An EWMA Control Chart for Monitoring the Mean of Skewed Populations Using Weighted Variance

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Abstract – **This paper discusses the use of weighted variance (WV) in setting up the limits of the exponentially weighted moving average (EWMA) chart for the monitoring of the mean of a process from a skewed population. This chart, called the WV–EWMA chart hereafter, reduces to the standard EWMA chart when the underlying distribution is symmetric. The Type-I and Type-II errors of the WV–EWMA chart are compared with that of the existing charts for skewed populations. Simulation results show that the new method gives a considerable improvement over the existing methods when the underlying distribution is skewed.**

Keywords – **EWMA chart, $\bar{X}$ chart, skewed populations, Type-I error, Type-II error, weighted variance, weighted standard deviation, skewness correction**

I. INTRODUCTION

A control chart is used to monitor a process. Standard variable charts assume that the quality characteristic is normally distributed. In many situations, this assumption does not hold. For example, chemical, semiconductor and cutting tool wear processes are often skewed [1]. For a skewed population, the Type–I error of standard charts increases with the skewness. Three different approaches that are currently used to deal with skewed populations are transformation, increasing the sample size and heuristic control charts. The heuristic charts that are available include the $\bar{X}$ and R charts based on the weighted variance (WV) [2], weighted standard deviation (WSD) [1] and skewness correction (SC) [3] methods as well as the EWMA chart based on the WSD method [1]. Other works on skewed univariate charts are reported in [4] – [10]. This paper considers the use of weighted variance (WV) to compute the limits of an EWMA chart. This method is found to perform well when the distribution is skewed.

II. LITERATURE REVIEW OF HEURISTIC CHARTS FOR MEAN

A. Weighted Variance (WV) $\bar{X}$ Chart

The WV method is based on the idea that a skewed distribution can be splitted into two segments at its average and each segment is used for creating a new symmetric distribution. The WV method uses both created symmetric distributions in setting up the chart's limits. The limits of the WV $\bar{X}$ chart are [2]

$$UCL_{\text{WV}} = \mu + 3\frac{\sigma}{\sqrt{n}}\sqrt{2P_X} \tag{1a}$$

and

$$LCL_{\text{WV}} = \mu - 3\frac{\sigma}{\sqrt{n}}\sqrt{2(1-P_X)}, \tag{1b}$$

where P_X is the probability that a random variable X will be less than or equal to its mean μ. Note that the WV $\bar{X}$ chart reduces to the standard $\bar{X}$ chart when $P_X = 0.5$. Here, σ denotes the standard deviation of X while n represents the sample size. The notations defined here will also be used in the later sections. When parameters are unknown, the control limits are computed using the following formulae [2]:

$$UCL_{\text{WV}} = \bar{\bar{X}} + \frac{3\bar{R}}{d_2'\sqrt{n}}\sqrt{2\hat{P}_X} \tag{2a}$$

and

$$LCL_{\text{WV}} = \bar{\bar{X}} - \frac{3\bar{R}}{d_2'\sqrt{n}}\sqrt{2(1-\hat{P}_X)}, \tag{2b}$$

where

$$\hat{P}_X = \frac{\sum_{i=1}^{k}\sum_{j=1}^{n}\delta\left(\bar{\bar{X}} - X_{ij}\right)}{n \times k} \tag{3}$$

with k and n denoting the number of samples and the number of observations in a sample, respectively, and $\delta(x) = 1$ for $x \geq 0$ or $\delta(x) = 0$ for $x < 0$. In Equations (2a) and (2b), $\bar{\bar{X}}$ denotes the grand mean, $\bar{R}$ the mean of the sample ranges and d_2' the constant for a given skewed population corresponding to the constant d_2 for a normal distribution. The constant d_2' can be computed via numerical integration for a given skewed distribution [2].

