NUMERICAL ANALYSIS OF NATURAL CONVECTION IN A TWO-DIMENSIONAL ENCLOSURE: THE EFFECTS OF ASPECT RATIO AND WALL TEMPERATURE VARIATION

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by

FAIROSIDI BIN IDRUS

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"In the Name of ALLAH, the Most Compassionate and the Most Merciful,

Lord of the Universe"

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TABLE OF CONTENTS

Page

CHAPTER THREE – THEORETICAL BACKGROUND

CHAPTER FOUR – METHODOLOGY

CHAPTER FIVE – RESULTS AND DISCUSSION

CHAPTER SIX – CONCLUSIONS

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

Nomenclature

Pr	Prandtl number
Ra	Rayleigh number
Ha	Hartmann number
Gr	Grashoff number
Nu	Local Nusselt number
$\overline{Nu}$	Average Nusselt number
A_H	Aspect ratio, *H/W*
A_W	Aspect ratio, *W/H*
A_R	Aspect ratio, *L/H*
H	Height of the enclosure [m]
W	Width of the enclosure [m]
L	Length of the enclosure [m]
t	Time [s]
T	Temperature [K]
ΔT	Temperature differences between heating and cooling zone
F	Force [N]
A	Body force per unit volume in *x*-direction [N/m^3]
B	Body force per unit volume in *y*-direction [N/m^3]
p	Pressure [N/m^2]
P	Dimensionless pressure
h	Convection heat transfer coefficient [W/m^{2}K]

$\bar{h}$	Average of convection heat transfer coefficient [W/m^{2}K]
q	Heat transfer [J]
$\dot{q}$	Heat transfer rate [W]
q''	Heat flux [W/m^2]
$\overline{q''}$	Average of heat flux [W/m^2]
w	Work transfer [J]
e	Specific internal energy [J/kg]
h	Specific enthalpy [J/kg]
C_p	Specific heat capacity at constant pressure [J/kgK]
u	Velocity component in x-direction [m/s]
U	Dimensionless velocity component in x-direction
v	Velocity component in y-direction [m/s]
V	Dimensionless velocity component in y-direction
x, y	Cartesian coordinates [m]
X, Y	Dimensionless Cartesian coordinates
Δx	Horizontal control volume size
Δy	Vertical control volume size
g	Gravitational acceleration [m/s^2]
$\dot{m}$	Mass flow rate [kg/s]
M_{cv}	Instantaneous mass inventory of a control volume
k	Thermal conductivity [W/mK]
K	Anisotropic permeability ratio
S	Slope of the wall temperature linear distribution
D	Distance between two heat transfer surfaces [m]
I	Inverse transformation

J	Jacobian determinant

Greek symbols

λ	Thermal diffusivity ratio
α	Thermal diffusivity [m^2/s]
ε	Dimensionless amplitude
$\in$	Dimensionless heat source length
τ	Dimensionless time
θ	Dimensionless temperature
ρ	Density [kg/m^3]
ν	Kinematic viscosity [m^2/s]
μ	Dynamic viscosity [kg/ms]
σ_x	Normal stress [N/m^2]
τ_{xy}	Tangential stress [N/m^2]
Φ	Viscous dissipation function [s^{-2}]
β	Volumetric thermal expansion coefficient [1/K]
ξ, η	Orthogonal computational space

Subscripts

H	Hot wall
C	Cold wall
cv	Control volume
i,j	Indices coordinates
min	Minimum
max	Maximum

ANALISIS BERANGKA TERHADAP PEROLAKAN TABIE DI DALAM SATU RUANG TERTUTUP DUA-DIMENSI: KESAN NISBAH BIDANG DAN PERUBAHAN SUHU DINDING

ABSTRAK

Penyelidikan ini tertumpu kepada pengkajian berangka ke atas perolakan tabie yang beraliran mantap bagi suatu aliran udara dengan nombor Prandtl, $Pr = 0.71$ di dalam satu ruang tertutup dua-dimensi. Untuk análisis tersebut, dua kes dengan keadaan sempadan yang berbeza telah diselidiki; dua dinding menegak yang sesuhu dan dua dinding menegak yang tidak sesuhu. Bagi kedua-dua keadaan, dinding mengufuk adiabatik dan nombor Rayleigh, Ra di dalam julat rendah, 10^3 - 10^5 telah dipertimbangkan. Di dalam kes yang pertama, satu ruang tertutup di mana suhu dinding sisinya berkeadaan seragam telah dipertimbangkan bagi pelbagai nisbah bidang, A_H iaitu 0.5, 1.0 and 2.0. Berpandukan kepada kaedah berangka, kesan nisbah bidang terhadap pemindahan haba dan aliran bendalir di dalam ruang tertutup tersebut telah diselidiki. Pengkajian kes yang kedua tertumpu kepada satu ruang tertutup segiempat sama dengan pelbagai keadaan suhu dinding menegaknya tidak seragam. Di dalam analisis ini, tiga parameter dinding tidak sesuhu telah dianalisa. Dengan melakukan kaedah berangka yang sama, pengaruh perbezaan suhu dinding terhadap pemindahan haba dan aliran bendalir di dalam ruang tersebut juga dianalisa. Penyelidikan ini dijalankan secara berangka dengan mengaplikasikan kaedah kebezaan terhingga. Tiga algoritma yang berbeza termasuklah kaedah formula kebezaan, upwind dan Runge-Kutta telah digunakan bagi penghampiran terbitan. Penyelesaian berangka berdasarkan kepada algoritma-algoritma di atas telah disahkan dengan membandingkan kontur sesuhu dan nombor Nusselt purata, $\overline{Nu}$

