The Beneficial Effects of Principal-Agent Problem

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ABSTRACT

This paper focus on a principal-agent problem in which the agent first learns an estimate of his production costs, then he has to decide whether to sink a fixed entry investment, and finally, if he has invested, the principal offers a price on a take-it-or-leave-it basis. If the cost estimate is precise, we show that, in equilibrium, the agent will never invest, because then the investment itself signals low costs, which in turn will make the principal propose a lower price. If this hold-up problem is resolved when the cost estimate is totally uninformative, then improving the quality of the cost estimate always implies (weakly) reduced welfare. If, on the other hand, the agent will not invest if the signal is useless, then improving the estimate quality may be good, because it can lead low-estimate types of the agent to invest. Then welfare is maximised for an intermediary level of estimate quality.

I. INTRODUCTION

The world is flooded with potential hold-up problems, i.e., situations in which socially beneficial investments are not undertaken because the investing party has to pay all the investment costs but receives only a fraction of the returns. Fundamentally, the origin of the hold-up problem is the deterioration of the investor's bargaining position once the investment is sunk: Virtually any ex ante division of the net gain from trade would suffice to make the investing party willing to invest, as long as this division prevails after the investment has been made. Therefore, measures to alleviate hold-up problems can be interpreted as measures to strengthen the investing party's bargaining position after he has invested.

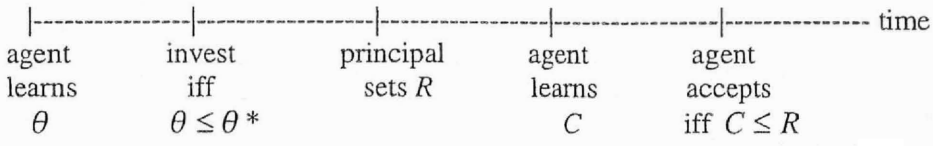
Lots of potential hold-up problems can be resolved by writing contracts before the investment decision is made. Obviously, if complete contracts can be written and enforced, the hold-up problem is easily eliminated. Less obviously, also incomplete contracts (implicit contracts included) may often make the hold-up problem vanish, by moving the parties' threat points and thereby altering their bargaining positions compared to a situation with no contract at all.

This paper will investigate the role of private information to alleviate hold-up problems. The basic idea is that if the investing party has some payoff-relevant private information, this can give rise to an information rent, which may then suffice to cover the investment cost. To highlight the power of private information to resolve hold-up problems, we start from the worst possible situation: one party – the agent – make the investment, while the other party – the principal – dictates the terms of trade. Moreover, we assume that no contract can be written before the investment is undertaken. Consequently, the agent's only source of rent is his private information about production costs. If the expected information rent exceeds the investment cost, the agent will invest.

II. THE MODEL

There is a principal who wants an indivisible task to be carried out by an agent. The timing of events is shown in Figure 1.

Figure 1. The timing of events



First the agent learns an estimate θ of his production costs. Then, possibly depending on the estimate, he decides whether to sink the entry investment $I>0$ or not. When the estimate has some information value, the agent's expected returns from the investment will be (at least weakly) decreasing in θ. Characterise the agent's investment strategy by a cut-off value θ^*, with the property that the firm will invest if and only if $\theta \leq \theta^*$.

If the principal finds that the agent has invested, she will offer a price R on a take-it-or-leave-it basis. Finally, the agent learns his true production costs and accepts the proposed price if and only if it covers the production costs. The principal's payoff W and the agent's payoff U are given by

$$W = \begin{cases} S - R & ,if\ production \\ 0 & ,otherwise \end{cases}$$

$$U = \begin{cases} R-C-I, & if \quad production \\ -I, & if \quad investment\ but\ no\ production \\ 0, & otherwise \end{cases}$$

where I is investment, C is production cost and S is the principal's valuation of having the project carried out and both parties reservation utilities are normalised to zero.

