STATIC MAGNETIC MONOPOLES CONFIGURATIONS*

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Abstract

Recently, we have reported on the existence of some monopoles, multimonopole, and antimonopoles configurations. In this paper we would like to give a brief review of the different exact monopoles, multimonopole, antimonopoles and vortex rings configurations of the magnetic ansatz of Ref.[9] when the parameters p and b of the solutions take different serial values. These exact solutions are a different kind of BPS solution. They satisfy the first order Bogomol'nyi equation but possess infinite energy. They can have radial, axial, or rotational symmetry about the z-axis. We classified these serial solutions as (i) the multimonopole at the origin; (ii) the finitely separated 1-monopoles; (iii) the screening solutions of multimonopole and (iv) the axially symmetric monopole-vortex ring solutions. Half-integer topological magnetic charge multimonopole also exist in some of these series of solutions.

1 Introduction

The SU(2) Yang-Mills-Higgs (YMH) field theory, with the Higgs field in the adjoint representation are known to possess both the magnetic monopole and multimonopole solutions. The famous 't Hooft-Polyakov monopole solution, [1] with non zero Higgs mass and Higgs self-interaction is a spherically symmetric, numerical monopole solution of unit magnetic charge. In general, configurations of the YMH field theory with a unit magnetic charge are spherically symmetric [1]-[3]. However we have presented a unit magnetic charge configuration in Ref.[12] that do not even possess axial symmetry but only mirror symmetry.

In the limit when the Higgs mass and the Higgs self-interaction tend to zero with the vacuum expectation value non vanishing, the Higgs field becomes massless and is non self-interacting. This model, with non-zero vacuum expectation is known as the Bogomol'nyi-Prasad-Sommerfield (BPS) limit as exact solutions can be obtained by solving the first order Bogomol'nyi equations [3], [6]. These BPS solutions possess minimal energies. Exact BPS multimonopole configurations with magnetic charges greater than unity and possessing axial and mirror symmetries were reported in the early 80's [4] and it has been shown that these solutions cannot possess spherical symmetry [5].

However when the Higgs potential is finite only numerical monopole solutions [1], [2] and numerical axially symmetric multimonopole solutions are known [7] . Asymmetric multimonopole solutions with no rotational symmetry also exist [8], however these solutions are numerical solutions even in the BPS limit.

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From observation, we notice that it is possible to construct the corresponding dyons solutions from these four classes of multimonopole–antimonopole solutions using the usual technique.

Finally we would like to comment that with the ansatz of Eqs. (13) and (14), there is no solution to the Bogomol'nyi equation of Eq. (12) with the negative sign. This is unlike the BPS solutions where the plus and minus sign of Eq. (12) correspond to monopoles and antimonopoles respectively.[14]

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References

1. R. Teh, *Int. J. Mod. Phys.* **A16**, 3479 (2001); *J. Fiz. Mal.* **23**, 196 (2002).
2. G. 't Hooft, *Nucl. Phys.* **B79**, 276 (1974).
3. A. M. Polyakov, *Sov. Phys. - JETP* **41**, 988 (1975); *Phys. Lett.* **B59**, 82 (1975); *JETP Lett.* **20**, 194 (1974).
4. M. K. Prasad and C. M. Sommerfield, *Phys. Rev. Lett.* **35**, 760 (1975).
5. E. B. Bogomol'nyi and M. S. Marinov, *Sov. J. Nucl. Phys.* **23**, 357 (1976).
6. C. Rebbi and P. Rossi, *Phys. Rev.* **D22**, 2010 (1980).
7. R. S. Ward, *Commun. Math. Phys.* **79**, 317 (1981).
8. P. Forgacs, Z. Horvarth and L. Palla, *Phys. Lett.* **B99**, 232 (1981); *Nucl. Phys.* **B192**, 141 (1981).
9. M. K. Prasad, *Commun. Math. Phys.* **80**, 137 (1981); M. K. Prasad and P. Rossi, *Phys. Rev.* **D24**, 2182 (1981).
10. P. M. Sutcliffe, *Int. J. Mod. Phys.* **A12**, 4663 (1997).
11. C. J. Houghton, N. S. Manton and P. M. Sutcliffe, *Nucl. Phys.* **B510**, 507 (1998).
12. E. B. Bogomol'nyi, *Sov. J. Nucl. Phys.* **24**, 449 (1976).
13. B. Kleihaus, J. Kunz and D. H. Tchrakian, *Mod. Phys. Lett.* **A13**, 2523 (1998).
14. P. Rossi, *Phys. Rep.* **86**, 317 (1982).
15. E. J. Weinberg and A. H. Guth, *Phys. Rev.* **D14**, 1660 (1976).
16. B. Kleihaus and J. Kunz, *Phys. Rev.* **D61**, 025003 (2000).
17. C. N. Yang and R. L. Mills, *Phys. Rev.* **96**, 191 (1954); R. Shaw, Ph.D. Thesis (Cambridge University, UK, 1955).
18. S. Coleman, *Commun. Math. Phys.* **55**, 113 (1977); S. Deser, *Phys. Lett.* **B64**, 463 (1976); H. Pagels, *Phys. Lett.* **B68**, 466 (1977); A. Actor, *Rev. Mod. Phys.* **51**, 461 (1979).
19. M. Georgi and S. L. Glashow, *Phys. Rev. Lett.* **28**, 1494 (1972).
20. J. Arafune, P. G. O. Freund and C. J. Goebel, *J. Math. Phys.* (*N.Y.*) **16**, 433 (1975); G. M. Murphy, *Ordinary Differential Equations and their Solutions* (Van Nostrand Reinhold Company, 1960).
21. N. S. Manton, *Nucl. Phys.* (*N.Y.*) **B126**, 525 (1977).

Axially symmetric monopoles-antimonopoles chain solutions which do not satisfy the Bogomol'nyi condition were also constructed numerically. These non-Bogomol'nyi solutions exist both in the limit of a vanishing Higgs potential as well as in the presence of a finite Higgs potential. BPS axially symmetric vortex rings solutions have also been constructed numerically [7].

Recently we have shown the existence of a different kind of BPS static monopole solutions [9]. Other than the Wu-Yang type 1-monopole, these configurations possess at most axial symmetry and they represent different combinations of monopoles, multimonopole, and antimonopoles, with mirror symmetry about the z-axis.

In this paper we would like to present more monopoles, multimonopole, antimonopoles, and vortex rings configurations of the magnetic ansatz of Ref.[9] when
the solutions take different serial values. Similarly, these exact solut
BPS solutions. They satisfy the first order Bogomol'nyi equation but possess infinite energy. They can have radial, axial, or rotational symmetry about the z-axis.

It is also our purpose in this paper to attempt to summarize all the possible monopole configurations that the magnetic anzatz of Ref.[9] are able to support. These serial monopole solutions can be classified into four categories of monopole solutions. They are (i) the multimonopole at the origin; (ii) the finitely separated 1-monopoles; (iii) the screening solutions of multimonopole by antimonopoles and (iv) the axially symmetric (AS) monopole-vortex ring configurations. The multimonopole in some of these series of solutions can have half-integer topological magnetic charge. The magnetic ansatz also admits isolated one-half topological magnetic charge monopoles [12].