bagi satu ruang tertutup bersegiempat sama yang dipanaskan daripada permukaan bawahnya dengan keputusan daripada satu kajian eksperimen dan ianya menunjukkan persetujuan-persetujuan yang memuaskan. Keputusan yang diperolehi dipersembahkan di dalam bentuk jadual nombor Nusselt purata, $\overline{Nu}$ dan bentuk grafik kontur sesuhu dan kontur vektor halaju. Daripada pengkajian kes yang pertama, kesan nisbah bidang terhadap pemindahan haba telah diperhatikan di mana ianya meningkat dengan kenaikan nisbah bidang, A_H dan nombor Rayleigh, Ra. Corak aliran udara tidak berubah begitu ketara dengan kenaikan nisbah bidang A_H tetapi ianya berubah begitu berkesan dengan kenaikan nombor Rayleigh, Ra. Di dalam pengkajian kes kedua, didapati bahawa perbezaan suhu bagi pelbagai dinding tidak sesuhu yang dipertimbangkan di dalam analisis ini telah memberikan kesan yang sedikit terhadap pemindahan haba dan corak aliran udara di dalam ruang tertutup tersebut. Walaubagaimanapun, penggantungan kenaikan nombor Rayleigh, Ra terhadap pemindahan haba dan corak aliran udara telah dapat dibuktikan daripada analisis yang sama.

NUMERICAL ANALYSIS OF NATURAL CONVECTION IN A TWO-DIMENSIONAL ENCLOSURE: THE EFFECTS OF ASPECT RATIO AND WALL TEMPERATURE VARIATION

ABSTRACT

This research is focused on the numerical investigation of a steady laminar natural convection flow of air with Prandtl number, Pr = 0.71 in a two-dimensional enclosure. For the analysis, two cases with different boundary conditions are investigated; two vertical walls, which are isothermal and two verticals walls, which are non-isothermal. For both conditions, adiabatic horizontal walls and Rayleigh number, Ra in low range of 10^3 - 10^5, are taken into consideration. In the first case, an enclosure having uniform temperature condition at the sidewalls is considered for various aspect ratios, A_H of 0.5, 1.0 and 2.0. Based on the numerical approach, the effect of aspect ratio on the heat transfer and fluid flow inside the enclosure is investigated. The second case of the study is focused on a square enclosure having various non-uniform temperatures condition at the vertical walls. In this analysis, three different parameters of non-isothermal wall are analyzed. By implementing the same numerical method, the influence of wall temperature variation on the heat transfer and fluid flow inside the enclosure is also analyzed. This research is conducted numerically by applying the finite difference method (FDM). Three difference algorithms including the difference formula, the upwind and the Runge-Kutta methods are used for the derivatives approximations. The numerical solutions based on the above algorithms are validated by comparing the isotherms contour and the average Nusselt number, $\overline{Nu}$ of a square enclosure, which is heated from the below surface with the results of an experimental study and reveals good agreements.

The results are presented in the tabular form of the average Nusselt number, $\overline{Nu}$ and the graphical form of the isotherms and velocity vector contours. From the first case of the study, the effect of aspect ratio on the heat transfer is observed where it increases with increasing aspect ratio, A_H and Rayleigh number, Ra. Flow patterns of the air do not vary by very much with the increment in the aspect ratio, A_H but they change significantly with the increasing Rayleigh number, Ra. In the second case of the study, it is determined that the temperature variation of the various non-isothermal walls in the considered analysis has given a slightly effect on the heat transfer and flow patterns of the air inside the enclosure. However, the dependence of the increasing Rayleigh number, Ra on the heat transfer and flow patterns of the air has been proven from the same analysis.

CHAPTER 1
INTRODUCTION

1.0 Overview

This chapter will first consider some of the important information for an overview understanding of the research backgrounds. The topics covered are written in five sections, which include the introduction of natural convection and the research objectives. It is followed by the scope and the significance of the study and finally the outline of the dissertation.

1.1 Introduction of Natural Convection

Heat is defined as energy transferred due to temperature difference. It flows from regions of higher temperature to regions of lower temperature. There are three basic modes of heat transfer mechanisms and convection is one of the modes. Convection relates to the transfer of heat from a bounding surface to a fluid in motion or to the heat transfer across a flow plane within the interior of the flowing fluid (Rohsenow et al., 1998). Convective heat transfer has grown to the status of a contemporary science because of people's desire to understand and predict how a fluid flow will act as a carrier for energy and matter (Bejan, 2004). Convection in nature occurs in two different forms, the so-called natural convection and forced convection. Natural convection is the motion that results from the interaction of gravity with density differences within a fluid. The differences may result from gradients in temperature, concentration or composition (Rohsenow et al., 1998). In natural convection, the fluid flows naturally as it is driven by the effect of buoyancy-driven motion caused by the body force field.

The natural convection flow phenomena inside an enclosed space, is an interesting example of very complex fluid systems that may yield to analytical, empirical and numerical solutions (Holman, 2001). The enclosure is defined as the confined space bounded by walls of any shape and filled with fluids (Yang, 1987). Natural convection in enclosures is always created by the complex interaction between the fluid and the heat with all the walls. Internal interactions will cause a diversity of flows that can appear inside the enclosures. In many engineering applications and naturally occurring processes, natural convection plays an important role as a dominating mechanism. Natural convection in enclosures has been widely used in many thermal applications such as in solar collectors, cooling devices for electronic instruments, building insulation, energy storage devices, furnace design and many others.

