

UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang 1992/93

Oktober/November, 1992

ASP300 - STATISTIK PERNIAGAAN

Masa: [3 jam]

ARAHAN

Sila pastikan bahawa kertas peperiksaan ini mengandungi Lapan Belas muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab ENAM soalan. Soalan 1 dan 2 adalah WAJIB. Jawab EMPAT soalan yang lain.

1. (a) (i) Bagaimakah regresi berganda berbeza dari regresi mudah?
 - (ii) Dalam analisis regresi berganda, jika model secara keseluruhan adalah signifikan, mengapakah kita masih perlu menguji setiap koefisien individu?
 - (iii) Terangkan dengan ringkas, multikekolinearan dan kenalpasti satu cara yang mudah untuk mengurangkan atau mengelakkannya.

[6 markah]

...2/-

- (b) Analisis regresi direka untuk membantu ramalan kadar muatan penumpang, Y, bagi sebuah pesawat berdasarkan enam pembolehubah peramal. Sebahagian output komputer diberikan di bawah:

Correlation matrix

Y	1.00	0.48	-0.26	0.74	0.48	0.72	0.56
X ₁	0.48	1.00	0.32	0.55	0.85	0.26	0.18
X ₂	-0.26	0.32	1.00	0.16	0.37	0.27	0.14
X ₃	0.74	0.55	0.16	1.00	0.18	0.64	0.36
X ₄	0.08	0.85	0.37	0.18	1.00	0.21	0.14
X ₅	0.72	0.26	0.27	0.64	0.21	1.00	0.32
X ₆	0.56	0.18	0.19	0.36	0.14	0.32	1.00

R - squared 0.921

Adj R - squared 0.896

Standard error 3.862

Analisis Jadual Varians

Source	DF	Sum of square	Mean square	F
Regression	6	7904,884		
Residual	20	3008,968		
Total	26	10913,852		

Regression Coefficients Table

Parameter	Value	Std. error	t-value
Intercept	240.8	80.666	
X ₁ Ticket cost	-0.452	0.327	
X ₂ Check-in min	-0.763	0.642	
X ₃ Depart-time	-0.224	0.998	
X ₄ Food items	-3.326	1.443	
X ₅ Arrival-time	0.32	0.357	
X ₆ Bag-retrieval	-0.841	1.09	

...3/-

- (i) Tuliskan persamaan garis regresi berganda.
- (ii) Adakah regresi ini bererti pada aras 0.05?
- (iii) Apakah peratusan varians bagi kadar muatan penumpang yang dapat dijelaskan oleh enam pembolehubah itu?
- (iv) Apakah pembollehubah-pembollehubah peramal yang mempunyai hubungan yang kadar bererti dengan muatan penumpang? Guna tahap 0.05.
- (v) Apakah pembollehubah-pembollehubah peramal yang berkemungkinan besar menyebabkan masalah multikolinearan.

[14 markah]

2. (a) Sampel rawak yang tidak bersandar dipilih dari 2 populasi dengan keputusan seperti berikut:

Sampel 1	10	7	8	11	10	9	9
Sampel 2	12	8	13	0	10	11	

Dengan menggunakan ANOVA, uji hipotesis yang sampel ini dipilih dari dua populasi yang mempunyai min yang sama.

[10 markah]

- (b) Analisis varians bagi analisis dua arah memberi jadual ANOVA (separa) yang seperti berikut:

Source	DF	SS	MS	F
Treatments	3	27.1	-	-
Rows	5	-	14.90	-
Error	-	33.4	-	-

- (i) Lengkapkan jadual ANOVA di atas dengan mengisi tempat-tempat kosong.
- (ii) Uji hipotesis yang kesan baris dan kesan 'treatment' adalah tidak bererti dari segi statistik.

...4/-

- (iii) Andaikan data di atas berkaitan dengan keluaran ladang. Tulis suatu laporan ringkas kepada Kementerian Pertanian mengenai keputusan kamu.

[10 markah]

3. Pengurus Besar sebuah pasaraya tempatan sedang meneliti perbelanjaan pengiklanan bulanan. Oleh kerana perbelanjaan pengiklanan di surat khabar merupakan butiran perbelanjaan utama, Pengurus Besar ingin menganalisis seterusnya untuk memaksimumkan keberkesanan kos. Dia ingin mengkaji perhubungan antara jumlah pengiklanan dengan bilangan orang dewasa (dalam ribu) yang mengunjungi pasaraya kerana pengiklanan. Untuk tujuan ini, data berikut dikumpul:

Bilangan iklan (X) 5 6 7 5 1 8 10 2 6 7 8 5 9 7 8 2

Bilangan orang
(dalam ribu) (Y) 33 37 42 32 10 40 61 8 35 39 48 30 51 45 41 7

$$\Sigma X = 96$$

$$\Sigma Y = 359$$

$$\Sigma X^2 = 676$$

$$\Sigma Y^2 = 23037$$

$$\Sigma XY = 3930$$

- (i) Bina garis regresi yang dianggarkan yang menjelaskan bilangan orang yang mengunjungi pasaraya itu sebagai fungsi bilangan iklan.
- (ii) Adakah terdapat bukti yang cukup untuk menyimpulkan bahawa wujud perhubungan langsung antara kedua-dua pembolehubah? ($\alpha = 0.05$).
- (iii) Apakah peratusan kebolehubahan dalam bilangan orang yang mengunjungi pasaraya yang dapat dijelaskan oleh kebolehubahan dalam jumlah iklan?
- (iv) Cari selang ramalan 95% bagi bilangan orang (dalam ribu) yang mengunjungi pasaraya jika pasaraya ingin mengeluarkan lima iklan bulan depan.

