\right\|_0^2 \\ &+ \|u - P_N \widehat{u}\|_0^2 + \left\|\frac{\partial^2}{\partial x^2}(u - P_N \widehat{u}\right\|_0^2\Bigg). \end{aligned} \tag{4.3}$$

We estimate the right hand side of (4.3)

$$\|f(u_N) - f(u)\|_0 \le c(\|u - u_N\|_0) \le c(\|\xi\|_0 + \|\psi\|_0) \tag{4.4}$$

The second term on the right hand side of equation (4.3) can be estimated as below:

$$\left\|\frac{\partial u}{\partial t}(t) - P_N u_{\hat{t}}(t)\right\|_0^2 \le \left\|\frac{\partial u}{\partial t}(t) - u_{\hat{t}}(t)\right\|_0^2 + \|u_{\hat{t}}(t) - P_N u_{\hat{t}}(t)\|_0^2, \tag{4.5}$$

By using Taylor's Theorem with integral remainder

$$\left\|\frac{\partial u}{\partial t}(t) - u_{\hat{t}}(t)\right\|_0^2 \le c\tau^{3/2}\left(\int_{t-\tau}^{t+\tau} \left\|\frac{\partial^3 u}{\partial t^3}(s)\right\|_0^2 ds\right)^{1/2},$$

$$\begin{aligned} \|u_{\hat{t}}(t) - P_N u_{\hat{t}}(t)\|_0^2 &= \frac{1}{2\tau}\|(u(t+\tau) - P_N u(t+\tau)) - (u(t-\tau) - P_N u(t-\tau))\|_0 \\ &= \frac{1}{2\tau}\|\xi(t+\tau) - \xi(t-\tau)\|_0. \end{aligned}$$

By using Taylor's Theorem with integral remainder

$$\frac{1}{2\tau}\|\xi(t+\tau)-\xi(t-\tau)\|_0 \leq \frac{c}{\tau}\left(\int_{t-\tau}^{t+\tau}\left\|\frac{\partial \xi}{\partial t}(s)\right\|_0^2 ds\right)^{1/2}$$

Substituting the above estimate in to (4.5), we obtained

$$\left\|\frac{\partial u}{\partial t}(t) - P_N u_{\hat{t}}(t)\right\|_0 \leq \frac{c}{\tau}\left(\int_{t-\tau}^{t+\tau}\left\|\frac{\partial \xi}{\partial t}(s)\right\|_0^2 ds\right)^{1/2} + c\tau^{3/2}\left(\int_{t-\tau}^{t+\tau}\left\|\frac{\partial^3 u}{\partial t^3}(s)\right\|_0^2 ds\right)^{1/2} \quad (4.6)$$

The third and fourth term on the right hand side of (4.3) can be estimated in a similar manner

$$\|u - P_N\widehat{u}\|_0 \leq \frac{1}{2}(\|\xi(t+\tau)-\xi(t-\tau)\|_0) + \left\|u(t) - \frac{1}{2}u(t+\tau) - \frac{1}{2}u(t-\tau)\right\|_0$$

$$\|u - P_N\widehat{u}\|_0 \leq \frac{1}{2}(\|\xi(t+\tau)-\xi(t-\tau)\|_0) + c\tau^{3/2}\left(\int_{t-\tau}^{t+\tau}\left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_0^2 ds\right)^{1/2} \quad (4.7)$$

and

$$\left\|\frac{\partial^2}{\partial x^2}(u - P_N\widehat{u})\right\|_0 \leq \frac{1}{2}(\|\xi(t+\tau)-\xi(t-\tau)\|_2) + c\tau^{3/2}\left(\int_{t-\tau}^{t+\tau}\left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_2^2 ds\right)^{1/2} \quad (4.8)$$