There are numerous studies in the literature regarding natural convection in enclosures, a considerable amount of which was reviewed by Ostrach (1988). Most of the previous studies on natural convection in enclosures are related to either side heating or bottom heating (Aydin et al., 1999). These studies are mainly focused on the investigations of energy transport and flow profiles of fluid of different Prandtl number, Pr for increasing Rayleigh number, Ra, which are conducted at various temperatures and geometries of the enclosure. There are several types of two-dimensional enclosures, which receive considerable attention. The various types of the enclosures are square, rectangular, triangle, sphere, cylinder inclined and partition. The most common case studies are the square and the rectangular enclosures, which are heated and cooled uniformly either at the two vertical or horizontal walls while the remaining two walls are thermally insulated.

1.2 Objectives of Study

The objectives of this research are listed below:

i) To develop a program based on a finite difference method (FDM) and to validate the applied numerical method for the classical uniform wall temperatures of a two-dimensional square cavity.

ii) To study the phenomena of natural convection inside a two-dimensional enclosure, which is differentially heated and cooled from the vertical walls for two different cases, employing the same numerical method.

iii) To investigate the effect of aspect ratio on heat transfer and fluid flow inside a two-dimensional enclosure, which is uniformly heated and cooled from the vertical walls.

iv) To determine the influence of wall temperature variation on heat transfer and fluid flow inside a two-dimensional square cavity, which is non-isothermally heated and cooled from the vertical walls.

1.3 Scope of Work

This research presented a computational method of study used to obtain the solutions of the buoyancy-driven laminar flow and heat transfer in a two-dimensional (2-D), natural convection of an air-filled cavity heated vertically for two different cases. The first case is restricted to a study of an enclosure having uniform temperature condition at the vertical walls for various aspect ratios, $A_H = H/W$ of 0.5, 1.0 and 2.0. Based on the numerical predictions, the effects of aspect ratio on the heat transfer and flow patterns of the air are investigated. The second case is focused on a square cavity, where the left and right sidewalls are held at various non-uniform temperatures condition, having a linearly temperature distribution. The purpose is to determine the influence of wall temperature variation on the heat transfer and flow patterns of the air. For both cases, Prandtl number, *Pr* of 0.71 and low Rayleigh number, *Ra* ranging from 10^3 to 10^5 are considered for the cavity, where the top and bottom walls are assumed to be adiabatic.

The coordinate system is defined so that the *x*-axis is in the horizontal direction and the *y*-axis in the vertical direction with the origin at the bottom corner on the left sidewall of the cavity. The flow is considered as a Newtonian fluid and is assumed that the fluid satisfies the Boussinesq approximation. The numerical technique based on the finite difference method (FDM) is generally applied in the computations of a uniform 20 x 20 grid size. The results are presented in tabular form of the average Nusselt number, $\overline{Nu}$, which represents the rate of heat being transferred. In addition, the results are also presented in graphical forms of isotherms and velocity vector contours, which demonstrate the thermal and fluid flow distributions inside the enclosure.

1.4 Significance of Study

There has been an ever-increasing awareness that fluid motions and transport processes generated by buoyancy are of interest and significance in many fields of science and technology. As a result, this subject is currently discussed over the world covering such diverse areas of geophysics, nuclear reactor systems, materials processing, energy storage and metallurgical industries as well as in the more conventional fields of the fluid and thermal sciences (Ostrach, 1988). Research on natural convection has centered on many configurations for the past few years and due to motivations from diverse applications, the research scope has been expanded in many ways.

Although there have been several studies on natural convection, only a limited number deal with various aspect ratios and non-isothermal conditions. Most of the preceding investigations either experimental or numerical are concerned with isothermally heated square cavities. This study is motivated by the need to understand the thermal and fluid flow distributions caused by the variations of the vertical walls temperature and geometry of the enclosure.

Further study is still needed since the non-uniform characteristic of temperature appears in many practical and industrial devices such as in the applications of electronic boards, energy storage, solar receivers and others, where the majority of the applications are considered as non-isothermal.

1.5 Outline of Dissertation

This thesis is written in six chapters. The first chapter covers backgrounds, principles and foundations of natural convection. Research objectives are also discussed in this chapter. The scope and the significance of study are presented later. An overview of previous work in the field of interest is presented in Chapter 2. In this chapter, related research on the natural convective study in various conditions, characteristics and geometries of enclosure are reviewed. Chapter 3 introduces the theoretical backgrounds and the mathematical modeling of the problem, which are considered in this study. The basic governing differential equations and the important formulations of heat transfer are also shown in details. Basic theory of the chosen numerical method is included at the end of this chapter.

The methodology applied in this research is explained in Chapter 4. This chapter demonstrates the method of analysis and the numerical algorithms used in generating the solutions of the specified problems. The respective physical models with boundary conditions and mesh structures are also described in this chapter. The fifth chapter presents the results and the discussion obtained from the respective analysis. The results are highlighted in tabular and graphical forms, as correlation of the variations of the aspect ratio, A_H Rayleigh number, Ra and wall temperature. Finally, Chapter 6 summarizes the conclusions drawn from the study followed by some recommendations for the future work.

CHAPTER 2
LITERATURE REVIEW

2.0 Overview

The information regarding the principles and foundations of natural convection and the study backgrounds have been explained in the preceding chapter. In the present chapter, the concern is to review the previous findings, which are relevant to the natural convection study. The selected research involved the experimental and the numerical solutions for the study of heat transfer in enclosures of different boundary conditions and temperature characteristics in various methods of study. Generally, the enclosures phenomena can be organized into two large categories, enclosures heated from the side and enclosures heated from the bottom. Considering the number of potential engineering applications, literature reviews in both categories are presented in this chapter on three main constraints of the research. Factors of various aspect ratios, increasing Rayleigh number and varying wall temperature, which all influenced in determining the heat transfer rate and fluid flow of natural convection in enclosures are presented in this chapter.