[15 markah]

...5/-

4. (a) (i) Bagaimanakah regresi mudah berbeza dari korelasi?
(ii) Bincangkan dengan ringkas dua cara yang digunakan untuk membina satu regresi linear mudah.

[5 markah]

- (b) Sebuah agensi perumahan ingin meramal harga jualan rumah satu tingkat. Satu kajian yang dijalankan menunjukkan pembolehubah yang mempunyai hubungan rapat dengan harga rumah adalah luas kawasan rumah. Data dari sampel rawak 15 rumah yang baru dijual diambil. Data itu mengenai harga jualan, Y (dalam ribu \$) dan saiz X (dalam ratus kaki persegi) dan dimasukkan ke dalam komputer untuk dianalisis. Output kumputer adalah seperti berikut:

Dependent variable: Y

Analysis of Variance

Source	DF	Sum of squares	Mean square	F-value	Prob < F
Model	1	4034.4144	4034.4144	23885	0.0003
Error	13	2185.8215	168.9093		

Root MSE 12.9965 R-Square 0.6476
Dep. mean 88.840 Adj. R-Square 0.6204
C. V. 14.6291

Parameter Estimates

Variable	DF	Parameter estimate	Standard error	t for Ho parameter=0	Prob < T
Intercept	1	18.3538	14.8077		
X	1	3.8785	0.7936		

... 6/-

- (i) Cari persamaan garis regresi.
- (ii) Uji secara keseluruhan utiliti model ini.
- (iii) Apakah peratusan varians dalam harga jualan yang dapat dijelaskan oleh persamaan ini?
- (iv) Ramalkan dengan aras keyakinan 90%, harga jualan sebuah rumah dengan luas kawasan rumah sebenar 1600 kaki persegi.

[10 markah]

5. (a) 45 kereta digunakan dalam satu ujian yang dijalankan untuk mengukur kerosakan yang berlaku di lebuhraya. Susunan di mana kereta tempatan (L) dan kereta luar negeri (F) mengalami kerosakan di lebuhraya dicatat.

LLFFFFFFLLFFLLLFLLLLLFFFFFLLLFLLLFFFPLLFF

Uji sama ada kereta-kereta dipilih secara rawak dengan menggunakan aras keertian 0.10.

[7 markah]

- (b) Seorang penyelidik pasaran memilih 24 kedai runcit secara rawak untuk menguji dua minuman ringan baru yang diimpot dan memberi markah 0 (tidak baik) ke 9 (terbaik). Markah-markah yang dicatatkan diberi di bawah:

Kedai runcit	1	2	3	4	5	6	7	8	9	10	11	12
--------------	---	---	---	---	---	---	---	---	---	----	----	----

Markah:

Jus epal	4	6	6	7	8	4	9	5	5	4	8	2
Jus anggur	6	8	7	4	9	2	8	7	4	4	2	4

Kedai runcit	13	14	15	16	17	18	19	20	21	22	23	24
--------------	----	----	----	----	----	----	----	----	----	----	----	----

Markah:

Jus epal	7	4	8	4	6	9	9	6	3	7	4	6
Jus anggur	4	2	5	4	5	8	6	7	5	4	1	6

....7/-

Gunakan ujian tanda untuk mengkaji sama ada terdapat perbezaan sebenar dalam keutamaan pelanggan bagi kedua-dua minuman ringan. Gunakan aras keertian 5%.

[8 markah]

6. (a) Sebuah syarikat insurans memperkenalkan tiga skim gaji yang berbeza bagi pegawai jualan: komisen, gaji tetap dan komisen dan gaji. Untuk melihat kesan skim-skim ini, syarikat ini memilih tiga sampel tidak bersandar dengan $n_1 = 7$, $n_2 = 7$, $n_3 = 6$. Pegawai-pegawai jualan dipilih dan jualan (dalam ribu \$) yang diperolehi dari mereka bagi suku tahun dicatat. Data yang diperolehi seperti berikut:

Sampel I (komisen)	65	98	130	210	195	187	240
Sampel II (gaji tetap)	120	115	90	126	107	155	80
Sampel III (komisen dengan gaji tetap)	140	156	220	112	104	235	

Gunakan ujian Kruskal-Wallis H untuk menentukan sama ada data memberi bukti yang cukup untuk menunjukkan aras jualan suku tahun bergantung kepada jenis ganjaran yang diterima oleh pegawai jualan. Gunakan 5% aras keertian.