B. Weighted Standard Deviation (WSD) $\bar{X}$ Chart

The WSD method uses the same approach as the WV method, i.e., by splitting a skewed distribution into two segments. The limits of the WSD $\bar{X}$ chart are [1]

$$UCL_{\text{WSD}} = \mu + 3\frac{\sigma}{\sqrt{n}}2P_X \tag{4a}$$

and

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$$LCL_{\text{WSD}} = \mu - 3\frac{\sigma}{\sqrt{n}} 2(1-P_X). \tag{4b}$$

When parameters are unknown, the control limits are computed using the following formulae [1]:

$$UCL_{\text{WSD}} = \bar{\bar{X}} + 3\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\left(2\hat{P}_X\right) \tag{5a}$$

and

$$LCL_{\text{WSD}} = \bar{\bar{X}} - 3\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\left[2(1-\hat{P}_X)\right]. \tag{5b}$$

Here,

$$d_2^{\text{WSD}} \equiv P_X d_2\left(2n(1-P_X)\right) + (1-P_X)\, d_2\left(2nP_X\right), \tag{6}$$

where $d_2(n)$ is d_2 for the normal distribution when the sample size is *n*. If P_X is unknown, we can use $\hat{P}_X$ in Equation (3) to compute d_2^{WSD}. Note that $d_2^{WSD} = d_2$ if the underlying distribution is symmetric [1].

C. Skewness Correction (SC) $\bar{X}$ Chart

Reference [3] proposes the SC method based on the Cornish–Fisher expansion. The SC $\bar{X}$ chart is based on the following limits when parameters are known [3]:

$$UCL_{\text{SC}} = \mu + \left(3 + c_4^*\right)\frac{\sigma}{\sqrt{n}} \tag{7a}$$

and

$$LCL_{\text{SC}} = \mu + \left(-3 + c_4^*\right)\frac{\sigma}{\sqrt{n}}. \tag{7b}$$

Here,

$$c_4^* = \frac{\frac{4}{3}\kappa_3(\bar{X})}{1 + 0.2\kappa_3^2(\bar{X})}, \tag{8}$$

where $\kappa_3(\bar{X})$ is the skewness coefficient of the sample mean $\bar{X}$. Note that $\kappa_3(\bar{X}) = \frac{\kappa_3}{\sqrt{n}}$, where κ_3 is the skewness coefficient of *X*. When parameters are unknown, the control limits are computed using the following formulae [3]:

$$UCL_{\text{SC}} = \bar{\bar{X}} + \left(3 + \frac{4\kappa_3/\left(3\sqrt{n}\right)}{1 + 0.2\,\kappa_3^2/n}\right)\frac{\bar{R}}{d_2'\sqrt{n}} \tag{9a}$$

and

$$LCL_{\text{SC}} = \bar{\bar{X}} + \left(-3 + \frac{4\kappa_3/\left(3\sqrt{n}\right)}{1 + 0.2\,\kappa_3^2/n}\right)\frac{\bar{R}}{d_2'\sqrt{n}} \tag{9b}$$

D. Weighted Standard Deviation (WSD) EWMA chart

The WSD–EWMA chart statistics are [1]

$$E_i = \lambda\bar{X}_i + (1-\lambda)E_{i-1},\ \text{for } i = 1, 2, \ldots, \tag{10}$$

where $0 < \lambda \le 1$ is a smoothing constant and $E_0 = \mu$. The limits of the WSD–EWMA chart are [1]

$$UCL_{\text{WSD-EWMA}} = \mu + L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left(2P_X\right) \tag{11a}$$

and

$$LCL_{\text{WSD-EWMA}} = \mu - L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left[2(1-P_X)\right]. \tag{11b}$$

The selection of λ and L is based on the approach discussed in [11]. The WSD–EWMA chart reduces to the standard EWMA chart when $P_X = 0.5$. When parameters are unknown, the control limits are computed using the following formulae [1]:

$$UCL_{\text{WSD-EWMA}} = \bar{\bar{X}} + L\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left(2\hat{P}_X\right) \tag{12a}$$

and

$$LCL_{\text{WSD-EWMA}} = \bar{\bar{X}} - L\frac{\bar{R}}{d_2^{\text{WSD}}\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\left[2(1-\hat{P}_X)\right]. \tag{12b}$$