To facilitate the analysis, choose the following technology and information structure. First, the estimate is drawn from a uniform distribution on [0,1]. With probability π the estimate is correct (i.e., it equals the agent's true cost). With the complementary probability $(1-\pi)$ the estimate is pure noise, in which case the true production cost is redrawn from the same uniform. The parameter π is assumed to be common knowledge and will be called the quality of the cost estimate. At this stage the agent does not learn whether the estimate is good or noisy.

Finally, the estimate and the true cost are observed by the agent only (while the technology and the information structure are common knowledge). Note that there are only three exogenous parameters in this model – S, I and π.

Clearly, if the principal believes that all types of the agent will invest (i.e., $\theta^* = 1$), then she will just face the original uniform distribution of costs. More interesting is the case in which she thinks that different estimates yield different investment decisions, in particular, if she believes that the agent's investment strategy is given by θ^*. Then, with probability π the agent had accurate cost information at the investment stage. Therefore the cost lies in the interval $[0,\theta^*]$. Moreover, since the original distribution was uniform, so is the truncated distribution. With probability $(1-\pi)$

however, the agent's information at the investment stage was just noise. In this case the agent's cost is uniformly distributed on [0,1]. Combining this information will find that the distribution of production costs for an agent that has invested can be described by the following density function:

$$f(C) = \begin{cases} \dfrac{\pi}{\theta^*} + 1 - \pi & if \quad C \le \theta^* \\ 1 - \pi & otherwise \end{cases} \tag{1}$$

Figure 2. Cost density faced by the principal

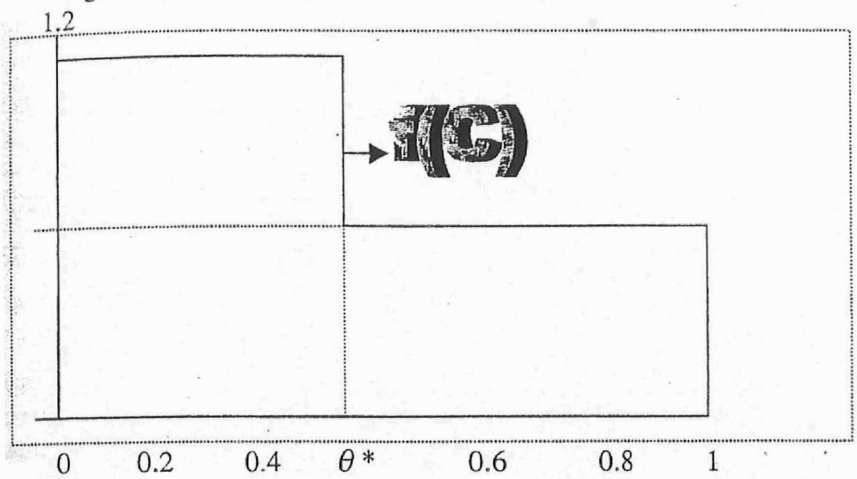


Figure 2 shows an example of such a density function, where $\pi = 1/3$ and $\theta^* = 1/2$. Note that the investment transforms the initial uniform distribution by putting more weight on costs lower than the cut-off point and less weight on costs higher than this point. Integrating f yields the following cumulative distribution function:

$$F(C) = \begin{cases} \left[\dfrac{\pi}{\theta^*} + 1 - \pi\right] C & if \quad C \le \theta^* \\ \pi + (1-\pi)C & otherwise \end{cases} \tag{2}$$

III. EQUILIBRIUM OF THE BASIC MODEL

There are two types of equilibrium to consider – one in which the hold-up problem is completely resolved and - one in which it is not.