[9 markah]

- (b) Analisis rekod-rekod lampau menunjukkan bahawa kilang telah mengalami 50 kemalangan sepanjang dua tahun yang lalu. Dari maklumat yang diberi di bawah, adakah berpatutan untuk mempercayai bahawa kejadian kemalangan tertabur secara seragam (uniform) di antara kelima-lima hari bekerja ataupun adakah kita boleh menjangka lebih banyak kemalangan berlaku dipertengahan minggu berbanding dengan hari-hari lain? Gunakan $\alpha = 0.05$.

Hari	Isnin	Selasa	Rabu	Khamis	Jumaat
Bilangan kemalangan	8	10	12	14	6

[6 markah]

...8/-

7. (a) Superitenden Hospital ingin mengkaji ketibaan pesakit-pesakit dalam wad kecemasan untuk meningkatkan kemudahan perubatan dalam wad. Data yang berikut menunjukkan ketibaan pesakit-pesakit sejam:

Jumlah ketibaan/jam	0	1	2	3	4	5	6 dan ke atas
Frekuensi	13	7	12	16	15	10	7

Adakah berpatutan untuk mempercayai bahawa ketibaan pesakit di wad kecemasan bertaburan Poisson? Guna $\alpha = 0.05$.

[9 markah]

- (b) Persatuan Alumni sebuah sekolah perniagaan telah mengumpul data bagi alumni yang telah tamat pengajian 5 tahun yang lalu. Mereka telah mengumpul data mengenai gaji tahunan masa kini untuk mengetahui sama ada siswazah bidang pemasaran memperolehi pendapatan kini yang lebih dari siswazah bidang kewangan. Matlamat yang diberi adalah seperti berikut:

Siswazah pemasaran	22,400;	17,800;	26,500;	19,300;	18,200	
	21,100;	19,100;	43,500			
Siswazah kewangan	21,900;	16,800;	18,700;	19,400;	17,300	
	32,900					

Jalankan satu ujian Mann-Whitney untuk mengesahkan hipotesis bahawa tiada perbezaan yang bererti antara gaji kedua-dua kumpulan siswazah ini.

[6 markah]

...9/-

LAMPIRAN 1

$$\hat{b} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$\hat{a} = \bar{y} - b\bar{x}$$

$$\hat{\alpha} = \sqrt{\frac{\sum (y - \hat{y})^2}{n-2}}$$

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2}$$

$$s_b^2 = \frac{\hat{\alpha}^2}{\sum x^2 - n\bar{x}^2}$$

$$p_i = \sum_{i=0}^{x(n)} p_i q^{n-i}$$

$$\chi^2 = \sum_{i=1}^k \frac{(o_i - E_i)}{E_i} \quad \text{or} \quad \chi^2 = \frac{(|B-C|-1)^2}{B+C}$$

$$D = \max \{|F_0(x) - S_n(x)|\}$$

$$D^* = \frac{1.36}{\sqrt{n}}$$

$$E(r) = \frac{2n_1 n_2}{n_1 + n_2} + 1$$

$$\text{Var}(r) = \frac{2n_1 n_2 (2n_1 n_2 - n_1 - n_2)}{(n_1 + n_2)^2 (n_1 + n_2 - 1)}$$

...10/-

ASP300

$$U = \frac{n_1 n_2 + n_1(n_1 + 1)}{2} - R_1$$

$$E(U) = \frac{n_1 n_2}{2}$$

$$\text{var}(U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

$$H = \frac{12}{n(n+1)} \sum_{i=1}^k \frac{r_i^2}{n_i} - 3(n + 1)$$

...11/-

$$\text{Probabilities } P[X \leq c] = \sum_{x=0}^c \frac{e^{-m} m^x}{x!}$$

Table 2: CUMULATIVE POISSON PROBABILITIES

c	m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0.333	.301	.273	.247	.223	.202	.183	.165	.150	.135						
1	.699	.663	.627	.592	.558	.525	.493	.463	.434	.406						
2	.900	.879	.857	.833	.809	.783	.757	.731	.704	.677						
3	.974	.966	.957	.946	.934	.921	.907	.891	.875	.857						
4	.995	.992	.989	.986	.981	.976	.970	.964	.956	.947						
5	.999	.998	.998	.997	.996	.994	.992	.990	.987	.983						
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000						
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000						
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000						
9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000						

69

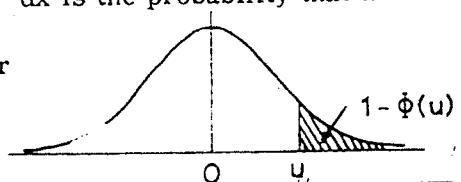
Table 2:i(Continued)

c	m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	.045	.041	.037	.033	.030	.027	.025	.022	.020	.018					
1	1	.185	.171	.159	.147	.136	.126	.116	.107	.099	.092					
2	2	.401	.380	.359	.340	.321	.303	.285	.269	.253	.238					
3	3	.625	.603	.580	.558	.537	.515	.494	.473	.453	.433					
4	4	.798	.781	.763	.744	.725	.706	.687	.668	.648	.629					
5	5	.906	.885	.863	.841	.811	.788	.764	.730	.701	.675					
6	6	.961	.955	.949	.942	.935	.927	.918	.909	.899	.889					
7	7	.986	.983	.980	.977	.973	.969	.965	.960	.955	.949					
8	8	.995	.994	.993	.992	.990	.988	.986	.984	.981	.979					
9	9	.999	.998	.997	.996	.995	.994	.993	.992	.991	.990					
10	10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000					
11	11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000					
12	12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000					