By applying Lemma 1, we obtain

$$\|\widehat{\xi}(t)\| \leq cN^{-m}\left(\|u(t+\tau)+u(t-\tau)\|\right)$$

Putting the above estimates into (4.4), (4.6), (4.7) and (4.8) in to (4.3), we obtain

$$\begin{aligned}(\|\psi(t)\|_0^2)_{\hat{t}} + 2\alpha\left\|\widehat{\psi}(t)\right\|_0^2 &\leq cN^{-2m}(\|u(t+\tau)\|_0^2 + \|u(t-\tau)\|_0^2) + c\|\widehat{\psi}(t)\|_0^2 \\ &+\frac{c}{\tau}\int_{t-\tau}^{t+\tau}\left\|\frac{\partial \xi}{\partial t}(s)\right\|_0^2 ds + cN^{4-2m}\left(\|u(t+\tau)\|_2^2 + \|u(t-\tau)\|_2^2\right) \\ &+c\tau^3\left(\int_{t-\tau}^{t+\tau}\left(\left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_0^2 + \left\|\frac{\partial^3 u}{\partial t^3}(s)\right\|_0^2 + \left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_2^2\right) ds\right).\end{aligned} \quad (4.9)$$

By summing up (4.9) for all $t \in S_\tau$ and $t' \leq t-\tau$, we obtain

$$\begin{aligned}\|\psi(t)\|_0^2 \leq \|\psi(0)\|_0^2 + \|\psi(\tau)\|_0^2 &+ c\tau N^{4-2m}\left(\sum_{t'=\tau}^{t-\tau}\|\widehat{u}(t')\|_2^2\right) + c\int_0^T\left\|\frac{\partial \xi}{\partial t}(s)\right\|_0^2 ds \\ &+c\tau^4\left(\int_0^T\left(\left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_0^2 + \left\|\frac{\partial^3 u}{\partial t^3}(s)\right\|_0^2 + \left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_2^2\right) ds\right).\end{aligned} \quad (4.10)$$

and

$$\|\psi(\tau)\|_0^2 \leq c\tau^4\|u\|_{C^2(0,T;L^2)}^2 \qquad \|\psi(0)\|_0^2 \leq cN^{-2m}\|u(0)\|_m^2$$

By applying Gronwall's Lemma, we have the following theorem

Theorem 2. *Assume τ is sufficiently small, $m > 2$ and the solution of (1.2)-(1.5) satisfies* $u \in L^2(0,T;H^m(\Omega)) \bigcap C^2(0,T;L^2(\Omega))$, $\frac{\partial u}{\partial t} \in L^2(0,T;L^2(\Omega))$, $\frac{\partial^2 u}{\partial t^2} \in L^2(0,T;L^2(\Omega)) \bigcap H^2(0,T;L^2(\Omega))$, $\frac{\partial^3 u}{\partial t^3} \in L^2(0,T;L^2(\Omega))$ *where u_N is the solution of (4.1). Then there exists a constant c, independent of τ and N such that*

$$\|u(t) - u_N(t)\|_0 \le c\left(N^{-m}K_1 + \tau^2 K_2\right)$$

where

$$K_1 = \left(\|u(0)\|_m^2 + \sum_{t'=\tau}^{t-\tau} \|\widehat{u}(t')\|_2^2\right)^{1/2}$$

$$K_2 = \left(\int_0^T \left(\left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_0^2 + \left\|\frac{\partial^3 u}{\partial t^3}(s)\right\|_0^2 + \left\|\frac{\partial^2 u}{\partial t^2}(s)\right\|_2^2\right) ds\right)^{1/2}$$

5 Numerical Results

In this section, we present some numerical results of our scheme for Sivashinsky equation (1.2)-(1.5). The numerical computations were carried out in MATLAB. We employ the standard spectral method (discrete Fourier transform) for the spatial discretization of the equation, while for the time integration, we use modified leapfrog scheme in such a way that linear terms are treated implicitly and the nonlinear terms explicitly.