Complete Resolution of The Hold-Up Problem

If the principal believes that the agent has chosen strategy $\theta^* = 1$ (i.e., always invest), then the cost distribution he is facing is the original uniform distribution on [0,1]. She seeks to maximise her expected utility, which can be written as follows:

$$W = \int_0^R (S - R)dC = (S - R)R \tag{3}$$

with respect to the price R. Equation (3) clearly illustrates the principal's dilemma: Reducing t price increases her surplus $S - R$ from production but decreases the probability R that the agent w accept the offered price. Let $R_1 \in Arg\max_R\{(S-R)R\}$ denote the value of R that maximises (3 Straightforward maximisation yields $R_1 = \min\{1, S/2\}$.

Let consider the agent's problem. For {always invest} to be his optimal strategy, it must b the case that he will invest even if he has learned the worst possible cost estimate, i.e., $\theta = 1$. B then his only hope for profit stems from the possibility of having the cost redrawn (since the princip never offers $R > 1$). Formally, a necessary and sufficient condition for {always invest} to be a equilibrium strategy is that

$$E[U \mid \theta = 1] = (1-\pi)\int_0^R [R - C]dC - I = (1-\pi)R^2/2 - I \geq 0 \qquad (4)$$

Rewriting (4) proves the following result:

Proposition 1. *A necessary and sufficient condition for existence of an equilibrium in which the hold up problem is completely resolved (i.e., in which $\theta^* = 1$) is that*

$$\pi \leq 1 - 2I/R_1^2. \qquad (5)$$

Proposition 1 implies that there is a threshold value for the qualityof pre-investmen information; if the estimate quality is better than this level, then the hold-up problem cannot possibl be completely resolved by the existence of private information. Note that Proposition 1 does no warrant uniqueness of the equilibrium, and as we will see below for some values of π satisfyin condition (5) there may also another equilibrium in which the hold-up problem is not completel resolved.

If the right hand side of (5) is negative, the hold-up problem cannot possibly be completel resolved for any value of π. This yields:

Corollary 1: *The hold-up problem cannot be completely resolved for any $\pi \in [0,1]$ if $I > R_1^2/2$.*

Partial Resolution of The Hold-Up Problem

Now we turn to the more complicated case in which the hold-up problem is not completel resolved, i.e., $\theta^* < 1$ in equilibrium. First we look at the agent's strategy. If the agent expects th principal to set a price equal to R, what is his optimal investment strategy when observing a co estimate θ? If the estimate exceeds R, then the agent's only hope of profit stems from the possibilit of having his costs drawn again, which happens with probability $1-\pi$. His expected profit in thi case then equals

$$E[U \mid \theta \geq R] = (1-\pi)\int_0^R [R - C]dC - I = (1-\pi)R^2/2 - I \qquad (6)$$

and in this case he should invest iff $(1-\pi)R^2/2 - I \geq 0$. If this last condition is satisfied, howeve then the agent should always invest, and this contradicts the assumption that $\theta^* < 1$ in equilibriu Therefore, if the hold-up problem is not completely resolved, then any equilibrium must involv $\theta^* < R$.

If, on the other hand, the agent observes an estimate $\theta < R$, then even if his cost not redrawn he can earn the rent equal $(R-\theta)$ with probability π. This rent arises addition to the rent stemming from the prospects of having the costs redrawn. Hence

$$E[U \mid \theta < R] = (1-\pi)R^2/2 + \pi(R-\theta) - I \tag{7}$$

Now we turn to the principal's problem. She seeks to maximise

$$W = \int_0^R (S-R)dF(C) = (S-R)F(R) \tag{8}$$

with respect to R, where $F(R)$ is given by expression (2). From the discussion above we know that we do not need to consider $R \le \theta^*$. Maximisation then yields $R = R_2(\pi) \equiv \min\{1, [S-\pi/(1-\pi)]/2\}$.