III. A PROPOSED WEIGHTED VARIANCE (WV) EWMA CHART

Let $\bar{X}_i$, for $i = 1, 2, \ldots$, denote a sequence of sample means for a process. The successive EWMA statistics can be described by Equation (10) and the limits of the WV–EWMA chart can be computed as follows when parameters are known:

$$UCL_{\text{WV-EWMA}} = \mu + L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{2P_X} \tag{13a}$$

and

$$LCL_{\text{WV-EWMA}} = \mu - L\frac{\sigma}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left[2(1-P_X)\right]}. \tag{13b}$$

The WV method decomposes the variance into two parts while the WSD method decomposes the standard deviation into two parts. In the WV method, $(2P_X)\sigma^2$ and $\left[2(1-P_X)\right]\sigma^2$ are used in place of σ^2 for computing $UCL_{\text{WV-EWMA}}$ and $LCL_{\text{WV-EWMA}}$, respectively. On the contrary, in the WSD method, $(2P_X)\sigma$ and $\left[2(1-P_X)\right]\sigma$ are used in place of σ for computing $UCL_{\text{WSD-EWMA}}$ and $LCL_{\text{WSD-EWMA}}$, respectively. When process parameters are unknown, the limits are computed as follows:

$$UCL_{\text{WV-EWMA}} = \bar{\bar{X}} + L\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left(2\hat{P}_X\right)} \tag{14a}$$

and

$$LCL_{\text{WV-EWMA}} = \bar{\bar{X}} - L\frac{\bar{R}}{d_2'\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\left[2\left(1-\hat{P}_X\right)\right]} \quad (14b)$$

IV. PERFORMANCE OF THE WV–EWMA CHART

The WV–EWMA chart is compared with the existing heuristic charts for skewed data, the standard $\bar{X}$ chart and the standard EWMA chart, in terms of the Type-I and Type-II error rates, computed via a Monte Carlo simulation using SAS. The Type-I error rate is defined as the probability of signaling an out-of-control even though the process is actually in-control. On the contrary, the Type-II error rate is defined as the probability of failing to signal an out-of-control although the process has shifted. The existing heuristic charts considered are the WV $\bar{X}$, WSD $\bar{X}$, SC $\bar{X}$ and WSD–EWMA charts. In this paper, the WSD–CUSUM chart suggested in [1] is not considered in the performance comparison, because both the WSD–CUSUM and the WSD–EWMA charts have almost the same performance [1]. Hence, we only consider the WSD–EWMA chart.

The skewed distributions considered are Weibull and gamma because they represent a wide variety of shapes from symmetric to highly skewed. For the sake of comparison, the standard normal distribution is also considered. For convenience, a scale parameter of one is used for the Weibull and gamma distributions. Note that P_X for the Weibull [2] and gamma [12] distributions are

$$P_X = 1 - \exp\left[-\left\{\Gamma\left(1+\frac{1}{\beta}\right)\right\}^{\beta}\right] \quad (15)$$

and

$$P_X = F(\eta), \quad (16)$$

respectively, where β and η are both shape parameters. Here, $\Gamma(\cdot)$ is the gamma function and $F(\cdot)$ is the gamma distribution function.

The skewness coefficients considered for the Weibull and gamma distributions are $\alpha_3 \in \{0, 1, 2, 3\}$. The shape parameters corresponding to these values of α_3 are $\beta \in \{3.6286, 1.5688, 0.9987, 0.7637\}$ and $\eta \in \{38000, 3.913, 0.983, 0.442\}$ for the Weibull and gamma distributions, respectively. The in-control means of the Weibull and gamma distributions are

$$\mu = \Gamma\left(1+\frac{1}{\beta}\right) \quad (17)$$

and

$$\mu = \eta, \quad (18)$$

respectively, while their in-control standard deviations are

$$\sigma = \sqrt{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2} \quad (19)$$

and

$$\sigma = \sqrt{\eta}, \quad (20)$$

respectively. The out-of-control mean is $\mu_1 = \mu + \delta\sigma$, where $\delta \in \{0.25, 0.5, 1, 2\}$.