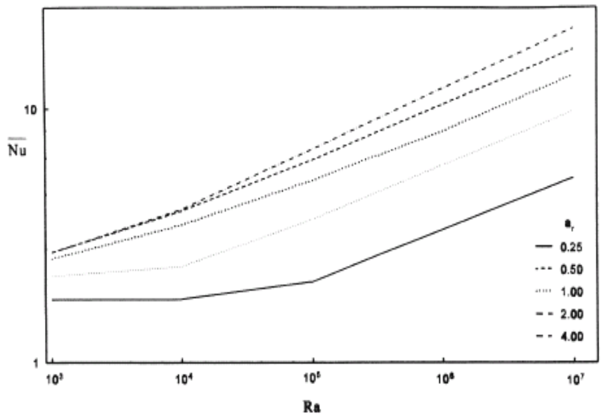
2.1 Various Aspect Ratios

Studies in free convective flow have been conducted comprehensively in both experimental and numerical analysis by previous researchers to investigate the phenomena of convection inside cavities, which was affected by the aspect ratio. It covered studies from the lower aspect ratio of enclosures, $A < 1$ and unity aspect ratio, $A = 1$, to the higher aspect ratio, $A > 1$ in many cases, adapting different algorithms and methods of analysis.

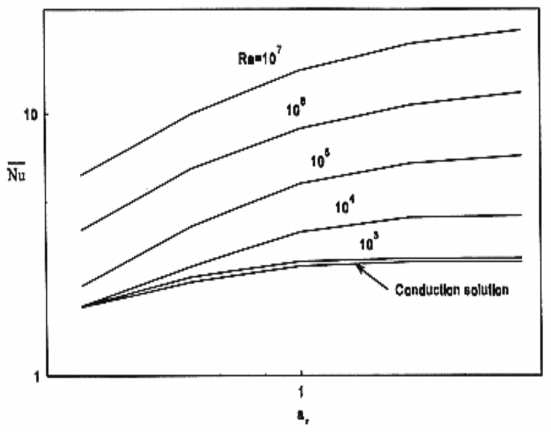
Research on steady natural convection in a two-dimensional rectangular enclosure of aspect ratio, L/H varied from 1 to 9 was conducted by Ganzarolli and Milanez (1995). The stream function-vorticity formulation had been applied in this numerical analysis. The cavity was heated from the below and symmetrically cooled from the sides at uniform wall temperatures. The Allen discretization scheme had been adapted and the discretized equations were solved in a line by line basis (Allen and Southwell, 1955). Rayleigh number, Ra ranging from 10^3 to 10^7 and Prandtl number, Pr of 0.7 and 7.0, had been considered in this analysis. The results had shown a little influence of the Prandtl number, Pr on the heat transfer and flow circulation inside the cavity. For the square cavity, $L/H = 1$, the flow or the isotherms contours were not strongly affected by the boundary condition at the cavity floor, uniform surface temperature or uniform heat flux. However, for the shallow cavity, $L/H > 1$, the isotherms and streamlines occupied the whole cavity more uniformly when the surface temperature or the heat flux was prescribed.

A transient three-dimensional free convection in a rectangular enclosure was tested by Hsieh and Yang (1996) in their experimental study. The study was presented in the Rayleigh number, Ra range of 6.9×10^7 - 4.12×10^8 for aspect ratio of $A_H = 3$ and $A_W = 1.2$ inside a cavity with silicone oil of Prandtl number, $P_r \cong 457$ as the working medium. The two vertical walls were heated on the right wall and cooled on the left wall as $t \geq 0$, respectively while the remaining four walls were adiabatic. It was found that a sudden application of a temperature difference to the sidewalls of a rectangular enclosure induced a strong convection flow immediately. There was a three-dimensional effect on flow patterns for the case of $Ra = 4.12 \times 10^8$ at $A_H = 3$ and $A_W = 1.2$. At $Ra = 10^6$, the period of the three-dimensional oscillations appeared to be about six times higher than those from the analytical prediction of the two-dimensional flows whereas the Nusselt number, Nu was a little higher.

Aydin et al. (1999) performed research on steady natural convection of air in a two-dimensional enclosure isothermally heated from one side and cooled from the ceiling. The study had been analyzed numerically using a stream function-vorticity formulation. The effects of Rayleigh number, Ra and aspect ratio, A_R on flow patterns and energy transports were investigated for Ra ranging from 10^3 to 10^7 and for A_R of 0.25 to 4.0. Both the hot wall and cold ceiling temperatures were assumed to be uniform. It was observed that for $Ra \leq 10^6$, a clockwise rotating single cell was occurred for each aspect ratio. A secondary cell was observed for all cases at $Ra = 10^7$ except for $A_R = 0.25$. For the case of tall enclosure, $A_R < 1$, average Nusselt number, $\overline{Nu}$ had a strong dependence only in high Rayleigh range, $Ra \geq 10^5$ where the warmer fluid occupied nearly the entire enclosure. For square and shallow enclosures, $A_R \geq 1$, the average Nusselt number, $\overline{Nu}$ increased with increasing Ra and major part of the enclosure was occupied by colder fluid especially at high Ra.



(a)



(b)

Figure 2.1: Variation of average Nusselt number at the heated wall with:

(a) Rayleigh number (b) aspect ratio (Aydin et al., 1999).