We briefly review the SU(2) YMH field theory in the next section. We present the magnetic ansatz and some of its basic properties in section 3. In section 4, we discussed the purely monopole configurations with only monopoles and multimonopole of positive topological magnetic charge. We briefly introduced the screening of multimonopole by antimonopoles configurations in section 5.1. In section 5.2, we discussed the axially symmetric monopole-vortex rings configurations. We summarized our results and give some comments in section 6.

2 The SU(2) Yang-Mills-Higgs Theory

The SU(2) YMH theory admits the triplet gauge fields A^a_μ which are the Yang-Mills vector fields coupled to a scalar Higgs triplets field Φ^a in 3+1 dimensions. The index a is the SU(2) internal space index and for a given a, Φ^a is a scalar whereas A^a_μ is a vector under Lorentz transformation. The Lagrangian in 3+1 dimensions is given by

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi^a - \frac{1}{4}\lambda(\Phi^a\Phi^a - \frac{\mu^2}{\lambda})^2, \tag{1}$$

where the Higgs field mass, μ, and the strength of the Higgs potential, λ, are constants. The vacuum expectation value of the Higgs field is then given by $\mu/\sqrt{\lambda}$. The Lagrangian (1) is gauge invariant under the set of independent local SU(2) transformations at each space-time point. The covariant derivative of the Higgs field and the gauge field strength tensor are given respectively by

$$\begin{aligned} D_\mu\Phi^a &= \partial_\mu\Phi^a + \epsilon^{abc}A^b_\mu\Phi^c, \ \text{ and} \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc}A^b_\mu A^c_\nu. \end{aligned} \tag{2}$$

Since the gauge field coupling constant g can be scaled away, we can set g to one without any loss of generality. The metric used is $g_{\mu\nu} = (-+++)$. The SU(2) internal group indices a, b, c run from 1 to 3 and the spatial indices are $\mu, \nu, \alpha = 0, 1, 2,$ and 3 in Minkowski space.

The equations of motion that follow from the Lagrangian (1) are

$$\begin{aligned} D^\mu F^a_{\mu\nu} &= \partial^\mu F^a_{\mu\nu} + \epsilon^{abc} A^{b\mu} F^c_{\mu\nu} = \epsilon^{abc} \Phi^b D_\nu \Phi^c, \\ D^\mu D_\mu \Phi^a &= -\lambda \Phi^a (\Phi^b \Phi^b - \frac{\mu^2}{\lambda}). \end{aligned} \tag{3}$$

The Abelian electromagnetic field tensor as proposed by 't Hooft [1], is given by

$$F_{\mu\nu} = \hat{\Phi}^a F^a_{\mu\nu} - \epsilon^{abc} \hat{\Phi}^a D_\mu \hat{\Phi}^b D_\nu \hat{\Phi}^c = \partial_\mu A_\nu - \partial_\nu A_\mu - \epsilon^{abc} \hat{\Phi}^a \partial_\mu \hat{\Phi}^b \partial_\nu \hat{\Phi}^c, \tag{4}$$

where $A_\mu = \hat{\Phi}^a A^a_\mu$, the Higgs field unit vector $\hat{\Phi}^a = \Phi^a / |\Phi|$ and the Higgs field magnitude $|\Phi| = \sqrt{\Phi^a \Phi^a}$. The Abelian electric field is $E_i = F_{0i}$, and the Abelian magnetic field is $B_i = -\frac{1}{2} \epsilon_{ijk} F_{jk}$. The topological magnetic current [10] which is also the topological current density of the system is defined to be

$$k_\mu = \frac{1}{8\pi} \epsilon_{\mu\nu\rho\sigma} \epsilon_{abc} \partial^\nu \hat{\Phi}^a \partial^\rho \hat{\Phi}^b \partial^\sigma \hat{\Phi}^c, \tag{5}$$

and the corresponding conserved topological magnetic charge is

$$\begin{aligned} M &= \int d^3x \, k_0 = \frac{1}{8\pi} \int \epsilon_{ijk} \epsilon^{abc} \partial_i \left(\hat{\Phi}^a \partial_j \hat{\Phi}^b \partial_k \hat{\Phi}^c \right) d^3x \\ &= \frac{1}{8\pi} \oint d^2\sigma_i \left(\epsilon_{ijk} \epsilon^{abc} \hat{\Phi}^a \partial_j \hat{\Phi}^b \partial_k \hat{\Phi}^c \right) = \frac{1}{4\pi} \oint d^2\sigma_i \, B_i. \end{aligned} \tag{6}$$

Our work is restricted to the static case where $A^a_0 = 0$. Hence the conserved energy of the system which is obtained from the Lagrangian (1) in the usual way, reduces for the static case to

$$E = \int d^3x \left(\frac{1}{2} B^a_i B^a_i + \frac{1}{2} D_i \Phi^a D_i \Phi^a + \frac{1}{4} \lambda (\Phi^a \Phi^a - \frac{\mu^2}{\lambda})^2 \right). \tag{7}$$

Here i, j, k which are the three space indices run from 1, 2, and 3. This energy vanishes when the gauge potential, A^a_i is zero or when A^a_i is a pure gauge, and when $\Phi^a \Phi_a = \mu^2/\lambda$ and $D_i \Phi^a = 0$.

In the model we are considering, the Higgs field is massless and with vanishing self-interaction. Hence the Lagrangian (1) is just simply

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} D^\mu \Phi^a D_\mu \Phi^a. \tag{8}$$

The magnitude of the Higgs field vanishes as $1/r$ at large r. It is in this limit that we are able to obtain explicit, exact magnetic monopoles solutions to the YMH equations. These solutions can be solved using both the second order Euler-Lagrange equations and the Bogomol'nyi equations,

$$B^a_i \pm D_i \Phi^a = 0. \tag{9}$$

The $\pm$ sign corresponds to monopoles and antimonopoles respectively for the usual BPS solutions. In our case the magnetic monopoles of Ref.[9] as well as the solutions presented here are solved with the $+$ sign and their anti-configurations [14] are solved with the $-$ sign. The configurations obtained correspond to different combinations of monopole, multimonopoles, antimonopoles, and vortex rings. In this paper, a multimonopole of magnetic charge M with all its magnetic charges superimposed at one point in space is denoted by a M-monopole. When $M = 1$, it is possible for the monopoles to have finite separations in space. However we failed to find M-monopoles with finite separations when $M > 1$.