All the charts considered in this study are designed based on an in-control ARL of 370 when the process follows a normal distribution. The combinations of $(\lambda, L) \in \{(0.1, 2.6952), (0.2, 2.8537), (0.3, 2.9286), (0.4, 2.9614)\}$ for the standard EWMA, WSD–EWMA and WV–EWMA charts are determined using the plots in [11]. The simulated results are tabulated in Tables I and II for the Type-I and Type-II error rates, respectively. Here, the Type-I error rate of a chart is computed as the proportion of sample points (based on 1,000,000 samples) plotting beyond its control limits when the process is in-control. On the contrary, the computation of the Type-II error rate is made based on the proportion of sample points (from 1,000,000 samples) plotting within the limits of a chart when the process is out-of-control. Only the Type-II error rates for the gamma distribution are shown because of space constraint.

The average run length (ARL) performance is not considered because the ARL values can be easily computed from the Type-I and Type-II errors in Tables I and II, respectively. Here, the in-control ARL is $\text{ARL}_0 = 1/\alpha$, while the out-of-control ARL is $\text{ARL}_1 = 1/(1-\beta)$, where α and β are the Type-I and Type-II error rates, respectively. Since the interpretation based on ARL or Type-I and Type-II errors are the same, we only consider the Type-I and Type-II errors in this study.

In general, Table I shows that the proposed WV–EWMA chart with $\lambda \le 0.2$ gives a lower Type-I error rate compared to the other charts when $n = 3$ or 5 (see the boldfaced values). When $n = 1$, the WV–EWMA chart with $\lambda = 0.1$ has the lowest Type-I error rate (also, see the boldfaced values). Since $0.05 < \lambda < 0.25$ is recommended [13] and due to the fact that the WV–EWMA chart provides a fovourable rate of Type-I error compared with the other charts when $0.05 < \lambda < 0.25$, the WV–EWMA chart is recommended. Table II shows that the WV–EWMA chart has the lowest Type-II error rate compared with the WSD–EWMA charts for all δ and $\lambda \in \{0.1, 0.2, 0.3, 0.4\}$. The WV–EWMA chart also has lower Type-II error rates than the $\bar{X}$ charts for skewed populations and the standard $\bar{X}$ chart when $\delta \le 1$. However, the WV–EWMA chart has a slightly higher Type-II error rate than the standard EWMA chart. Since the standard EWMA chart has a very high Type-I error rate compared with the WV–EWMA and WSD–EWMA charts when the skewness is large, the standard EWMA chart is less attractive. Furthermore, when $\lambda = 0.3$ and $\lambda =$

0.4, the standard EWMA chart has a higher Type-I error rate than the $\bar{X}$ charts based on the WSD and SC methods.

V. CONCLUSION

This paper proposes the WV–EWMA chart for the mean. The new chart gives a more favourable performance than all existing heuristic charts for skewed populations, in terms of the Type-I and Type-II errors.