The effect of vertical boundary conditions on the rate of heat transfer from a discrete heat source on the bottom of a horizontal enclosure was determined numerically by Sezai and Mohamad (2000). The enclosure was cooled from the above and insulated from the bottom. The multi-grid technique had been employed in solving the three-dimensional form of Navier-Stokes equations. Rayleigh number, *Ra* based on the enclosure height was varied from 10^3 until unstable flow was predicted for a fixed Prandtl number, *Pr* of 0.71. Aspect ratio of the source was varied until it fully covered the entire width of the bottom plate. It was encountered that the vertical wall boundary conditions did not influence the rate of heat transfer. The limit of the maximum *Ra* to obtain a convergent solution decreased with increasing aspect ratio of the source. The heat transfer was maximum at the edges of the discrete heater and minimum at the center. The edge effects decreased as the length of the heater increased.

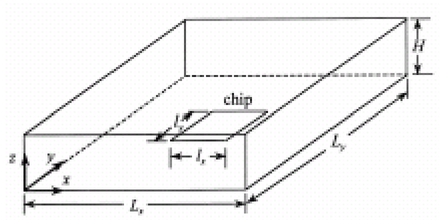


Figure 2.2: Schematic diagram of physical configuration and the coordinate system (Sezai and Mohamad, 2000).

In the investigation of two-dimensional free convection in a cylindrical enclosure filled with heat generating anisotropic porous medium, Dhanasekaran et al. (2002) had demonstrated a numerical study to observe the effects of anisotropic permeability ratio, K and thermal diffusivity ratio, λ. The heat generating porous medium was contained in a vertical cylinder enclosure with isothermal wall while the top and bottom were perfectly insulated surfaces. It had been proven that the flow field and heat transfer were significantly influenced by anisotropy. The non-dimensional maximum cavity temperature increased with the increment in permeability ratio. For aspect ratio, $A \geq 2$, the non-dimensional maximum cavity temperature increased with increasing thermal diffusivity ratio. For $A = 1$, there was a critical value of thermal diffusivity ratio at which the maximum cavity temperature was minimum. This critical value increased with increasing permeability ratio.

The dependence of aspect ratio of a container, A_R on magnetic damping of natural convection in low-conducting aqueous solution was analyzed by Wang and Wakayama (2002). The fluid in a three-dimensional rectangular container was heated from the vertical wall and cooled from the opposing vertical wall while the other walls were thermally insulated. It was studied numerically for $0 \leq Ha \leq 36.67$, $1000 \leq Gr \leq 39810$ and $0.1 \leq A_R \leq 1.0$. The ratio of average Nusselt number, $\overline{Nu}$

with and without magnetic field had been used to quantify the damping efficiency of natural convection. The results revealed that the magnetic damping of natural convection was strongly dependent on the aspect ratio of the container and it had an optimum value for minimizing natural convection. The optimum value of the aspect ratio was almost independent of Hartmann number, *Ha* and it showed the tendency to decrease with increasing Grashof number, *Gr*.

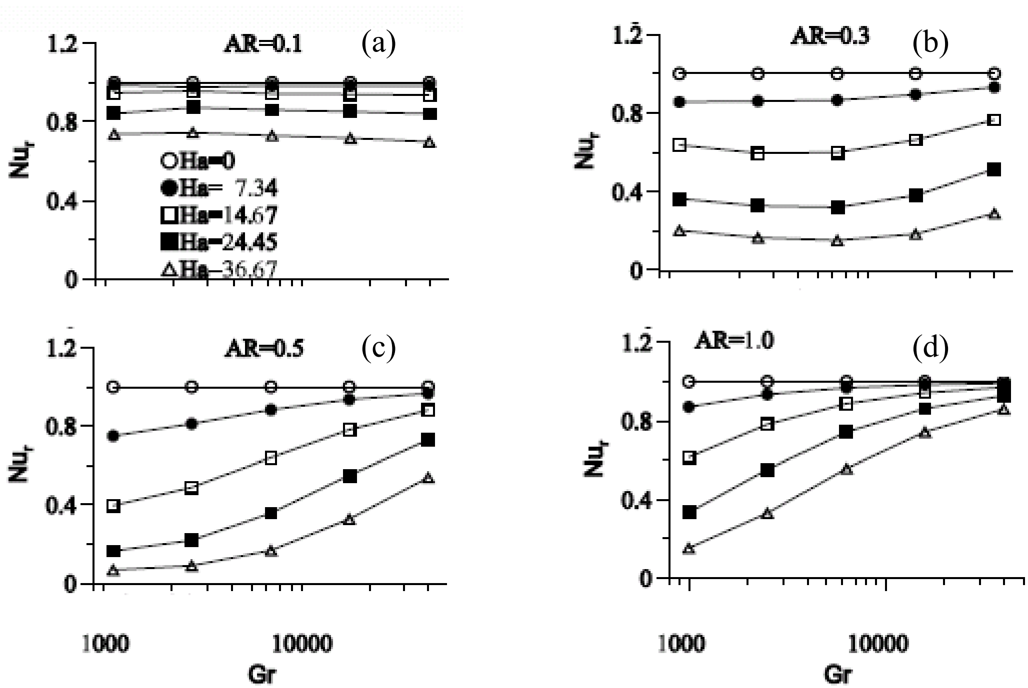


Figure 2.3: Dependence of Nu_r on Gr and Ha for various A_R: (a) 0.1 (b) 0.3 (c) 0.5 (d) 1.0 (Wang and Wakayama, 2002).