3 The Magnetic Ansatz

We make use of the static magnetic ansatz [9] to solve for the monopoles solutions here. The gauge fields and the Higgs field are given respectively by

$$\begin{aligned}
A^a_\mu &= \frac{1}{r}\psi(r)\left(\hat{\theta}^a\hat{\phi}_\mu - \hat{\phi}^a\hat{\theta}_\mu\right) + \frac{1}{r}R(\theta)\left(\hat{\phi}^a\hat{r}_\mu - \hat{r}^a\hat{\phi}_\mu\right) + \frac{1}{r}G(\theta,\phi)\left(\hat{r}^a\hat{\theta}_\mu - \hat{\theta}^a\hat{r}_\mu\right),\\
\Phi^a &= \Phi_1\,\hat{r}^a + \Phi_2\,\hat{\theta}^a + \Phi_3\,\hat{\phi}^a,
\end{aligned} \tag{10}$$

where $\Phi_1 = \frac{1}{r}\psi(r)$, $\Phi_2 = \frac{1}{r}R(\theta)$, $\Phi_3 = \frac{1}{r}G(\theta,\phi)$. The spherical coordinate orthonormal unit vectors, $\hat{r}_i$, $\hat{\theta}_i$, and $\hat{\phi}_i$ are defined by

$$\begin{aligned}
\hat{r}_i &= \sin\theta\,\cos\phi\,\delta_{1i} + \sin\theta\,\sin\phi\,\delta_{2i} + \cos\theta\,\delta_{3i},\\
\hat{\theta}_i &= \cos\theta\,\cos\phi\,\delta_{1i} + \cos\theta\,\sin\phi\,\delta_{2i} - \sin\theta\,\delta_{3i},\\
\hat{\phi}_i &= -\sin\phi\,\delta_{1i} + \cos\phi\,\delta_{2i},
\end{aligned} \tag{11}$$

where $r = \sqrt{x^i x_i}$, $\theta = \cos^{-1}(x_3/r)$, and $\phi = \tan^{-1}(x_2/x_1)$. The gauge field strength tensor and the covariant derivative of the Higgs field are given respectively by

$$\begin{aligned}
F^a_{\mu\nu} &= \frac{1}{r^2}\hat{r}^a\left\{\dot{R} + R\cot\theta + 2\psi - \psi^2 + G^\phi\csc\theta\right\}(\hat{\phi}_\mu\hat{\theta}_\nu - \hat{\phi}_\nu\hat{\theta}_\mu)\\
&+ \frac{1}{r^2}\left\{\hat{\theta}^a R(1-\psi) + \hat{\phi}^a G(1-\psi)\right\}(\hat{\phi}_\mu\hat{\theta}_\nu - \hat{\phi}_\nu\hat{\theta}_\mu)\\
&+ \frac{1}{r^2}\left\{\hat{r}^a R(1-\psi) + \hat{\phi}^a G(\cot\theta - R)\right\}(\hat{r}_\mu\hat{\phi}_\nu - \hat{r}_\nu\hat{\phi}_\mu)\\
&+ \frac{1}{r^2}\hat{\theta}^a\left\{r\psi' + R\cot\theta - R^2 + G^\phi\csc\theta\right\}(\hat{r}_\mu\hat{\phi}_\nu - \hat{r}_\nu\hat{\phi}_\mu)\\
&+ \frac{1}{r^2}\left\{-\hat{r}^a G(1-\psi) + \hat{\theta}^a(\dot{G} + RG)\right\}(\hat{r}_\mu\hat{\theta}_\nu - \hat{r}_\nu\hat{\theta}_\mu)\\
&+ \frac{1}{r^2}\left\{-\hat{\phi}^a(r\psi' + \dot{R} - G^2)\right\}(\hat{r}_\mu\hat{\theta}_\nu - \hat{r}_\nu\hat{\theta}_\mu),\\
D_\mu\Phi^a &= \frac{1}{r^2}\left\{\hat{r}^a(r\psi' - \psi - R^2 - G^2) - \hat{\theta}^a R(1-\psi) - \hat{\phi}^a G(1-\psi)\right\}\hat{r}_\mu\\
&+ \frac{1}{r^2}\left\{-\hat{r}^a R(1-\psi) + \hat{\theta}^a(\dot{R} + \psi - \psi^2 - G^2) + \hat{\phi}^a(\dot{G} + RG)\right\}\hat{\theta}_\mu\\
&+ \frac{1}{r^2}\left\{-\hat{r}^a G(1-\psi) - \hat{\theta}^a G(\cot\theta - R)\right\}\hat{\phi}_\mu\\
&+ \frac{1}{r^2}\left\{\hat{\phi}^a(\psi - \psi^2 + R\cot\theta - R^2 + G^\phi\csc\theta)\right\}\hat{\phi}_\mu,
\end{aligned} \tag{12}$$

where prime means $\partial/\partial r$, dot means $\partial/\partial\theta$ and superscript ϕ means $\partial/\partial\phi$. The gauge fixing condition that we used here is the radiation or Coulomb gauge, $\partial^i A^a_i = 0$, $A^a_0 = 0$.

By substituting the ansatz (10) into the equations of motion (3) as well as the Bogomol'nyi equations (9) with the positive sign, these equations can be simplified to just four first order differential equations,

$$r\psi' + \psi - \psi^2 = -p, \tag{13}$$

$$\dot{R} + R\cot\theta - R^2 = p - b^2\csc^2\theta, \tag{14}$$

$$\dot{G} + G\cot\theta = 0, \quad G^\phi\csc\theta - G^2 = b^2\csc^2\theta, \tag{15}$$

where p and b^2 are arbitary constants. Eq.(13) is exactly solvable for all real values of p and the integration constant can be scaled away by letting $r \to r/c$, where c is the arbitrary integration constant. In order to have solutions of ψ with $(2m+1)$ powers of r, we write $p = m(m+1)$. Eqs.(15) are also exactly solvable and a general physical solution is $G(\theta, \phi) = b \csc\theta \tan(b\phi)$, where b is restricted to take half-integer values for G to be a single value function. Eq.(14) is a Riccati equation and $R(\theta)$ can be exactly solved for different combinations of p and b.

Therefore we can write $b = m \pm s$ where $s = 0, 1, 2, 3, \ldots$ and m can take half-integer values. For non zero values of $b = m - s$, we have the 1_s series of monopole solutions and for non zero values of $b = m + s$, we have the 2_s series of monopole solutions. When $b = 0$, we have the axially symmetric monopole solutions which is reported in a separate work when m is restricted to be an integer [11] and in another paper [12] when $m = \frac{1}{2}$. The C series of monopole solutions is obtained when p is set to zero with $b = m$.

The solutions for the profile functions $\psi(r)$ and $G(\theta, \phi)$ are standard,

$$\psi(r) = \frac{(m+1) - mr^{2m+1}}{1 + r^{2m+1}}, \quad G(\theta, \phi) = (m \pm s) \csc\theta \tan(m \pm s)\phi, \tag{16}$$

where we fixed s to be a positive integer and when b is non vanishing. For $\psi(r)$ to have integer powers of r, and $G(\theta, \phi)$ to be a single value function, the value of m is restricted to a half-integer, where $m \geq -\frac{1}{2}$.

It is the Riccati equation (14) that give rise to the different monopoles configurations as the solution for $R(\theta)$ is non-unique. The profile function $R(\theta)$ for non vanishing $b = m + s$, is given by

$$\begin{aligned} R(\theta) &= (m+1)\cot\theta + (s-1)\csc\theta \frac{Q_{m+1}^{m+s}(\cos\theta)}{Q_m^{m+s}(\cos\theta)}, \\ s &= 1,\ 2,\ 3,\ \ldots, \quad m = -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \end{aligned} \tag{17}$$

where $Q_m^{m+s}(\cos\theta)$ is the associated Legendre function of the second kind of degree m and order $m+s$. We labelled the monopoles configurations of Eq.(17) as the 2_s series of monopole solutions with no antimonopoles in the systems. We further subdivide the 2_s solutions into the $A2_s$ series when s is even and the $B2_s$ series when s is odd. The $A2_s$ configuration is that of a multimonopole at the origin, $r = 0$. The $B2_s$ series is the configuration where the monopoles are finitely separated and arranged in a circle about the z-axis.