AREAS IN TAIL OF THE NORMAL DISTRIBUTION

The function tabulated is $1 - \Phi(u)$ where $\Phi(u)$ is the cumulative distribution function of a standardised Normal variable u . Thus $1 - \Phi(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-x^2/2} dx$ is the probability that a

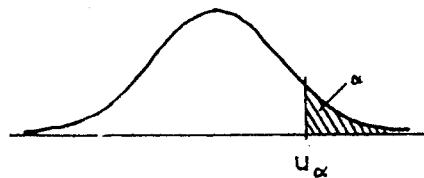
standardised Normal variable selected at random will be greater than a value of u ($= \frac{x-\mu}{\sigma}$) .



$\frac{(x - \mu)}{\sigma}$.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
3.0	.00135									
3.1	.00097									
3.2	.00069									
3.3	.00048									
3.4	.00034									
3.5	.00023									
3.6	.00016									
3.7	.00011									
3.8	.00007									
3.9	.00005									
4.0	.00003									

PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

The table gives the 100α percentage points, u_α , of a standardised Normal distribution where $\alpha = \frac{1}{\sqrt{2\pi}} \int_{u_\alpha}^{\infty} e^{-x^2/2} dx$. Thus u_α is the value of a standardised Normal variate which has probability α of being exceeded.



α	u_α								
.50	0.0000	.050	1.6449	.030	1.8808	.020	2.0537	.010	2.3263
.45	0.1257	.048	1.6646	.029	1.8957	.019	2.0749	.009	2.3656
.40	0.2533	.046	1.6849	.028	1.9110	.018	2.0969	.008	2.4089
.35	0.3853	.044	1.7060	.027	1.9268	.017	2.1201	.007	2.4573
.30	0.5244	.042	1.7279	.026	1.9431	.016	2.1444	.006	2.5121
.25	0.6745	.040	1.7507	.025	1.9600	.015	2.1701	.005	2.5758
.20	0.8416	.038	1.7744	.024	1.9774	.014	2.1973	.004	2.6521
.15	1.0364	.036	1.7991	.023	1.9954	.013	2.2262	.003	2.7478
.10	1.2816	.034	1.8250	.022	2.0141	.012	2.2571	.002	2.8782
.05	1.6449	.032	1.8522	.021	2.0335	.011	2.2904	.001	3.0902
									.00005 4.4172

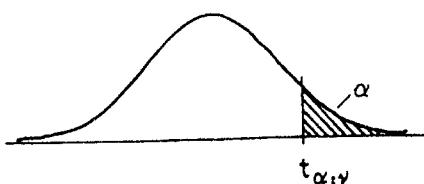
PERCENTAGE POINTS OF THE t DISTRIBUTION

The table gives the value of $t_{\alpha; \nu}$ — the 100 α percentage point of the t distribution for ν degrees of freedom.

The values of t are obtained by solution of the equation:-

$$\alpha = \Gamma\left(\frac{1}{2}(\nu+1)\right) \{\Gamma(\frac{1}{2}\nu)\}^{-1} (\nu\pi)^{-1/2} \int_t^{\infty} (1 + x^2/\nu)^{-(\nu+1)/2} dx$$

Note. The tabulation is for one tail only i.e. for positive values of t . For $|t|$ the column headings for α must be doubled.



$\alpha =$	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
$\nu = 1$	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

This table is taken from Table III of Fisher & Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers and also from Table 12 of Biometrika Tables for Statisticians, Volume 1, by permission of the Biometrika Trustees.

PERCENTAGE POINTS OF THE χ^2 DISTRIBUTIONTable of $\chi^2_{\alpha; \nu}$ — the 100 α percentage point of the χ^2 distribution for ν degrees of freedom