A numerical simulation had been carried-out by Das et al. (2003) to investigate the effects of surface waviness and aspect ratio on heat transfer characteristics inside a wavy walled of a two-dimensional enclosure. The enclosure consisted of two parallel wavy walls and two straight walls. The top and bottom walls were wavy and kept isothermal while the two straight vertical sidewalls were considered adiabatic. The range of surface waviness ratio, λ = 0.00 - 0.25, aspect ratio, A = 0.25 - 0.50 and Rayleigh number, $Ra = 10^0 - 10^7$ had been simulated for a fluid having Prandtl number, Pr = 1.0. The results determined that at different Ra, aspect ratio did not play any important role on the heat transfer characteristics at fixed surface waviness but it dominated when the surface waviness changed from zero to a certain value. The lower the surface waviness, the higher was the heat transfer for lower aspect ratio. At higher aspect ratio, heat transfer increased with increasing surface waviness. For a constant Ra and at low aspect ratio, heat transfer decreased gradually with increasing surface waviness up to a certain value whereas for higher aspect ratio, heat transfer gradually increased.

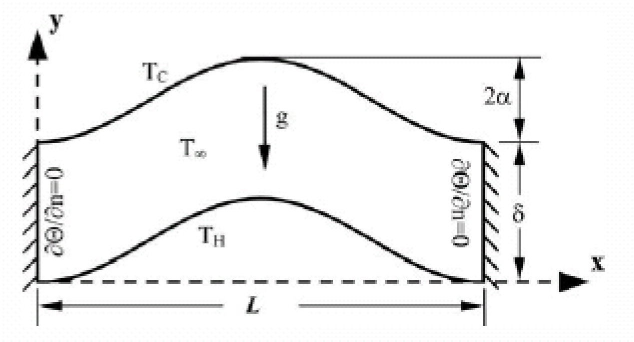


Figure 2.4: Schematic representation of the computational geometry under consideration with boundary conditions (Das et al., 2003).

D'Orazio et al. (2004) presented a study to find out the influence of aspect ratio towards the transient Rayleigh-Benard free convection inside a two-dimensional tall enclosure. The enclosure having the range of aspect ratio, A of $2 \leq A \leq 6$ and Rayleigh number, Ra of $10^3 \leq Ra \leq 2 \times 10^6$ was heated from the bottom and cooled from the top. The control volume formulation had been applied numerically and by using the Power Law scheme, the fluxes across the control volume were evaluated. It was clarified that the changing of Nusselt number, Nu was dependent on the flow transition either in sudden or more gradual. From the Nusselt number curve of the single-cell steady state, it showed that the distribution of critical *Ra* was increased with the increment in aspect ratio. For $A \geq 3$, in the course of increasing *Ra*, the flow pattern had been found evolving directly from the one-cell steady solution to the two-cell oscillatory solution.

Different aspect ratio, A_R of 1.0 and A_R of 3.0 had been focused by Joshi et al. (2006) in the analytical study of two-dimensional natural convection in a cavity with uniform volumetric heat generation. Two different boundary conditions were investigated for the cavity. In the first problem, the horizontal walls were considered to be adiabatic and the vertical walls were isothermal and in the second problem, all walls were considered isothermal. A stream function-vorticity formulation had been adapted where the variables were expanded in terms of Rayleigh number, *Ra*. It was observed that the horizontal component of velocity was smaller than the vertical component near the center and vertical walls of the cavity. For a square cavity with all walls isothermal and with horizontal walls insulated, the stream function plots appeared qualitatively similar but the actual values of the stream function were different in both cases. For the cavity with aspect ratio, $A_R = 3.0$, the stream function values for all walls isothermal were approximately half the corresponding values for the case of adiabatic horizontal walls.

In the investigation of coupled heat and mass transfer by natural convection occurring in a trapezoidal cavity, Hammami et al. (2007) had conducted a numerical study to evaluate the thermal and hydrodynamic behaviour of the system. The influence of the cavity dimensions on heat and mass transfer rates was also examined. The cavity vertical walls were thermally isolated and impermeable. On the lower surface, it was imposed with a constant high temperature and concentration while the upper surface was cooled at a constant temperature and had a zero concentration. In this study, the governing equations were solved by a finite volume technique providing the temperature, concentration and velocity fields. The obtained results proved that the flow configuration depended strongly on the aspect ratio. The flow changed from a predominantly mono-cellular pattern to a multi-cellular structure when the aspect ratio of the cavity was increased. The average Nusselt number, $\overline{Nu}$ was also found to increase when the length ratio increased.

The effect of aspect ratio on entropy generation in a rectangular cavity with differentially heated vertical walls was analyzed numerically by Ilis et al. (2008). The vertical walls of the cavities were at different constant temperatures while the horizontal walls were adiabatic. Heat transfer between vertical walls occurred by laminar natural convection. Based on the obtained dimensionless velocity and temperature values, the distributions of local entropy generation due to heat transfer and fluid friction are determined and related maps were plotted. The variation of the total entropy generation and average Bejan number for the whole cavity volume at different aspect ratio for different Rayleigh number, Ra was also evaluated. It was found that for a cavity with high Ra (i.e., $Ra = 10^5$), the fluid friction irreversibility was dominant and total entropy generation increased with aspect ratio, attained a maximum and it decreased. The total entropy generation in a cavity increased with Ra, however the increment rate depended on the aspect ratio.

2.2 Increasing Rayleigh Number

Several studies have been reported extensively in the literature concerning the influence of Rayleigh number, *Ra* on natural convection flow. During this phenomena, heat transfer through the walls of the cavity causes density changes to the fluid and leads to the buoyancy-driven recirculation, which resulting the flow to be treated as laminar or turbulent depending on the *Ra* (Markatos and Pericleous, 1984). Different convective forms were obtained from the flow patterns either at increasing or decreasing *Ra*.