The profile function $R(\theta)$ for non vanishing $b = m - s$, is given by

$$\begin{aligned} R(\theta) &= (m+1)\cot\theta - (s+1)\csc\theta \frac{P_{m+1}^{m-s}(\cos\theta)}{P_m^{m-s}(\cos\theta)}, \\ s &= 0, 1, 2, 3, \ldots, \quad m = s + \frac{1}{2}, s+1, s+1\frac{1}{2}, \ldots, \end{aligned} \tag{18}$$

where $P_m^{m-s}(\cos\theta)$ is the associated Legendre function of the first kind of degree m and order $m-s$. The solution (18) is the screening solutions of a multimonopole by antimonopoles which we labelled as the 1_s series of solutions. Similarly, we further subdivide the 1_s solutions into the $B1_s$ configurations when s is zero or even and the $A1_s$ configurations when s is odd. The axially symmetric monopoles solutions is the series when $b = 0$, hence the function $G(\theta, \phi)$ vanishes, and $p = m(m+1)$ [11].

The energy of the system of solutions here is not finite due to the singularity of the solutions at the origin, $r = 0$. Also the vacuum expectation values of our solutions tend to zero at large r. Hence unlike the normal BPS solutions, the energy of our solutions is not bounded from below.

The net topological charge of the system is given by the integration of the radial component of the Abelian magnetic field over the sphere at infinity,

$$M_\infty = \frac{1}{8\pi}\oint d^2\sigma_i \left(\epsilon_{ijk}\epsilon^{abc}\hat{\Phi}^a\partial_j\hat{\Phi}^b\partial_k\hat{\Phi}^c\right)\Big|_{r\to\infty}. \tag{19}$$

Hence the monopoles and antimonopoles of our solutions here can then be associated with the number of point zeros of $r\Phi^a$ enclosed by the sphere at infinity. The positions of the monopoles and antimonopoles of our solutions correspond to the point zeros of the Higgs field but the multimonopole is always located at the origin of the coordinate axes where the Higgs field is singular. The definition for the magnetic charges as given by Eq.(6) and Eq.(19) is not affected by the fact that the magnitude of the Higgs field, $|\Phi|$, vanishes at large r. It only depends on the direction of the unit vector of the Higgs field, $\hat{\Phi}^a$, in internal space.

From the ansatz (10), $A_\mu = \hat{\Phi}^a A^a_\mu = 0$. Hence the Abelian electric field is always zero and the Abelian magnetic field is independent of the gauge fields, A^a_μ. To calculate for the Abelian magnetic field B_i, we rewrite the Higgs field of Eq.(10) from the spherical to the Cartesian coordinate system, [7], [9]

$$\Phi^a = \Phi_1\,\hat{r}^a + \Phi_2\,\hat{\theta}^a + \Phi_3\,\hat{\phi}^a = \tilde{\Phi}_1\,\delta^{a1} + \tilde{\Phi}_2\,\delta^{a2} + \tilde{\Phi}_3\,\delta^{a3}, \tag{20}$$

$$\begin{aligned} \text{where}\quad \tilde{\Phi}_1 &= \sin\theta\cos\phi\;\Phi_1 + \cos\theta\cos\phi\;\Phi_2 - \sin\phi\;\Phi_3 = |\Phi|\cos\alpha\sin\beta \\ \tilde{\Phi}_2 &= \sin\theta\sin\phi\;\Phi_1 + \cos\theta\sin\phi\;\Phi_2 + \cos\phi\;\Phi_3 = |\Phi|\cos\alpha\cos\beta \\ \tilde{\Phi}_3 &= \cos\theta\;\Phi_1 - \sin\theta\;\Phi_2 = |\Phi|\sin\alpha. \end{aligned} \tag{21}$$

The Higgs unit vector is then simplified to

$$\hat{\Phi}^a = \cos\alpha\sin\beta\;\delta^{a1} + \cos\alpha\cos\beta\;\delta^{a2} + \sin\alpha\;\delta^{a3}, \tag{22}$$

$$\text{where,}\ \sin\alpha = \frac{\psi\cos\theta - R\sin\theta}{\sqrt{\psi^2 + R^2 + G^2}},\ \ \beta = \gamma - \phi,\ \ \gamma = \tan^{-1}\left(\frac{\psi\sin\theta + R\cos\theta}{G}\right), \tag{23}$$

and the Abelian magnetic field is found to be

$$\begin{aligned} B_i &= \frac{1}{r^2\sin\theta}\left\{\frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial\phi} - \frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial\theta}\right\}\hat{r}_i \\ &+ \frac{1}{r\sin\theta}\left\{\frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial r} - \frac{\partial\sin\alpha}{\partial r}\frac{\partial\beta}{\partial\phi}\right\}\hat{\theta}_i + \frac{1}{r}\left\{\frac{\partial\sin\alpha}{\partial r}\frac{\partial\beta}{\partial\theta} - \frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial r}\right\}\hat{\phi}_i. \end{aligned} \tag{24}$$

Defining the Abelian field magnetic flux as

$$\Omega = 4\pi M = \oint d^2\sigma_i B_i = \int B_i(r^2\sin\theta d\theta)\hat{r}_i\;d\phi, \tag{25}$$

the magnetic charge enclosed by the sphere centered at $r = 0$ and of fixed radius r_1, is found to be

$$M_{r_1} = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\left(\frac{\partial\sin\alpha}{\partial\theta}\frac{\partial\beta}{\partial\phi} - \frac{\partial\sin\alpha}{\partial\phi}\frac{\partial\beta}{\partial\theta}\right)d\theta d\phi\Big|_{r=r_1}. \tag{26}$$

Hence the magnetic charge enclosed by the sphere of infinite radius is denoted by M_∞ and the magnetic charge enclosed by the sphere of vanishing radius is denoted by M_0. We also denote the net magnetic charge of the screening antimonopoles in the 1_s series of solutions by M_A.

4 Monopoles and Multimonopole

The configurations with finitely separated 1-monopoles are labelled as the $B2_s$ series of monopoles. These configurations are made up of one topological charge monopoles located on the point zeros of the Higgs field. They are all arranged symmetrically on a circle about the z-axis. The configurations with the multimonopole at the origin are the $A2_s$ series and the C series o monopoles. Both these series of multimonopole can possess half-integer topological magnetic charge. However the C series supports monopoles of $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$ topological magnetic charge at the origin, whereas the A2 series only contains multimonopoles of $2\frac{1}{2}$ topological magnetic charge and above.