$\alpha =$.995	.99	.98	.975	.95	.90	.80	.75	.70	.50	.30	.25	.20	.10	.05	.025	.02	.01	.005	.001	$\nu =$	
$\nu = 1$.0393	.03157	.03628	.03982	.00393	.0158	.0642	.102	.148	.455	1.074	1.323	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	1	
2	.0100	.0201	.0404	.0506	.103	.211	.446	.575	.713	1.386	2.403	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2	
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.366	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3	
4	.207	.297	.429	.484	.711	1.084	1.679	1.923	2.195	3.357	4.873	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4	
5	.412	.554	.752	.831	1.145	1.610	2.303	2.675	3.000	4.351	6.061	6.626	7.289	9.226	11.070	12.832	13.388	15.086	16.750	20.517	5	
6	.676	.872	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6	
7	.989	1.239	1.564	1.690	2.167	2.833	3.932	4.255	4.671	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.822	18.475	20.278	24.322	7	
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8	
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9	
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.342	9.121	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	39	10	
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.701	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11	
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	14.01	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12	
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.299	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13	
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.117	18.151	20.064	23.685	26.873	29.141	31.319	36.123	14		
15	4.601	5.229	5.985	6.262	7.261	8.547	10.367	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15		
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.502	26.296	28.845	29.633	32.000	34.267	39.252	16	
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	12.792	13.531	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17	
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.338	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18	
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.338	21.699	22.718	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19	
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.266	19.337	22.878	25.038	28.412	31.410	34.170	37.020	37.566	39.987	45.315	50	20		
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21	
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	30.813	33.924	37.781	37.659	40.289	42.268	48.268	22	
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23	
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24	
25	10.520	11.524	12.697	13.120	14.611	16.473	19.939	20.867	24.337	28.172	29.339	30.675	34.362	37.652	40.646	41.566	44.314	46.928	52.620	52	25	
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26	
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27	
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	22.657	23.647	27.356	31.391	32.620	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28	
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29	
30	13.787	14.953	16.506	16.791	20.599	23.364	24.508	29.336	33.520	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	60	30		
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	33.660	34.872	39.335	44.165	45.616	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40	
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.773	56.134	58.184	63.167	67.505	71.213	75.154	79.490	86.661	90.504	50	
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.227	66.981	68.972	74.397	79.082	81.298	84.580	88.379	91.952	99.607	60	
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	61.698	63.646	69.334	75.689	77.577	79.115	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70	
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	71.145	72.915	79.334	86.120	88.100	90.495	96.571	101.054	107.565	113.145	118.136	124.116	128.299	137.208	90
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	80.625	82.511	89.334	96.524	98.650	101.054	107.565	113.145	118.136	124.116	128.299	137.208	140.170	149.449	100
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	90.133	92.129	99.334	106.908	109.141	111.697	118.498	124.342	128.561	131.142	135.807	140.170	149.449	100	

For values of $\nu > 30$, approximate values for χ^2 may be obtained from the expression $\nu \left[1 - \frac{2}{\sqrt{\nu}} + \frac{x^2/2}{\nu} \right]^3$, where x is the normal deviate cutting off the corresponding tails of a normal distribution if x is taken at the 0.02 level, so that 0.01 of the normal distribution is in each tail, the expression yields χ^2 at the 0.99 and 0.01 points. For very large values of ν it is sufficiently accurate to compute $\sqrt{\nu} \cdot x^2$, the distribution of which is approximately normal around a mean of $\sqrt{2\nu - 1}$ and with a standard deviation of 1. This table is taken by consent from Statistical Tables for Biological, Agricultural, and Medical Research, by R.A. Fisher and F. Yates, published by Oliver and Boyd, Edinburgh, and from Table 8 of Biometrika Tables for Statisticians, Vol. I, by permission of the Biometrika Trustees.

PERCENTAGE POINTS OF THE F DISTRIBUTION

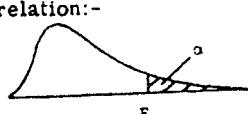
The table gives the values of $F_{\alpha; \nu_1, \nu_2}$, the 100 α percentage point of the F distribution having ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.

For each pair of values of ν_1 and ν_2 , $F_{\alpha; \nu_1, \nu_2}$ is tabulated for $\alpha = 0.05, 0.025, 0.01, 0.001$, the 0.025 values being bracketed.

The lower percentage points of the distribution may be obtained from the relation:-