Let us characterise the properties of the equilibrium in the case of partial resolution of the hold-up problem. Clearly, the cut-off value of investment, θ^*, must satisfy $E[U \mid \theta^* < R] = 0$. Rearranging (using (7) and the fact that $\theta^* < R$ implies that $R = R_2(\pi)$) we find that θ^* must satisfy $\theta^* = H(\pi)$, where $H(\pi) \equiv R_2(\pi) + \{(1-\pi)[R_2(\pi)]^2 - I\}/\pi$. Since R_2 is continuous for $\pi \in (0,1)$ and differentiable in the same interval except for the point $\pi = (S-2)/(S-1)$ in the case of $S > 2$, so is H (At the point $\pi = (S-2)/(S-1)$ the graph of $R_2(\pi)$ is kinked).

After rearranging, its derivative can be written

$$\frac{dH(\pi)}{d\pi} = \begin{cases} \left[I - \frac{S^2}{8}\right]\frac{1}{\pi^2} - \frac{3}{8(1-\pi)^2} & \text{if} \quad S < \frac{2-\pi}{1-\pi} \\ -\frac{1-2I}{2\pi^2} & \text{if} \quad S > \frac{2-\pi}{1-\pi} \end{cases} \tag{9}$$

The following result applies to cases in which the hold-up problem is completely resolved for $\pi = 0$.

Proposition 2. *If $I < R_1^2/2$, there exist estimate qualities $0 < \pi_1 < \pi_2 < 1$ such that*

i) there exists an equilibrium in which $\theta^* \in [0,1)$ iff $\pi \in (\pi_1, \pi_2]$,
ii) this equilibrium is unique and satisfies $\{\theta^* = H(\pi), R = R_2(\pi)\}$, and
iii) θ^* is strictly decreasing in π in this interval.

The equations defining π_1 and π_2 $(H(\pi_1) = R_2(\pi_1)$ and $H(\pi_2) = 0$, respectively, turn out to be second-degree equation in π. Not much insight is gained from inspecting their explicit solutions, however.

Proposition 2 has a very interesting immediate consequence (taking into account Proposition 1):

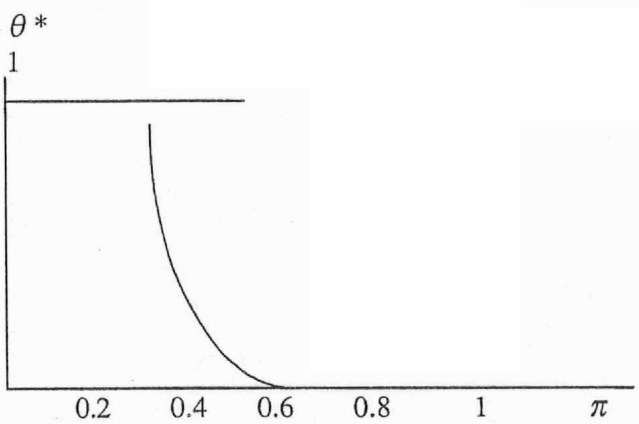
Corollary 2. *If the hold-up problem is completely resolved for $\pi = 0$, then improving the estimate quality will monotonically increase the hold-up problem.*

The equilibrium investment strategy is plotted against π in the figure below, for the case of $I =$ and $S = 2$ (it is easily verified that $\pi = 0$ implies $\theta^* = 1$ for these parameters).

Note that for some values of π there are two equilibrium: one in which the hold-up proble is completely resolved, and one in which it is not. The former equilibrium Pareto-dominates the lat one (due to strategic complimentarily). Increasing the estimate quality at best does not affect the be equilibrium (and at worst makes it in feasible), while it reduces the likelihood of investment in t other equilibrium.

What happens if the hold-up problem cannot be completely resolved for any π ? (Fro Corollary 1 we know that this is equivalent to $I > R_1^2/2$). Now, if the estimate is pure noise (i.e $\pi = 0$) the agent will not invest. Consequently, improving the estimate quality from this point cann do any harm. Can it have any positive effects? The answer might be yes, as can be shown by th following numerical example: Let $\pi = 0.5$, $I = 0.6$ and $S = 3$. Then we find that $R_1 = R_2 = 1$, an consequently that $I > \frac{R_1}{2}$. Using equation (10) we find $\theta^* = 0.3$, which proves that in this ca improved signal quality will alleviate the hold-up problem (although not eliminate it).