4.1 The 2_s Series Multimonopole

In the 2_s series of solutions there are totally no antimonopoles in the configurations. All the monopoles and multimonopole possess only positive topological magnetic charge. We further subdivide this series into the $A2_s$ solutions when s is even, and the $B2_s$ solutions when s is odd The solutions for the Riccati equation (14), when $p = m(m+1)$ and $b = m+s$, for $s = 1, 2, 3, 4$; are

$$\begin{aligned}
(\text{B}2_1 \text{ solution}),\ R(\theta) &= (m+1)\cot\theta; \\
(\text{A}2_2 \text{ solution}),\ R(\theta) &= \tan\theta + (m+2)\cot\theta; \\
(\text{B}2_3 \text{ solution}),\ R(\theta) &= (m+1)\cot\theta + \frac{4(m+2)\cot\theta}{2(m+2)\cos^2\theta + \sin^2\theta}; \\
(\text{A}2_4 \text{ solution}),\ R(\theta) &= \tan\theta + (m+2)\cot\theta + \frac{4(m+3)\cot\theta}{2(m+3)\cos^2\theta + 3\sin^2\theta}.
\end{aligned} \tag{27}$$

The boundary conditions of the 2_s series of solutions are

$$\begin{aligned}
\psi(r)|_{r\to\infty} &\to -m, \quad \psi(r)|_{r\to 0} \to (m+1); \quad G(\theta, 0) = G(\theta, 2\pi) = 0; \\
R(\theta)\sin\theta|_{\theta\to 0} &\to (m+s), \quad R(\theta)\sin\theta|_{\theta\to\pi} \to -(m+s).
\end{aligned} \tag{28}$$

The difference between the $A2_s$ and $B2_s$ solutions lies in the profile function, $R(\theta)$. For the $A2_s$ solutions, $R(\theta)\cos\theta|_{\theta=\pi/2} = 1$, whereas for the $B2_s$ solutions, $R(\frac{\pi}{2}) = 0$.

The $A2_s$ series of solutions are configurations with a single multimonopole at the origin. The A2 solutions [9] is the first member of the $A2_s$ series and when $m = -\frac{1}{2}$, the multimonopole charge is $2\frac{1}{2}$. In fact, the A1 and the A2 solutions actually converge into a single multimonopole solution when $m = -\frac{1}{2}$ in both solutions. A vector field plot of the Abelian magnetic field, B_i, for the $2\frac{1}{2}$-monopole is shown in Fig.(1). Hence half-integer topological charge multimonopoles do exist in the SU(2) YMH theory. When $m = 0$, the A2 multimonopole charge is 3. A vector field plot of the Abelian magnetic field, B_i, for the 3-monopole is shown in Fig.(2).

The B2 solutions [9] are the finitely separated 1-monopoles solutions. This series starts with $m = \frac{1}{2}$, where the solution is a three 1-monopoles configuration. The three 1-monopoles are located at the zeros of the Higgs field on the x-y plane at distance $r = \sqrt{3}$ from the origin. As the parameter m increases in steps of one-half, the number of monopoles in the configuration increases by one. When $m = 1$, the B2 configuration has four finitely separated 1-monopoles. A vector field plot of its Abelian magnetic field, Fig.(3), shows the four 1-monopoles located along the x and y axes at a distance $r = \sqrt[3]{2}$ from the origin. Hence in the B2 solutions, we can have odd as well as even numbers of finitely separated 1-monopoles. In Ref.[9], we reported on configurations with only even numbers of finitely separated monopoles in the B2 solutions.

The higher series of A2_s and B2_s solutions when $s = 3, 4, 5, \ldots$, start with higher multimonopole charge. Some properties of the lower 2_s series of solutions are tabulated in Table 1.

Table 1: The 2_s Series of Multimonopole Solutions. Here $b = m + s$, where $s = 1, 2, 3, 4$, and $M_A = 0$. For even s, we have the A2_s series where $m = -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$ and for the odd s, we have the B2_s series where $m = \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$.

2_s **Series**	s	M_0	M_∞	**Configuration**
B2_1	1	0	$2(m+1)$	$2(m+1)$ 1-monopoles
A2_2	2	$m+3$	$m+3$	$(m+3)$-monopole
B2_3	3	0	$2(m+3)$	$2(m+3)$ 1-monopoles
A2_4	4	$m+5$	$m+5$	$(m+5)$-monopole

4.2 The C Series Multimonopole

The C solution is a series of multimonopole solutions with half-integer topological magnetic charge [12]. The multimonopole is located at the origin, $r = 0$, and has positive topological magnetic charge, $M = m \in \{\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots\}$. This series of solutions is solved by writing, $p = 0$ and $b = m$ in Eq.(13) to (15). The solutions obtained are

$$\psi(r) = \frac{1}{1+r}, \quad R(\theta) = m \csc\theta, \quad G(\theta, \phi) = m \csc\theta \tan(m\phi). \tag{29}$$

The boundary conditions are $\psi(r)|_{r\to 0} = 1$, $\psi(r)|_{r\to\infty} = 0$; $R(\theta)|_{\theta\to 0,\pi} \to +\infty$; and $G(\theta, 0) = G(\theta, 2\pi) = 0$.

The magnetic charge of the monopole at $r = 0$ is calculated to be one-half of the normal t'Hooft-Polyakov monopole charge when $m = \frac{1}{2}$. This half-monopole solution of the C series possesses only mirror symmetry at the vertical plane through the x-z axes and a string singularity along the negative z-axis. A vector field plot of its Abelian magnetic field is shown in Fig.(4).

When $m = 0$ and 1, the C configurations possess topological magnetic charge one. The 1-monopole when, $m = 0$, is just the radially symmetric Wu-Yang type 1-monopole [9] whereas the 1-monopole when, $m = 1$, possesses only mirror symmetry at the vertical x-z plane. A 3-D surface plot of the Abelian magnetic energy density, $B_i B_i$, at small r reveals that this particular 1-monopole is actually made up of two half-monopoles. More detail discussions on half-monopoles can be found in Ref.[12] and [15].

The C series of solutions continue to support higher topological magnetic charge multimonopole as m increases in steps of one-half. Hence when $m = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$, the topological charge of the multimonopole is $M = 1, \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \ldots$ respectively. All these C multimonopoles except for the case when, $m = 0$, which is radially symmetrical, seem to be made up of half-monopoles [12].

5 Antimonopoles, Multimonopole, and Vortex Rings

In this section we discussed monopole configurations containing antimonopoles, multimonopole and vortex rings. We only give a brief introduction of the screening multimonopole solutions as the detail will be given in a separate paper [13]. The axially symmetric monopole-vortex ring configurations do support one-half topological charge magnetic monopole [12].

5.1 Screening of Multimonopole

The screening of multimonopole occurs in the 1_s series of monopole solutions. This series is further subdivided into two alternating series, the $A1_s$ series when the parameter s is odd and the $B1_s$ series when s is zero or even. The first two members of these two series are the A1 and the B1 solutions of Ref.[9] when m is a natural number. The boundaries conditions of the 1_s series of solutions are

$$\begin{aligned} \psi(r)|_{r\to\infty} &\to -m, \quad \psi(r)|_{r\to 0} \to (m+1); \quad G(\theta,0) = G(\theta,2\pi) = 0; \\ R(\theta)\sin\theta|_{\theta\to 0} &\to -(m-s), \quad R(\theta)\sin\theta|_{\theta\to\pi} \to (m-s). \end{aligned} \tag{30}$$

The difference between the $A1_s$ and $B1_s$ solutions is in the profile function, $R(\theta)$. For the $A1_s$ solutions, $R(\theta)\cos\theta|_{\theta=\pi/2} = 1$, whereas for the $B1_s$ solutions, $R(\frac{\pi}{2}) = 0$.

In these series, the value of M_∞ can be calculated by exact integration using Maple 9. The magnetic charge M_0 can be obtained by the use of the approximation methods in the Maple 9 software. The parameter m here can increase in steps of one-half starting from $s + \frac{1}{2}$ for each of the $A1_s$ and $B1_s$ series of solutions.