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TABLE I
TYPE-I ERROR RATES FOR THE VARIOUS CHARTS

Sample Size, n	Distribution	Skewness, α_3	EWMA Charts												$\overline{X}$ Charts			
			$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.3$			$\lambda = 0.4$						
			WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	SC Method	Standard Method
1	Normal	0	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026
	Weibull	0	**0.0026**	0.0026	0.0026	0.0025	0.0025	0.0025	0.0021	0.0021	0.0021	0.0019	0.0019	0.0019	0.0007	0.0007	0.0007	0.0007
		1	**0.0026**	0.0029	0.0029	0.0028	0.0024	0.0038	0.0033	0.0023	0.0047	0.0041	0.0027	0.0058	0.0072	0.0050	0.0015	0.0098
		2	**0.0027**	0.0034	0.0037	0.0039	0.0026	0.0062	0.0055	0.0034	0.0086	0.0071	0.0045	0.0108	0.0125	0.0084	0.0042	0.0183
		3	**0.0028**	0.0040	0.0044	0.0047	0.0028	0.0077	0.0068	0.0041	0.0105	0.0087	0.0054	0.0130	0.0144	0.0095	0.0074	0.0213
	Gamma	0	**0.0028**	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0026	0.0026	0.0027	0.0027	0.0027	0.0027	0.0027
		1	**0.0027**	0.0028	0.0030	0.0031	0.0026	0.0040	0.0037	0.0028	0.0051	0.0046	0.0033	0.0063	0.0079	0.0059	0.0020	0.0106
		2	**0.0027**	0.0033	0.0037	0.0039	0.0026	0.0063	0.0055	0.0033	0.0087	0.0072	0.0044	0.0110	0.0128	0.0084	0.0043	0.0186
		3	**0.0026**	0.0046	0.0045	0.0047	0.0025	0.0078	0.0067	0.0037	0.0108	0.0086	0.0050	0.0137	0.0148	0.0092	0.0071	0.0224
3	Normal	0	0.0028	0.0028	0.0028	0.0028	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0	**0.0027**	0.0027	0.0027	**0.0026**	0.0027	0.0026	0.0025	0.0025	0.0025	0.0024	0.0024	0.0024	0.0020	0.0021	0.0021	0.0021
		1	**0.0028**	0.0032	0.0028	**0.0027**	0.0030	0.0030	0.0027	0.0026	0.0034	0.0028	0.0024	0.0039	0.0038	0.0025	0.0017	0.0059
		2	**0.0029**	0.0046	0.0030	**0.0029**	0.0040	0.0043	0.0033	0.0032	0.0056	0.0039	0.0028	0.0069	0.0071	0.0039	0.0021	0.0118
		3	**0.0031**	0.0065	0.0035	**0.0032**	0.0054	0.0056	0.0042	0.0038	0.0076	0.0054	0.0032	0.0095	0.0097	0.0053	0.0035	0.0163
	Gamma	0	**0.0027**	0.0027	0.0027	**0.0027**	0.0027	0.0027	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026	0.0026
		1	**0.0028**	0.0032	0.0029	**0.0027**	0.0030	0.0031	0.0028	0.0027	0.0035	0.0030	0.0026	0.0040	0.0043	0.0029	0.0026	0.0063
		2	**0.0030**	0.0047	0.0030	**0.0029**	0.0042	0.0042	0.0032	0.0032	0.0055	0.0039	0.0028	0.0070	0.0072	0.0041	0.0023	0.0118
		3	**0.0033**	0.0078	0.0034	**0.0030**	0.0068	0.0055	0.0039	0.0045	0.0075	0.0050	0.0030	0.0096	0.0091	0.0046	0.0033	0.0165
5	Normal	0	0.0027	0.0027	0.0027	0.0027	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
	Weibull	0	**0.0027**	0.0027	0.0027	**0.0027**	0.0027	0.0027	0.0026	0.0026	0.0026	0.0025	0.0025	0.0025	0.0023	0.0023	0.0023	0.0023
		1	0.0029	0.0033	0.0028	**0.0027**	0.0032	0.0029	0.0026	0.0029	0.0030	0.0026	0.0026	0.0033	0.0031	0.0021	0.0021	0.0048
		2	0.0031	0.0051	0.0029	**0.0029**	0.0049	0.0036	0.0029	0.0042	0.0045	0.0031	0.0035	0.0056	0.0052	0.0026	0.0021	0.0092
		3	0.0034	0.0073	0.0032	**0.0032**	0.0071	0.0047	0.0034	0.0058	0.0063	0.0041	0.0045	0.0078	0.0073	0.0036	0.0027	0.0133
	Gamma	0	**0.0028**	0.0028	0.0028	**0.0028**	0.0028	0.0028	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027	0.0027
		1	**0.0028**	0.0032	0.0028	**0.0027**	0.0032	0.0030	0.0026	0.0029	0.0031	0.0027	0.0028	0.0034	0.0033	0.0024	0.0026	0.0049
		2	0.0032	0.0052	0.0028	**0.0029**	0.0050	0.0036	0.0028	0.0043	0.0046	0.0032	0.0035	0.0056	0.0053	0.0027	0.0022	0.0094
		3	0.0036	0.0088	0.0031	**0.0031**	0.0090	0.0047	0.0031	0.0075	0.0063	0.0037	0.0056	0.0078	0.0069	0.0032	0.0025	0.0134