We work on the periodic domain $\Omega = (-L, L)$. First we introduce some notation for the discrete Fourier transformation. We assume that the domain in space is equidiscretized with spacing $h = \frac{2\pi}{N}$, where the integer N is even. The spatial grid points are given by $x_j = L(jh - \pi)/\pi$ of the domain Ω with $j = 0, 1, ..., N$. The formula for discrete Fourier transform is

$$\widehat{u}_k = \frac{1}{N}\sum_{j=0}^{N} u_j e^{-ikx_j}, \qquad k = -N/2 + 1.., -1, 0, 1, ..., N/2, \tag{5.1}$$

and the inverse discrete Fourier transform is given by

$$u_j = \sum_{-N/2+1}^{N/2} \widehat{u}_k e^{ix_j}, \qquad j = 0, 1, ..., N \tag{5.2}$$

We set the parameters to be $N = 32$, $T = 1.0$ and the step length along time direction is taken to be $\tau = 0.001$. We consider the Sivashinsky equation (1.2) with initial condition

$$u(x, 0) = cos\left(\frac{x}{2}\right).$$

The exact solution of the equation (1.2)-(1.5) is unknown. Therefore we present the graph of the approximate solution in Figure 1, Figure 2, and in Figure 3 with different values of α. Momani [11] has used Adomain's decomposition method for the same problem with $\alpha = 0.5$ and the results indicate that the wave solution bifurcate into three waves. However the Fourier spectral method, we have delvoped does not indicate this (see Figure 2). Otherwise there is good agrement between the results of Fourier spectral method and the Adomain's decomposition method.

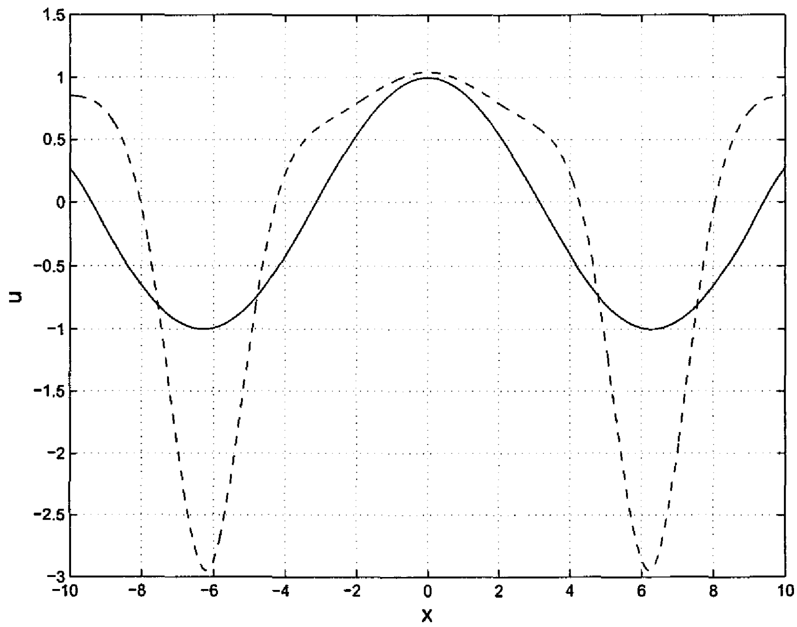

Figure 1: The graph of the approximate solution at t=0 (solid line) and t=1 (dotted line) at α=0.1

6 Conclusion

A Fourier spectral method for the one dimensional Sivashinsky equation has been proposed and the solution obtained for different values of the parameter α. Error estimates for semi-discrete and fully discrete schemes are obtained by the energy estimation method. Thus, the proposed method is an efficient technique for the numerical solutions of partial differential equations.

References

[1] G.I. Sivashinsky, On cellular instability in the solidification of a dilute binary alloy, North-Holland, Physica D, 8 (1983) 243-248.

[2] V.G. Gertsberg and G.I. Sivashinsky, Prog. Theo. Phys. 66 (1981), 1219-

[3] D.J. Wollkind and L.A. Sigel, Phil. Trans. Roy. Society (London) 268a (1970) 351-