Hence the B1 series can possess multimonopole at $r = 0$ with magnetic charge, $M_0 = 1, 2, 3, \ldots$ when $m = \frac{1}{2}, 1, \frac{3}{2}, \ldots$ respectively. This multimonopole is surrounded by an equal number of antimonopoles. In Ref.[9], we only discussed multimonopole of even monopole charge. Here we have noticed that m can take half-integer values and hence the multimonopole can also possesses odd values of monopole charge. Therefore when $m = \frac{1}{2}$, the B1 configuration is a pair of monopole and antimonopole. The monopole is at the origin, whilst the antimonopole is at the point $(\sqrt{3}, 0, 0)$.

The screening antimonopoles are of monopole charge -1 each and they are all symmetrically arranged on a circle of radius $r = \sqrt[2m+1]{\frac{m+1}{m}}$ about the multimonopole at the origin. Hence these configurations reside in the topologically trivial sector of the monopole solution and the net topological charge of the B1 solution is zero.

The magnetic charge of the multimonopole at $r = 0$ of the $A1_s$ solutions possess half-integer values of monopole charge when m is a half-odd-integer, whereas the magnetic charge of the multimonopole of the $B1_s$ solutions are always of integer values of monopole charge.

The surrounding screening antimonopoles of the A1 configurations are located on two horizontal circles. Hence the first member of the A1 solution when $m = 1\frac{1}{2}$ has a multimonopole of charge $+2\frac{1}{2}$ at $r = 0$ and partially screened by two antimonopoles located on the x-z plane. The next member has $M_0 = 4$ and $M_A = -4$, and hence the net magnetic charge is zero. As m increases in steps of one-half, M_0 increases by $1\frac{1}{2}$ monopole charge and the screening antimonopoles increases by two.

The subsequence 1_s series has multimonopole charge of $M_0 = (s+2)m-(s+1)s$, net magnetic charge of $M_\infty = s(s+1-m)$, and net antimonopole charge of $M_A = -2(s+1)(m-s)$, Table 2. The $A1_s$ screening series possesses half-integer multimonopole charge when the parameter m is a half-odd-integer, whereas the $B1_s$ screening series possesses only integer values of multimonopole charge. The horizontal layers of screening antimonopoles increase as $(s + 1)$. A more detail discussion of the higher 1_s screening solutions is given in a separate work [13].

5.2 Axially Symmetric Monopole-Vortex Rings

In the AS monopole-vortex ring solution, the parameter b in Eq.(14) and Eq.(15) is set to zero. Hence the profile function $G(\theta,\phi)$ vanishes and the YM gauge potentials and the Higgs field,

Table 2: The 1_s Series of Screening Solutions. Here $b = m - s$ where $s = 0, 1, 2, 3, 4, \ldots$. Each series starts with $m = s + \frac{1}{2}$ and m increases in steps of one-half. When s is odd, we have the $A1_s$ series and when s is zero or even we have the $B1_s$ series.

1_s Series	s	M_0	M_∞	M_A
$B1_0$	0	$2m$	0	$-2m$
$A1_1$	1	$3m - (2 \times 1)$	$1(2 - m)$	$-4(m - 1)$
$B1_2$	2	$4m - (3 \times 2)$	$2(3 - m)$	$-6(m - 2)$
$A1_3$	3	$5m - (4 \times 3)$	$3(4 - m)$	$-8(m - 3)$
$B1_4$	4	$6m - (5 \times 4)$	$4(5 - m)$	$-10(m - 4)$

Eq.(10), are ϕ independence. The solutions for Eq.(14) and (15) are then

$$\psi(r) = \frac{(m+1) - mr^{2m+1}}{1 + r^{2m+1}}, \quad R(\theta) = (m+1)\left\{\cot\theta - \frac{P_{m+1}(\cos\theta)}{P_m(\cos\theta)}\csc\theta\right\}, \tag{31}$$

where P_m is the Legendre polynomial of degree $m = 0, 1, 2, 3, \ldots$. Hence the boundary conditions of the solutions, Eq.(31), are $\psi(0) = m + 1$, $\psi(\infty) = -m$, $R(0) = R(\pi) = 0$. When $m = 0$, the solution is just the radially symmetric Wu-Yang type 1-monopole at the origin [9].

The Abelian magnetic field in the AS solutions reduces to only two components, the $\hat{r}_i$ and the $\hat{\theta}_i$ components,

$$B_i = -\frac{1}{r^2 \sin\theta}\left\{\frac{\partial \sin\alpha}{\partial\theta}\right\}\hat{r}_i + \frac{1}{r\sin\theta}\left\{\frac{\partial \sin\alpha}{\partial r}\right\}\hat{\theta}_i. \tag{32}$$

The topological magnetic charge enclosed by the sphere at infinity, M_∞, is found to be negative one whilst the net topological magnetic charge, M_0, when the radius of the enclosing sphere tends to zero at the origin, is found to be positive one. These results is always true when m is a natural number. Our analysis of these AS solutions [11] shows that there is a composite 1-monopole located at the origin where the Higgs field is singular with two antimonopoles located, one each at $z = \pm \sqrt[2m+1]{\frac{m+1}{m}}$. Hence the AS solution is an antimonopole-monopole-antimonopole (A-M-A) configuration for integer values of $m \geq 1$. When $m = 1$, there is no vortex ring in the A-M-A configuration [9], [11]. The composite 1-monopole has a monopole-antimonopole-monopole (MAM) structure at $r = 0$, Fig.(5).

Vortex ring starts to appear when $m = 2$. The structure of the composite 1-monopole is then MAMAM and the vortex ring is located along the ring where the Higgs field vanishes. This vortex ring centered at the origin has a radius of $r = 1.0845$ on the x-y plane and possess zero net magnetic charge. It seems to be composed of an inner negatively charged ring and an outer positively charged ring at zero distance from each other and yet the charges do not annihilate each other [7], [11]. A plot of the Abelian magnetic field lines along a vertical plane through the z-axis is shown in Fig.(6).

When $m = 3$, two vortex rings appear in the AS configuration. These two vortex rings are horizontally positioned at equidistances, $r = 1.0420$, as the two antimonopoles from the origin. The structure of the composite 1-monopole is then MAMAMAM. A plot of the Abelian magnetic field lines along a vertical plane through the z-axis is shown in Fig.(7).

The number of vortex rings appearing in the AS configuration is equal to $m - 1$ and they are located in space where the Higgs field vanishes along rings. By referring to the magnetic field lines diagrams, Fig.(5), Fig.(6) and Fig.(7), we notice that some kind of a "polarization" process seems to be taking place between the composite 1-monopole at the origin with the two antimonopoles, one on each side of the z-axis and the vortex rings, thus resulting in a MAMA...M structure for the 1-monopole. The number of "poles" in the structure is equal to $(2m+1)$. When m is odd the center of the structure corresponds an antimonopole and when m is even the center correspond to a monopole.