TABLE II
TYPE-II ERROR RATES FOR THE VARIOUS CHARTS WHEN THE UNDERLYING DISTRIBUTION IS GAMMA

Sample Size, n	Shape Parameter, η	Skewness, α_3	Shift, δ	EWMA Charts												$\overline{X}$ Charts			
				$\lambda = 0.1$			$\lambda = 0.2$			$\lambda = 0.3$			$\lambda = 0.4$						
				WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	Standard Method	WV Method	WSD Method	SC Method	Standard Method
1	38000	0	0.25	0.9888	0.9888	0.9888	0.9918	0.9918	0.9917	0.9934	0.9935	0.9934	0.9943	0.9943	0.9943	0.9964	0.9964	0.9964	0.9964
			0.50	0.9645	0.9646	0.9645	0.9723	0.9724	0.9723	0.9789	0.9790	0.9788	0.9833	0.9833	0.9832	0.9935	0.9936	0.9935	0.9935
			1.00	0.8972	0.8973	0.8971	0.8978	0.8980	0.8977	0.9088	0.9090	0.9086	0.9216	0.9218	0.9215	0.9771	0.9772	0.9770	0.9770
			2.00	0.7605	0.7607	0.7603	0.7213	0.7216	0.7211	0.7060	0.7063	0.7058	0.7023	0.7025	0.7020	0.8417	0.8422	0.8412	0.8412
	3.913	1	0.25	0.9898	0.9921	0.9872	0.9906	0.9931	0.9877	0.9904	0.9930	0.9874	0.9900	0.9927	0.9876	0.9885	0.9916	0.9971	0.9847
			0.50	0.9699	0.9745	0.9649	0.9754	0.9806	0.9695	0.9783	0.9835	0.9724	0.9796	0.9848	0.9736	0.9835	0.9879	0.9957	0.9783
			1.00	0.9088	0.9173	0.9000	0.9141	0.9256	0.9022	0.9253	0.9381	0.9116	0.9350	0.9478	0.9208	0.9670	0.9752	0.9911	0.9574
			2.00	0.7771	0.7918	0.7620	0.7460	0.7654	0.7259	0.7393	0.7634	0.7150	0.7456	0.7745	0.7170	0.8799	0.9075	0.9631	0.8486
	0.983	2	0.25	0.9907	0.9942	0.9865	0.9901	0.9939	0.9852	0.9889	0.9934	0.9833	0.9875	0.9921	0.9814	0.9836	0.9892	0.9945	0.9763
			0.50	0.9744	0.9818	0.9662	0.9781	0.9857	0.9689	0.9791	0.9866	0.9697	0.9800	0.9864	0.9694	0.9791	0.9862	0.9930	0.9695
			1.00	0.9194	0.9340	0.9038	0.9286	0.9470	0.9085	0.9391	0.9574	0.9175	0.9460	0.9633	0.9248	0.9655	0.9774	0.9885	0.9501
			2.00	0.7903	0.8164	0.7621	0.7664	0.8024	0.7282	0.7703	0.8139	0.7239	0.7863	0.8351	0.7335	0.9074	0.9392	0.9690	0.8650
	0.442	3	0.25	0.9916	0.9954	0.9863	0.9902	0.9946	0.9841	0.9886	0.9935	0.9816	0.9867	0.9923	0.9791	0.9820	0.9889	0.9909	0.9727
			0.50	0.9783	0.9866	0.9680	0.9807	0.9889	0.9697	0.9806	0.9888	0.9695	0.9798	0.9882	0.9685	0.9782	0.9866	0.9890	0.9667