$$F_{1-\alpha; \nu_1, \nu_2} = 1/F_{\alpha; \nu_2, \nu_1}$$

$$\text{e.g. } F_{.95; 12, 8} = 1/F_{.05; 8, 12} = 1/2.85 = 0.351$$



F_{α, ν_1, ν_2}

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	10	12	24	∞
1	161.4 (648)	199.5 (800)	215.7 (864)	224.6 (900)	230.2 (922)	234.0 (937)	236.8 (948)	238.9 (957)	241.9 (969)	243.9 (977)	249.0 (997)	254.3 (1018)
	4052	5000	5403	5625	5764	5859	5928	5981	6056	6106	6235	6368
	4053*	5000*	5404*	5625*	5764*	5859*	5929*	5981*	6056*	6107*	6235*	6368*
2	18.5 (38.5)	19.0 (39.0)	19.2 (39.2)	19.2 (39.2)	19.3 (39.3)	19.3 (39.3)	19.4 (39.4)	19.4 (39.4)	19.4 (39.4)	19.4 (39.4)	19.5 (39.5)	19.5 (39.5)
	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.5	99.5
	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	999.4	999.5	999.5
3	10.13 (17.4)	9.55 (16.0)	9.28 (15.4)	9.12 (15.1)	9.01 (14.9)	8.94 (14.7)	8.89 (14.6)	8.85 (14.5)	8.79 (14.4)	8.74 (14.3)	8.64 (14.1)	8.53 (13.9)
	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.6	26.1
	167.0	148.5	141.1	137.1	134.6	132.8	131.5	130.6	129.2	128.3	125.9	123.5
4	7.71 (12.22)	6.94 (10.65)	6.59 (9.98)	6.39 (9.60)	6.26 (9.36)	6.16 (9.20)	6.09 (9.07)	6.04 (8.98)	5.96 (8.84)	5.91 (8.75)	5.77 (8.51)	5.63 (8.26)
	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	13.9	13.5
	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.05	47.41	45.77	44.05
5	6.61 (10.01)	5.79 (8.43)	5.41 (7.76)	5.19 (7.39)	5.05 (7.15)	4.95 (6.98)	4.88 (6.85)	4.82 (6.76)	4.74 (6.62)	4.68 (6.52)	4.53 (6.28)	4.36 (6.02)
	16.26	13.27	12.06	11.39	10.97	10.87	10.46	10.29	10.05	9.89	9.47	9.02
	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	26.92	26.42	25.14	23.79
6	5.99 (8.81)	5.14 (7.26)	4.76 (6.60)	4.53 (6.23)	4.39 (5.99)	4.28 (5.82)	4.21 (5.70)	4.15 (5.60)	4.06 (5.46)	4.00 (5.37)	3.84 (5.12)	3.67 (4.85)
	13.74	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.31	6.88
	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.41	17.99	16.90	15.75
7	5.59 (8.07)	4.74 (6.54)	4.35 (5.89)	4.12 (5.52)	3.97 (5.29)	3.87 (5.12)	3.79 (4.99)	3.73 (4.90)	3.64 (4.76)	3.57 (4.67)	3.41 (4.42)	3.23 (4.14)
	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.07	5.65
	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.08	13.71	12.73	11.70
8	5.32 (7.57)	4.46 (6.06)	4.07 (5.42)	3.84 (5.05)	3.69 (4.82)	3.58 (4.65)	3.50 (4.53)	3.44 (4.43)	3.35 (4.30)	3.28 (4.20)	3.12 (3.95)	2.93 (3.67)
	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.28	4.86
	25.42	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.54	11.19	10.30	9.34
9	5.12 (7.21)	4.26 (5.71)	3.86 (5.08)	3.63 (4.72)	3.48 (4.48)	3.37 (4.32)	3.29 (4.20)	3.23 (4.10)	3.14 (3.96)	3.07 (3.87)	2.90 (3.61)	2.71 (3.33)
	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.73	4.31
	22.86	16.39	13.90	12.56	11.71	11.13	10.69	10.37	9.87	9.57	8.72	7.81
10	4.96 (6.94)	4.10 (5.46)	3.71 (4.83)	3.48 (4.47)	3.33 (4.24)	3.22 (4.07)	3.14 (3.95)	3.07 (3.85)	2.98 (3.85)	2.91 (3.82)	2.74 (3.37)	2.54 (3.08)
	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.33	3.91
	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.74	8.44	7.64	6.78
11	4.84 (6.72)	3.98 (5.26)	3.59 (4.63)	3.36 (4.28)	3.20 (4.04)	3.09 (3.88)	3.01 (3.76)	2.95 (3.66)	2.85 (3.53)	2.79 (3.43)	2.61 (3.17)	2.40 (2.88)
	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.54	4.40	4.02	3.60
	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	7.92	7.63	6.85	6.00
12	4.75 (6.55)	3.89 (5.10)	3.49 (4.47)	3.26 (4.12)	3.11 (3.89)	3.00 (3.73)	2.91 (3.61)	2.85 (3.51)	2.75 (3.37)	2.69 (3.28)	2.51 (3.02)	2.30 (2.72)
	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	3.78	3.36
	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.25	5.42
13	4.67 (6.41)	3.81 (4.97)	3.41 (4.35)	3.18 (4.00)	3.03 (3.77)	2.92 (3.60)	2.83 (3.48)	2.77 (3.39)	2.67 (3.25)	2.60 (3.15)	2.42 (2.89)	2.21 (2.60)
	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.10	3.96	3.59	3.17
	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.80	6.52	5.78	4.97