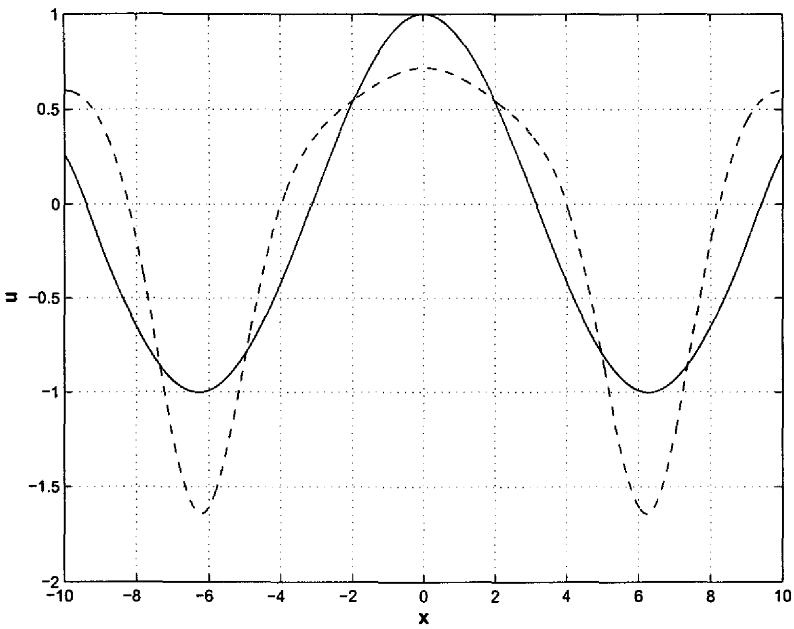

Figure 2: The graph of the approximate solution at t=0 (solid line) and t=1 (dotted line) at α=0.5

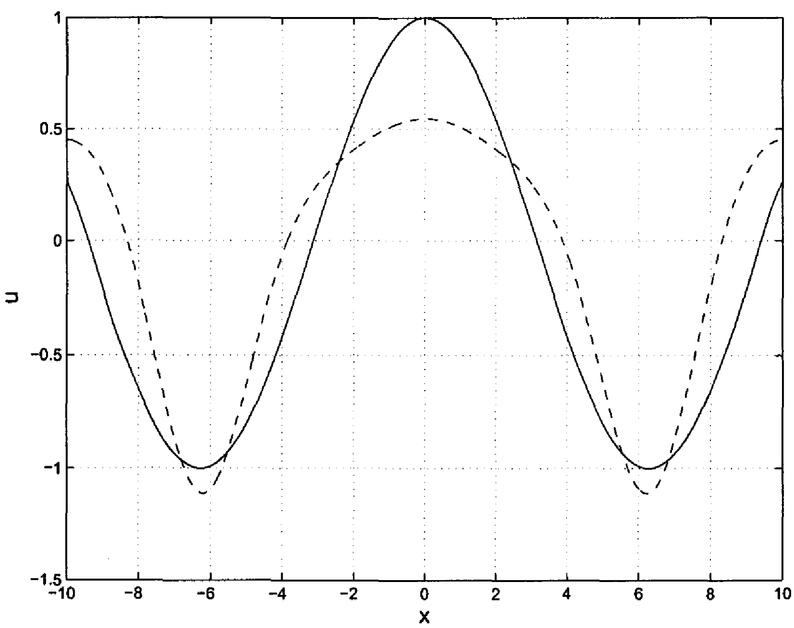

Figure 3: The graph of the approximate solution at t=0 (solid line) and t=1 (dotted line) at α=0.8

[4] K. Omrani, A second order spliting method for a finite difference scheme for the Sivashinsky equation, Appl. Math. Lett. 16 (2003) 441-445

[5] K. Omrani, Numerical methods and error analysis for the nonlinear Sivashinsky equation, Appl. Math. Comp. 189 (2007) 949-962.

[6] C. Canuto, M. Y. Hussaini, A. Quarteroni, T. A. Zang, Spectral methods (fundamental in single domains) Springer-Verlag, Berlin, 2006.

[7] R. Peyret, Spectral methods in incompressible viscous flow, Springer-Verlag, New York, 2002.

[8] J.P. Boyd, Chebyshev and Fourier spectral methods, Springer-Verlag, New York, 2002.

[9] M. Daniel, Elementary particles as solutions of the Sivashinsky equation, Non. Ph. Physica D, 44(1990) 502-556

[10] A.N. Cohen, L.A. Peletier, The steady state of one dimensional Sivashinsky equations, Appl.Math.Lett, 4 (1991) 69-71

[11] S. Momani, A numerical scheme for the solution of Sivashinsky equation, Appl. Math. Comp. 168 (2005) 1273-1280