			1.00	0.9283	0.9471	0.9076	0.9404	0.9615	0.9153	0.9496	0.9693	0.9247	0.9545	0.9727	0.9309	0.9675	0.9802	0.9840	0.9501
			2.00	0.8004	0.8350	0.7621	0.7798	0.8306	0.7274	0.7954	0.8553	0.7237	0.8212	0.8819	0.7476	0.9263	0.9561	0.9648	0.8833
5	38000	0	0.25	0.9573	0.9573	0.9573	0.9655	0.9656	0.9654	0.9732	0.9733	0.9732	0.9787	0.9787	0.9786	0.9925	0.9925	0.9925	0.9925
			0.50	0.8805	0.8805	0.8805	0.8771	0.8772	0.8769	0.8866	0.8868	0.8864	0.9001	0.9003	0.8999	0.9699	0.9699	0.9699	0.9699
			1.00	0.7301	0.7301	0.7301	0.6821	0.6823	0.6818	0.6596	0.6599	0.6593	0.6488	0.6492	0.6485	0.7778	0.7778	0.7778	0.7778
			2.00	0.4999	0.4999	0.4999	0.3822	0.3826	0.3817	0.2640	0.2646	0.2634	0.1819	0.1825	0.1814	0.0707	0.0707	0.0707	0.0707
	3.913	1	0.25	0.9632	0.9686	0.9575	0.9714	0.9778	0.9643	0.9774	0.9835	0.9702	0.9809	0.9866	0.9740	0.9898	0.9932	0.9953	0.9850
			0.50	0.8914	0.9008	0.8817	0.8927	0.9058	0.8791	0.9047	0.9204	0.8885	0.9181	0.9347	0.9004	0.9708	0.9800	0.9854	0.9590
			1.00	0.7471	0.7631	0.7306	0.7050	0.7259	0.6835	0.6880	0.7134	0.6620	0.6842	0.7146	0.6535	0.8344	0.8753	0.9023	0.7876
			2.00	0.5099	0.5282	0.4942	0.4265	0.4556	0.3899	0.3281	0.3758	0.2750	0.2408	0.2960	0.1862	0.0889	0.1377	0.1865	0.0547
	0.983	2	0.25	0.9632	0.9686	0.9575	0.9714	0.9778	0.9643	0.9774	0.9835	0.9702	0.9809	0.9866	0.9740	0.9898	0.9932	0.9953	0.9850
			0.50	0.8914	0.9008	0.8817	0.8927	0.9058	0.8791	0.9047	0.9204	0.8885	0.9181	0.9347	0.9004	0.9708	0.9800	0.9854	0.9590
			1.00	0.7471	0.7631	0.7306	0.7050	0.7259	0.6835	0.6880	0.7134	0.6620	0.6842	0.7146	0.6535	0.8344	0.8753	0.9023	0.7876
			2.00	0.5221	0.5715	0.4889	0.4525	0.4885	0.3988	0.3803	0.4426	0.2879	0.3019	0.3909	0.1891	0.1150	0.2739	0.3730	0.0296
	0.442	3	0.25	0.9721	0.9834	0.9589	0.9790	0.9899	0.9636	0.9820	0.9918	0.9667	0.9831	0.9925	0.9681	0.9861	0.9937	0.9948	0.9732
			0.50	0.9088	0.9304	0.8850	0.9174	0.9461	0.8850	0.9312	0.9610	0.8948	0.9419	0.9700	0.9050	0.9723	0.9872	0.9895	0.9471
			1.00	0.7727	0.8103	0.7313	0.7408	0.7916	0.6861	0.7355	0.7986	0.6688	0.7447	0.8193	0.6662	0.8953	0.9497	0.9584	0.8111
			2.00	0.5376	0.6125	0.4843	0.4663	0.5144	0.4078	0.4153	0.4731	0.3008	0.3526	0.4415	0.1886	0.1501	0.4568	0.5266	0.0001