Figure 3. Equilibrium investment when $\pi = 0$ yields investment



In the formal analysis we distinguish between the cases in which $S > 2$ and $S < 2$ respectively.

Proposition 3. *If* $S > 2$ *and* $I > R_1^2/2 = 1/2$, *equilibrium in which* $\theta^* \in [0,1)$ *exist iff* $I \leq \frac{S - 3/2}{S - 1}$

If this condition is satisfied, then

i) *the interval* $[2I - 1, \pi_2]$ *is non-empty (but possibly only a point, if the condition abo holds with equality),*

ii) $\{\theta^* = H(\pi), R = R_2(\pi)\}$ *is an equilibrium with* $\theta^* \geq 0$ *iff* $\pi \in [2I - 1, \pi_2]$, *and*

iii) *maximum investment equals (S – 3/2 – I)/(S – 2) and is attained wh* $\pi = (S-2)/(S-1)$.

The most interesting part of this result is the condition for existence of equilibrium in which positive fraction of agent types choose to invest. For such equilibrium to exist, the investment cos

must not be too large relative to the principal's valuation S. The reason is that high investment costs make the agent reluctant to invest, while low valuations make the principal more willing to risk lower prices, which also makes the agent reluctant to invest.

A similar, but more complicated result can be obtained for cases in which $S < 2$:

Proposition 4. *If* $S < 2$ *and* $I > R_1^2/2 = S^2/8$, *then* H *is a strictly concave function of* π, *with a unique maximum* π^{**} *in* $(0,1)$ *defined by the first-order condition* $dH(\pi)/d\pi = 0$. *If* $H(\pi^{**}) = 0$, *then* $\{\theta^{**} = 0, R = R_2(\pi^{**})\}$ *is the unique equilibrium with* $\theta^* \geq 0$. *If* $H(\pi^{**}) > 0$, *then there exists an interval* $[\pi_0, \pi_2]$ *such that*

i) $\pi^{**} \in (\pi_0, \pi_2)$.

ii) $\{\theta^* = H(\pi), R = R_2(\pi)\}$ *is an equilibrium with* $\theta^* \geq 0$ *iff* $\pi \in [\pi_0, \pi_2]$, *and*

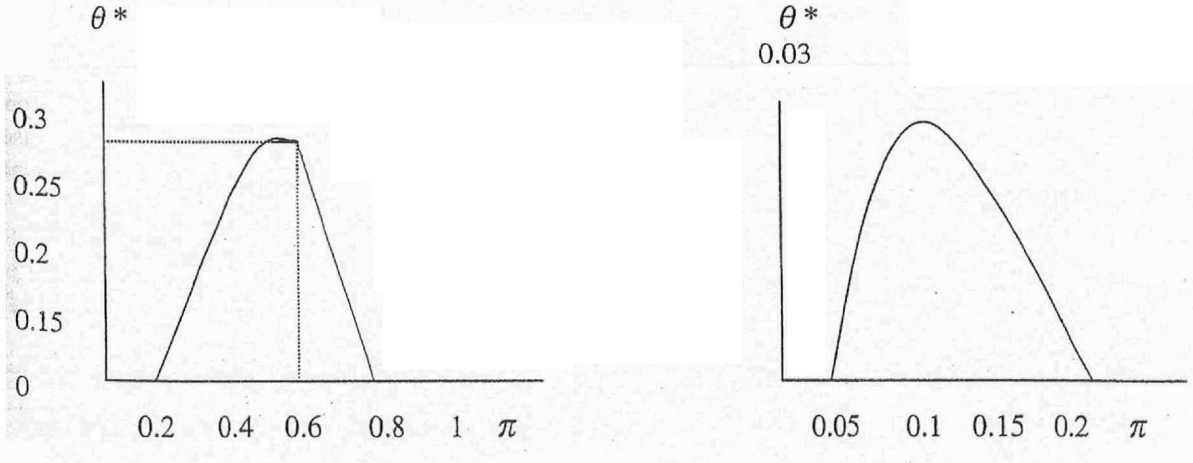
iii) θ^* *is increasing in* π *for* $\pi \in [\pi_0, \pi^{**})$ *and decreasing for* $\pi \in (\pi^{**}, \pi_2]$.