The axially symmetric half-monopole solution is obtained by writing $m = -\frac{1}{2}$ in Eq.(31),

$$\psi(r) = \frac{1}{2}, \quad R(\theta) = \frac{1}{2}\left\{\cot\theta - \frac{P_{\frac{1}{2}}(\cos\theta)}{P_{-\frac{1}{2}}(\cos\theta)}\csc\theta\right\}, \tag{33}$$

where $P_{\pm\frac{1}{2}}(\cos\theta)$ is the Legendre function of the first kind of degree $\pm\frac{1}{2}$. The boundary conditions of the solution, Eq.(33), are $R(0) = 0$, $R(\pi) = -\infty$ and the gauge potentials possess a string singularity along the negative z-axis. Also $\sin\alpha = \frac{\psi\cos\theta - R\sin\theta}{\sqrt{\psi^2+R^2}}$ and $\beta = \pi/2 - \phi$, are independent of distant r. Hence the topological magnetic charge does not depend on the radius of the enclosing sphere and

$$M = M_\infty = -\frac{1}{2}\sin\alpha\Big|_{0, r\to\infty}^{\pi} = \frac{1}{2}. \tag{34}$$

There are also no zeros of the Higgs field in solution (33) at finite r, and there exist only a half-monopole located at the origin, $r = 0$, where the Higgs field is singular. The magnetic field, $B_i = B_r\hat{r}_i$, where $B_r = -\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\alpha$, of the half-monopole is axially symmetric about the z-axis and solely radial in direction. This is in contrast to the Dirac 1-monopole which is radially symmetrical. The magnetic field, B_i, points radially outwards from $\theta = 0\ldots 2.82$ radian, after which it changes sign and points inwards. It blows up along the the negative z-axis giving rise to the string singularity.

The existence of smooth Yang-Mills potentials which correspond to monopole of one-half winding number in the SU(2) YMH field has also been demonstrated in Ref. [15]. The Abelian magnetic field of the half-monopole solution (33) is similar to that of Ref. [15] in that the magnetic field is axially symmetrical and possess a string singularity along the negative z-axis.

6 Comments

In Ref.[9], we have reported on configurations of even number of finitely separated monopoles in the B2 solutions. By noticing that m can take half-integer values with the function G still a single value function, the B2 solutions here can contain both odd and even numbers of finitely separated monopoles, starting with three 1-monopoles configuration when $m = \frac{1}{2}$. Similarly, the number of B1 solutions is doubled when m takes half-integer values and the B1 configuration when $m = \frac{1}{2}$ is just a pair of monopole and antimonopole. No half-integer multimonopole is found in the B series of solutions.

Half-integer topological charge multimonopoles are recorded in the A and C series of monopoles solutions. We have also notice the screening antimonopole are always of unit topological charge. The A2 multimonopole series starts with a multimonopole of topological charge $2\frac{1}{2}$, whereas the C multimonopole series has multimonopole charge of $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$, Fig.(8). Detail studies of the axially symmetric monopole solutions [11], half-monopole solutions [12], and higher series 1_s solutions [13], are reported separately.

We have found that the SU(2) YMH theory does support monopole of one-half topological charge [12], as well as multimonopole of half-integer topological charge [13], [14]. Also a 1-monopole need not be radially symmetrical as the 1-monopole of the C solution when $m = 1$, possesses only mirror symmetry and is actually made up of two one-half monopoles. There are only two isolated half-monopole solutions of the ansatz (10). The one-half topological charge of the AS solution when $m = -\frac{1}{2}$ is axially symmetrical about the z-axis with a string singularity along the negative z-axis. The one-half topological charge of the C solution when $m = \frac{1}{2}$ possesses only mirror symmety at the x-z plane and also has a string singularity along the negative z-axis. The magnetic field of this C half-monopole is stronger in the positive x and z region, Fig.(4).

The profile functions, $\psi(r)$, and $G(\theta, \phi)$ of Eq.(16) are standard solutions for all the monopoles solutions of the ansatz (10). It is the profile function, $R(\theta)$, that determines the different types of monopoles configurations of the gauge field potentials (10). The positions of the antimonopoles of the 1_s screening solutions and the monopoles of the $B2_s$ series about the origin are determined by the zeros of $\psi(r)$, $R(\theta)$, and $G(\theta, \phi)$.

We have also proved that for every monopoles solutions of ansatz (10), there always exist an anti-configuration of this solution where the directions of its Abelian magnetic field and hence its topological magnetic charge sign are reversed [14]. A point plot of the topological magnetic charges M_∞ versus M_0 which summarizes our work is shown in Fig.(8) for the A1, A2, B1, B2, C, AS series of solutions and their anti-configurations.

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References

[1] G. 't Hooft, Nucl. Phy. **B 79** (1974) 276; A.M. Polyakov, Sov. Phys. - JETP **41** (1975) 988; Phys. Lett. **B 59** (1975) 82; JETP Lett. **20** (1974) 194.

[2] E.B. Bogomol'nyi and M.S. Marinov, Sov. J. Nucl. Phys. **23** (1976) 357.

[3] M.K. Prasad and C.M. Sommerfield, Phys. Rev. Lett. **35** (1975) 760.

[4] C. Rebbi and P. Rossi, Phys. Rev. **D 22** (1980) 2010; R.S. Ward, Commun. Math. Phys. **79** (1981) 317; P. Forgacs, Z. Horvarth and L. Palla, Phys. Lett. **B 99** (1981) 232; Nucl. Phys. **B 192** (1981) 141; M.K. Prasad, Commun. Math. Phys. **80** (1981) 137; M.K. Prasad and P. Rossi, Phys. Rev. **D 24** (1981) 2182.

[5] E.J. Weinberg and A.H. Guth, Phys. Rev. **D 14** (1976) 1660.

[6] E.B. Bogomol'nyi, Sov. J. Nucl. Phys. **24** (1976) 449.

[7] B. Kleihaus and J. Kunz, Phys. Rev. **D 61** (2000) 025003; B. Kleihaus, J. Kunz, and Y. Shnir, Phys. Lett. **B 570** (2003) 237; Phys. Rev. **D 68** (2003) 101701.

[8] P.M. Sutcliffe, Int. J. Mod. Phys. **A 12** (1997) 4663; C.J. Houghton, N.S. Manton and P.M. Sutcliffe, Nucl.Phys. **B 510** (1998) 507.

[9] Rosy Teh, Int. J. Mod. Phys. **A 16** (2001) 3479; J. Fiz. Mal. **23** (2002) 196; Rosy Teh and K.M. Wong, Int. J. Mod. Phys. **A 19** (2004) 371.

[10] N.S. Manton, Nucl. Phys. (N.Y.) **B 126** (1977) 525.

[11] Rosy Teh and Khai-Ming Wong, *Monopole-Antimonopole and Vortex Rings*, USM Preprint 2004; Hep-th/0406075.

[12] Rosy Teh and Khai-Ming Wong, *Half-Monopoles and Multimonopole*, USM Preprint 2004; Hep-th/0406059; accepted for publication in the Int. J. Mod. Phys. A.

[13] Rosy Teh and Khai-Ming Wong, *Screening Solutions of Multimonopole by Unit Charge Antimonopoles*, Contributed paper to be presented in the "Persidangan Fizik Kebangsaan", PERFIK 04, Octber 5-7, 2004, Palace of the Golden Horses, Kuala Lumpur, Malaysia.

[14] Rosy Teh and Khai-Ming Wong, *Static Monopoles and their Anti-Configurations*, USM Preprint 2004; Hep-th/0406094.