* Entries marked thus must be multiplied by 100

ASP300

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	7	8	10	12	24	∞
14	4.60 (6.30)	3.74 (4.86)	3.34 (4.24)	3.11 (3.89)	2.96 (3.66)	2.85 (3.50)	2.76 (3.38)	2.70 (3.29)	2.60 (3.15)	2.53 (3.05)	2.35 (2.79)	2.13 (2.49)
	8.86 17.14	6.51 11.78	5.56 9.73	5.04 8.62	4.70 7.92	4.46 7.44	4.28 7.08	4.14 6.80	3.94 6.40	3.80 6.13	3.43 5.41	3.00 4.60
	4.49 (6.12)	3.63 (4.69)	3.24 (4.08)	3.01 (3.73)	2.85 (3.50)	2.74 (3.34)	2.66 (3.22)	2.59 (3.12)	2.49 (2.99)	2.42 (2.89)	2.24 (2.63)	2.01 (2.32)
16	8.53 16.12	6.23 10.97	5.29 9.01	4.77 7.94	4.44 7.27	4.20 6.80	4.03 6.46	3.89 6.19	3.69 5.81	3.55 5.55	3.18 4.85	2.75 4.06
	4.41 (5.98)	3.55 (4.56)	3.16 (3.95)	2.93 (3.61)	2.77 (3.38)	2.66 (3.22)	2.58 (3.10)	2.51 (3.01)	2.41 (2.87)	2.34 (2.77)	2.15 (2.50)	1.92 (2.19)
	8.29 15.38	6.01 10.39	5.09 8.49	4.58 7.46	4.25 6.81	4.01 6.35	3.84 6.02	3.71 5.76	3.51 5.39	3.37 5.13	3.00 4.45	2.57 3.67
20	4.35 (5.87)	3.49 (4.46)	3.10 (3.86)	2.87 (3.51)	2.71 (3.29)	2.60 (3.13)	2.51 (3.01)	2.45 (2.91)	2.35 (2.77)	2.28 (2.68)	2.08 (2.41)	1.84 (2.09)
	8.10 14.82	5.85 9.95	4.94 8.10	4.43 7.10	4.10 6.46	3.87 6.02	3.70 5.69	3.56 5.44	3.37 5.08	3.23 4.82	2.86 4.15	2.42 3.38
	4.30 (5.79)	3.44 (4.38)	3.05 (3.78)	2.82 (3.44)	2.66 (3.22)	2.55 (3.05)	2.46 (2.93)	2.40 (2.84)	2.30 (2.70)	2.23 (2.60)	2.03 (2.33)	1.78 (2.00)
22	7.95 14.38	5.72 9.61	4.82 7.80	4.31 6.81	3.99 6.19	3.76 5.76	3.59 5.44	3.45 5.19	3.26 4.83	3.12 4.58	2.75 3.92	2.31 3.15
	4.26 (5.72)	3.40 (4.32)	3.01 (3.72)	2.78 (3.38)	2.62 (3.15)	2.51 (2.99)	2.42 (2.87)	2.36 (2.78)	2.25 (2.64)	2.18 (2.54)	1.98 (2.27)	1.73 (1.94)
	7.82 14.03	5.61 9.34	4.72 7.55	4.22 6.59	3.90 5.98	3.67 5.55	3.50 5.23	3.36 4.99	3.17 4.64	3.03 4.39	2.66 3.74	2.21 2.97
24	4.23 (5.66)	3.37 (4.27)	2.98 (3.67)	2.74 (3.33)	2.59 (3.10)	2.47 (2.94)	2.39 (2.82)	2.32 (2.73)	2.22 (2.59)	2.15 (2.49)	1.95 (2.22)	1.69 (1.88)
	7.72 13.74	5.53 9.12	4.64 7.36	4.14 6.41	3.82 5.80	3.59 5.38	3.42 5.07	3.29 4.83	3.09 4.48	2.96 4.24	2.58 3.59	2.13 2.82
	4.20 (5.61)	3.34 (4.22)	2.95 (3.63)	2.71 (3.29)	2.56 (3.06)	2.45 (2.90)	2.36 (2.78)	2.29 (2.69)	2.19 (2.55)	2.12 (2.45)	1.91 (2.17)	1.65 (1.83)
26	7.64 13.50	5.45 8.93	4.57 7.19	4.07 6.25	3.75 5.66	3.53 5.24	3.42 4.93	3.29 4.69	3.09 4.35	2.96 4.11	2.58 3.46	2.13 2.69
	4.17 (5.57)	3.32 (4.18)	2.92 (3.59)	2.69 (3.25)	2.53 (3.03)	2.42 (2.87)	2.33 (2.75)	2.27 (2.65)	2.16 (2.51)	2.09 (2.41)	1.89 (2.14)	1.62 (1.79)
	7.56 13.29	5.39 8.77	4.51 7.05	4.02 6.12	3.70 5.53	3.47 5.12	3.30 4.82	3.17 4.58	2.98 4.24	2.84 4.00	2.52 3.36	2.06 2.59
30	4.08 (5.42)	3.23 (4.05)	2.84 (3.46)	2.61 (3.13)	2.45 (2.90)	2.34 (2.74)	2.25 (2.62)	2.18 (2.53)	2.08 (2.39)	2.00 (2.29)	1.79 (2.01)	1.51 (1.64)
	7.31 12.61	5.18 8.25	4.31 6.59	3.83 5.70	3.51 5.13	3.29 4.73	3.12 4.44	2.99 4.21	2.80 3.87	2.66 3.64	2.29 3.01	1.80 2.23
	4.00 (5.29)	3.15 (3.93)	2.76 (3.34)	2.53 (3.01)	2.37 (2.79)	2.25 (2.63)	2.17 (2.51)	2.10 (2.41)	1.99 (2.27)	1.92 (2.17)	1.70 (1.88)	1.39 (1.48)
60	7.08 11.97	4.98 7.77	4.13 6.17	3.65 5.31	3.34 4.76	3.12 4.37	2.95 4.09	2.82 3.86	2.63 3.54	2.50 3.32	2.12 2.69	1.60 1.89
	3.92 (5.15)	3.07 (3.80)	2.68 (3.23)	2.45 (2.89)	2.29 (2.67)	2.18 (2.52)	2.09 (2.39)	2.02 (2.30)	1.91 (2.16)	1.92 (2.05)	1.70 (1.76)	1.25 (1.31)
	6.85 11.38	4.79 7.32	3.95 5.78	3.48 4.95	3.17 4.42	2.96 4.04	2.79 3.77	2.66 3.55	2.47 3.24	2.34 3.02	2.12 2.40	1.38 1.54
120	3.84 (5.02)	3.00 (3.69)	2.60 (3.12)	2.37 (2.79)	2.21 (2.57)	2.10 (2.41)	2.01 (2.29)	1.94 (2.19)	1.83 (2.05)	1.75 (1.94)	1.52 (1.64)	1.00 (1.00)
	6.63 10.83	4.61 6.91	3.78 5.42	3.32 4.62	3.02 4.10	2.80 3.74	2.64 3.47	2.51 3.27	2.32 2.96	2.18 2.74	1.79 2.13	1.00 1.00