The intuition behind this proposition is similar to that for the previous result and it can be shown again that existence requires that I is not too large compared to S. Also in this case we can compute explicit expressions for π^{**} and π_0.

One may wonder what is the difference between the case in which improved quality of the estimate was always bad for investment (cf. Propositions 1 and 2), and this last case, in which some pre-investment information is essential for investment (cf. Propositions 3 and 4). If the hold-up problem is (completely) resolved for $\pi = 0$, then improving the agent's cost estimate has only costs: For a given expected price, early cost information makes high-cost forms less willing to invest (the direct effect), and this effect may be reinforced by the principal's reaction (the signaling effect). If the hold-up problem is not resolved for $\pi = 0$ we can identify a benefit from letting the agent learn his production costs earlier: If we start in a situation in which the agent's expected costs are higher than the (expected) price, then more reliable cost estimates can make low-cost firms willing to invest (the direct effect). However, the signaling effect is also present, and eventually dominates the direct effect.

Figure 4 below show how investment behaviour varies with π, for $S = 3$ and $I = 0.6$ (Fig. 4a) and $S = 1$ and $I = 0.13$. (Fig. 4b).

Figure 4. Equilibrium investment when $\pi = 0$ yields no investment



From the figure we see that early information increase investment up to a certain level, th decreases investment. What characterises the former part of the curve is that although the investme is signaling low costs, the signal is not precise enough to make the principal offer a (much) low price. In contrast, on the downward sloping part of the curve, increasing the signal precision leads t principal to lower the price significantly, and this effects dominates the beneficial effects of mo precise signals on the agent's willingness to invest.

The simple necessary condition for an equilibrium with $\theta^* \in [0,1)$ to exist will given in th next proposition. Obviously, if the expected price does not cover the investment cost, the agent wi not invest. Hence, solving $R_2 = I$ for π gives us an upper bound on the estimate quality:

Proposition 5. *A necessary condition for an equilibrium with* $\theta^* \in [0,1)$ to exist is that

$$\pi \leq \frac{S-2I}{S+1-2I} = 1 - \frac{1}{1+S-2I}$$

Combining Propositions 1 and 5 now gives us a necessary condition for equilibrium with an investment to exist.

IV. CONCLUDING REMARKS

Sometimes it is claimed that the reason why certain investments are not undertaken is that th investments are too risky. One example could be foreign investments in certain developing countrie Prospective investors often face two different sources of risk. First, they often know little about th (labour or product) market, which they consider to enter. Second, some governments have acquired reputation for breaking promises but they made to attract investments in the first place (e.g., after th entry investments are sunk, the government may rise taxes on the foreign establishments).

Suppose that both sources are present, then the example features a combination of priva information (assuming that the government in question can never observe the firm's profits) an postcontractual opportunism. In light of our analysis, the largest problem is the hold-up threat. Th existence of private information (and also the risk associated with uncertain demand and co conditions) can actually lead to investments rather than deter them, because private informatio creates information rent.

Moreover, suppose that the prospective investor can collect more precise information abou the market conditions before entry. This will certainly reduce the "market risk", but more importan it will also make the government able to infer more cost information from the entry decision itself. I our model we have seen that this signaling effect can dominate. In particular, if the firm receives "to accurate" information before entry, the only equilibrium is the one in which no entry occurs.

This example illustrates that the theory here developed have implications for policy issue The government in question, recognising its inability to commit not to extract profits in the futur should consider to restrict the prospective investor's access to information.

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