[15] E. Harikumar, I. Mitra, and H.S. Sharatchandra, Phys. Lett. **B 557** (2003) 303.

Figure Captions

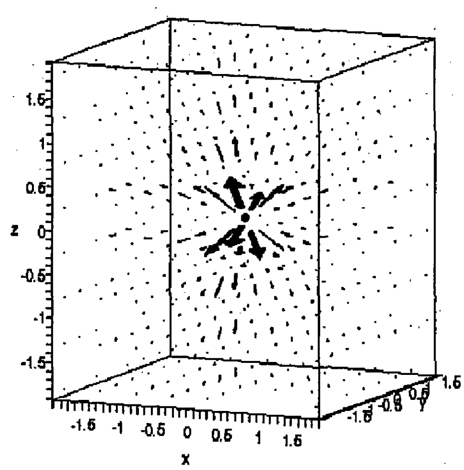

Figure 1: The Abelian magnetic field, B_i, of the A2 solution when $m = -\frac{1}{2}$. The $2\frac{1}{2}$-monopole is located at the origin of the coordinate axes.

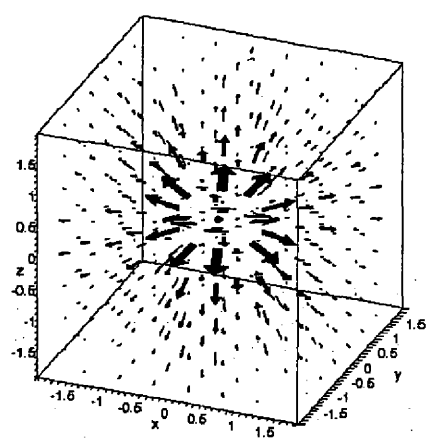

Figure 2: The Abelian magnetic field of the A2 solution when $m = 0$. The 3-monopole is located at the origin of the coordinate axes.

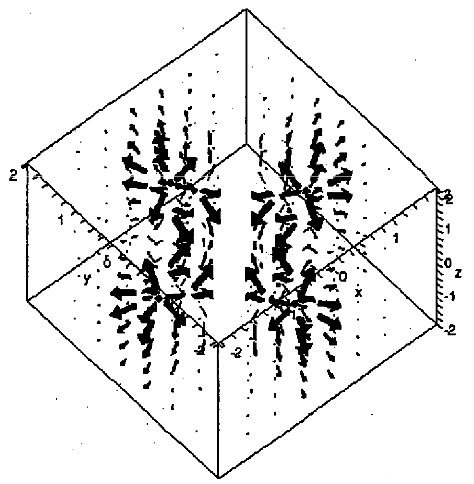

Figure 3: The Abelian magnetic field of the B2 solution when $m = 1$, showing the four 1-monopoles at $r = 1.2599$, $\theta = 0$ and $\phi = 0, \frac{1}{2}\pi, \pi, \frac{3}{2}\pi$.

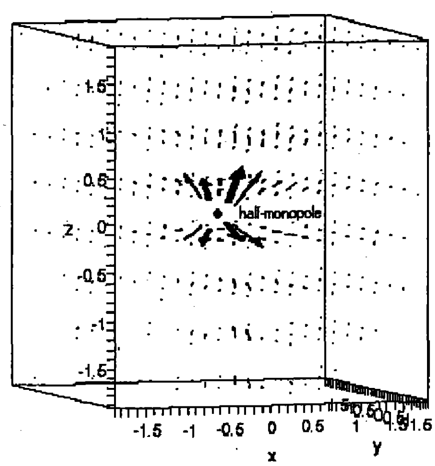

Figure 4: The Abelian magnetic field of the C half-monopole. The half-monopole is located at $r = 0$. The field is stronger on the positive half of the x and z-axes.

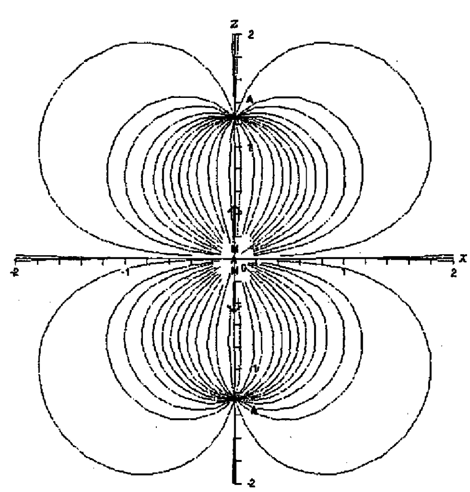

Figure 5: A plot of the magnetic field lines of the AS solution when $m = 1$. The two anti-monopoles are located at $z = \pm 1.2599$.

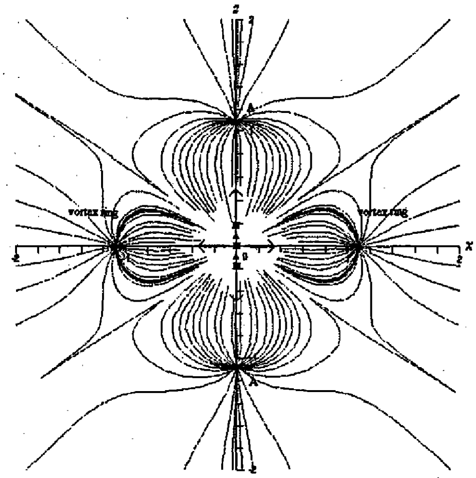

Figure 6: A plot of the magnetic field lines of the AS solution when $m = 2$. The vortex ring is and the two antimonopoles are at distances 1.0845 from the origin.

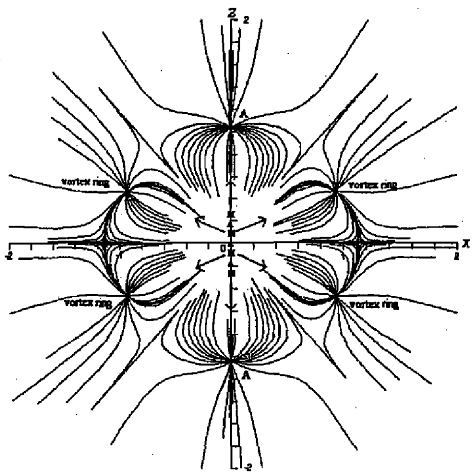

Figure 7: A plot of the magnetic field lines of the AS solution when $m = 3$. The two vortex rings and the two antimonopoles are at distances 1.0420 from the origin.

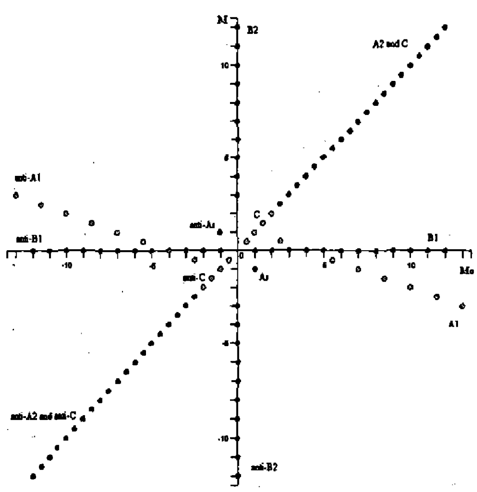

Figure 8: A point plot of the net magnetic charges M_∞ versus the magnetic charges M_0.