A two-dimensional square enclosure with steady state constant property homogeneous fluid was examined by Kimura and Bejan (1983). The heat line visualization of convective heat transfer had been utilized in defining the transfer of heat by fluid flow. The numerical algorithm was computed by using the Allen-Southwell method of finite difference scheme (Allen and Southwell, 1955). The analysis was conducted at Rayleigh number, $Ra = 1.4 \times 10$ and $Ra = 1.4 \times 10^5$. The heat line pattern was observed in order to distinguish the effects of conduction and convection phenomena on the heat transfer. It was defined that at low *Ra*, the fluid flow caused a little disturbance to the flow of energy but at high *Ra*, the heat line had indicated the rising of heat on the energy flow as the transfer of energy was dominated by convection.

Fusegi et al. (1992) performed a high-resolution finite difference numerical study on a three-dimensional steady-state free convection of air in a cubical enclosure. The cavity was heated differentially at two vertical sidewalls while the remaining four walls were thermally insulated. The Rayleigh number, *Ra* was studied at the range of $10^3 \leq Ra \leq 10^6$. Examinations of the perspective fields revealed that the convective activities were intensified and significant z-variations were confined into narrower area close to the end walls as the Ra increased. The w velocity was found to be of magnitude smaller than the dominant velocities, u and v over the entire Rayleigh number range. The non-zero values of the w velocity were noticed in the end wall regions and this area became smaller as Ra increased. At high Ra, the secondary vortices formed along the vertical edges, thus affected the average Nusselt number, $\overline{Nu}$ distribution.

A steady laminar of two-dimensional natural convection flow of air and water in a square heated cavity was calculated by Henkes and Hoogendoorn (1993) for increasingly large Rayleigh number, Ra up to 10^{11}. The flow was simplified numerically by solving both the Navier-Stokes equations and the boundary-layer equations. The cavity had a hot left vertical wall and a cold right vertical wall whereas the floor and ceiling were both adiabatic. Four different streamline patterns were predicted when the Ra was increased up to 10^{11}. The last streamline pattern of $Ra > 5 \times 10^6$ contained four asymptotic structures, a vertical boundary layer along the heated wall, a core region, a corner region and a horizontal layer. For increasing Ra, the core became thermally stratified and had horizontal streamlines. For $Ra \to \infty$, the Navier-Stokes solution along the vertical wall converged to the boundary layer solution. Finite Ra affected the Navier-Stokes solution for the vertical layers and it was restricted to influence at the corners.

The conservation equations for laminar and turbulent flows in a two-dimensional square cavity for a series of Rayleigh number, *Ra* reaching values up to 10^{10} were solved numerically by Barakos et al. (1994). The simulations were undertaken using the control volume method approaching uniform and non-uniform grids. The k-ε model was adapted in turbulence modeling with and without logarithmic wall functions. The vertical walls of the cavity were kept isothermal with the left wall at high temperature and the right wall at low temperature. The horizontal walls were assumed to be perfectly adiabatic. The turbulent solution was found to have a laminar approximation with a non-zero turbulent viscosity for low values of $Ra < 10^6$. This solution was followed by a turbulent for $Ra > 10^6$ when the logarithmic wall functions were used and for $Ra > 10^8$ when the functions were only used for k and ε. The average Nusselt number, $\overline{Nu}$ along the hot wall showed a sudden increased when the turbulent solution was reached.

In the investigation of development of multi-cellular solutions in free convection of an air filled vertical cavity, Wakitani (1997) demonstrated a numerical simulation to find out the dependence of the flow structure on the initial condition. Consideration was given to the multiple solutions of two-dimensional in a tall cavity for a wide range of Rayleigh number, *Ra*. The left and right side walls were held at constant temperatures whereas the top and bottom of the cavity were thermally insulated. A multi-cellular flow with a three-cell pattern first appeared at $Ra = 8 \times 10^3$ for the initial condition of a motionless and isothermal state and for the condition by gradually increasing *Ra*. The differences were encountered in the cell patterns obtained from their conditions at the range of $1 \times 10^4 \leq Ra < 5 \times 10^4$ while the solutions for $Ra \geq 5 \times 10^4$ did not depend on the initial conditions. A time periodic unsteady solutions first appeared at *Ra* = 4 x 10^5 after the reverse transition had occurred.

The effects of dimension ratio and Rayleigh number, *Ra* on the flow structure and heat transfer were analyzed by Asan (2000). A steady-state laminar two-dimensional natural convection in an annulus between two isothermal concentric square ducts was considered numerically. The stream function-vorticity formulation and the control volume integration solution technique had been applied in this study. Solutions were obtained up to a *Ra* of 10^6 with different dimension ratio. The results proved that both dimension ratio and *Ra* had influenced the flow field and temperature. With increasing *Ra* and dimension ratio, secondary vortices between upper sides of the squares started to develop. The heat transfer results determined that the average Nusselt number, $\overline{Nu}$ were mainly dependent on the *Ra* and dimension ratio. The influence of low *Ra* on average Nusselt number, $\overline{Nu}$ was not significant compared to the influence of dimension ratio.

A numerical simulation had been conducted by Aydin and Yang (2000) on free convection of air in a two-dimensional rectangular enclosure with localized heating from below and symmetrically cooling from the sides. Localized heating was simulated by a centrally located heat source on the bottom wall with four different values of dimensionless heat source length. Rayleigh number, *Ra* values from 10^3 to 10^5 had been evaluated for the analysis. It was observed that for small *Ra*, the heat transfer was dominated by conduction across the fluid layer while for high *Ra*, the process was primarily convection whereas the effect of conduction vanished. The heat transfer was enhanced by the increasing of the non-dimensional heat source thickness, $\in$ especially at high *Ra*.