This table is taken from Table V of Fisher & Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Oliver & Boyd Ltd., Edinburgh, and by permission of the authors and publishers and also from Table 18 of Biometrika Tables for Statisticians, Volume 1, by permission of the Biometrika Trustees.

TABLE K. TABLE OF CRITICAL VALUES OF U IN THE MANN-WHITNEY TEST*

Table K₁. Critical Values of U for a One-tailed Test at $\alpha = .001$ or for a Two-tailed Test at $\alpha = .002$

$n_2 \backslash n_1$	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	4	5	6	7	8	9	10	11	12	13	14
5	4	5	6	7	8	9	10	11	12	13	14	15
6	5	6	7	8	9	10	11	12	13	14	15	16
7	6	7	8	9	10	11	12	13	14	15	16	17
8	7	8	9	10	11	12	13	14	15	16	17	18
9	8	9	10	11	12	13	14	15	16	17	18	19
10	9	10	11	12	13	14	15	16	17	18	19	20
11	10	11	12	13	14	15	16	17	18	19	20	21
12	11	12	13	14	15	16	17	18	19	20	21	22
13	12	13	14	15	16	17	18	19	20	21	22	23
14	13	14	15	16	17	18	19	20	21	22	23	24
15	14	15	16	17	18	19	20	21	22	23	24	25
16	15	16	17	18	19	20	21	22	23	24	25	26
17	16	17	18	19	20	21	22	23	24	25	26	27
18	17	18	19	20	21	22	23	24	25	26	27	28
19	18	19	20	21	22	23	24	25	26	27	28	29
20	19	20	21	22	23	24	25	26	27	28	29	30

$n_2 \backslash n_1$	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	2	3	4	5	6	7	8	9	10	11	12
3	2	3	4	5	6	7	8	9	10	11	12	13
4	3	4	5	6	7	8	9	10	11	12	13	14
5	4	5	6	7	8	9	10	11	12	13	14	15
6	5	6	7	8	9	10	11	12	13	14	15	16
7	6	7	8	9	10	11	12	13	14	15	16	17
8	7	8	9	10	11	12	13	14	15	16	17	18
9	8	9	10	11	12	13	14	15	16	17	18	19
10	9	10	11	12	13	14	15	16	17	18	19	20
11	10	11	12	13	14	15	16	17	18	19	20	21
12	11	12	13	14	15	16	17	18	19	20	21	22
13	12	13	14	15	16	17	18	19	20	21	22	23
14	13	14	15	16	17	18	19	20	21	22	23	24
15	14	15	16	17	18	19	20	21	22	23	24	25
16	15	16	17	18	19	20	21	22	23	24	25	26
17	16	17	18	19	20	21	22	23	24	25	26	27
18	17	18	19	20	21	22	23	24	25	26	27	28
19	18	19	20	21	22	23	24	25	26	27	28	29
20	19	20	21	22	23	24	25	26	27	28	29	30

* Adapted and abridged from Tables 1, 3, 5, and 7 of Auble, D. 1953. Extended tables for the Mann-Whitney statistic. *Bulletin of the Institute of Educational Research at Indiana University*, 1, No. 2, with the kind permission of the author and the publisher.

TABLE K. TABLE OF CRITICAL VALUES OF U IN THE MANN-WHITNEY TEST*
(Continued)

Table K₁. Critical Values of U for a One-tailed Test at $\alpha = .01$ or for a Two-tailed Test at $\alpha = .02$

Table K₂. Critical Values of U for a One-tailed Test at $\alpha = .01$ or for a Two-tailed Test at $\alpha = .02$

Table K₃. Critical Values of U for a One-tailed Test at $\alpha = .01$ or for a Two-tailed Test at $\alpha = .02$

* Adapted and abridged from Tables 1, 3, 5, and 7 of Auble, D. 1953. Extended tables for the Mann-Whitney statistic. *Bulletin of the Institute of Educational Research at Indiana University*, 1, No. 2, with the kind permission of the author and the publisher.