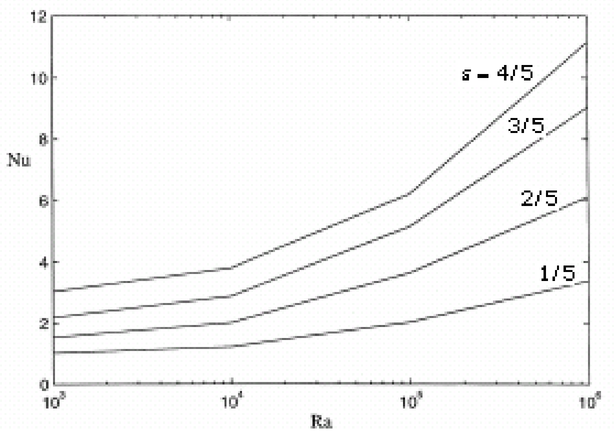


Figure 2.5: Variation of the average Nusselt number, $\overline{Nu}$ at the heated wall with the Rayleigh number for various non-dimensional heat source lengths, $\in$ (Aydin and Yang, 2000).

Chang and Tsay (2001) carried-out research on a laminar natural convection in an enclosure induced by a heated backward step. The study was restricted in defining the dependence of the geometry size, Rayleigh number, *Ra* and Prandtl number, *Pr* on the flow structure and heat transfer. The range of Rayleigh number, $10^3 \leq Ra \leq 10^7$ and *Pr* of 0.71 had been selected. The solutions were solved numerically by adapting the finite difference method. It was clarified that at $Ra = 10^7$, the presence of the backward step had influenced the heat transfer coefficient while it enhanced with decreasing *Ra*. At lower *Ra*, the influence of the dimensionless distance between the heated and cooled plate showed more significant on heat transfer rate.

Rayleigh number, *Ra* in the range of 10^2 to 10^8 had been focused by Sarris et al. (2002) in the numerical study of free convection in a two-dimensional rectangular enclosure with sinusoidal temperature profile on the upper wall. The bottom and sidewalls were in adiabatic conditions. The applied sinusoidal temperature was symmetric with respect to the mid-plane of the enclosure. The circulation patterns were determined to increase in intensity and their centers moved toward the upper wall corners with increasing *Ra*. As a result, the thermal boundary layer was confined near the upper wall regions. For small *Ra*, the heat transfer was dominated by conduction across the fluid layers. The increasing *Ra* caused the domination changed to convection as the values of the maximum and minimum local Nusselt number, *Nu* at the upper wall increased.

A time-dependent laminar natural convection of air-cooling in a vertical rectangular enclosure with three discrete flush-mountered heaters was investigated by Hyun and Bae (2004). The finite difference procedure had been employed to solve the system equations. The numerical analysis was performed for a tall enclosure at Rayleigh number, *Ra* varied from 10^5 to 10^7. The buoyant convection due to multiple heat sources was analyzed. It was revealed that the cycle average temperature of all heaters was affected by the periodic change at low *Ra*. As the period of change in thermal condition reached the immediate values at higher *Ra*, the temperatures of the heaters reached the peak values.

A transient free convection in a two-dimensional square cavity filled with a porous medium was numerically studied by Saeid and Pop (2004). The left and right vertical wall was suddenly heated and cooled respectively to a constant temperature by equal amount relative to an initially uniform temperature distribution. Both the horizontal walls were adiabatic. The finite volume method had been utilized in solving the non-dimensional governing equations for Rayleigh number, *Ra* values of 10^2 to 10^4. It was observed that the average Nusselt number, $\overline{Nu}$ had shown undershoot followed by a constant steady state value for all $Ra = 10^2$ to 10^4 during the transient period. It was also defined that the time required to reach the steady state was longer for low *Ra* and shorter for high *Ra*.

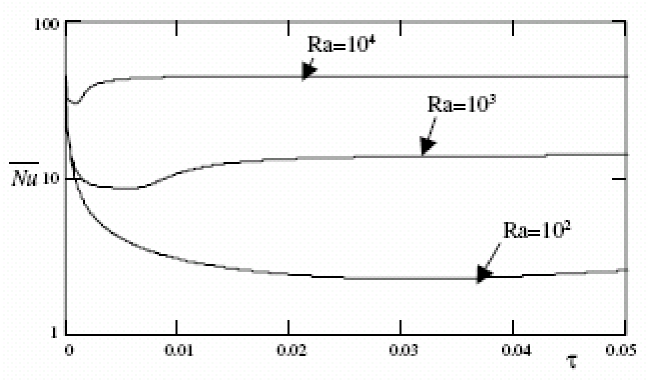


Figure 2.6: Variation of the transient average Nusselt number with τ at different Rayleigh number (Saeid and Pop, 2004).

The study analyzing the development of heat transfer inside a cavity at the increasing of the heat source length was tested experimentally and numerically by Calcagni et al. (2005). Natural convective heat transfer was considered in a two-dimensional square enclosure characterized by a discrete heater located on the lower wall and cooled from the lateral walls. Convection had been examined for Rayleigh number, Ra from 10^3 to 10^6. The local Nusselt number, Nu was evaluated on the heat source surface and it showed a symmetrical form raising near the heat source borders. Both the experimental and numerical investigation had pointed out that heat transfer was prevalently in conductive mode for $Ra \leq 10^4$ while the convective phenomena developed completely for $Ra \cong 10^5$. For high Ra, an increased of the heat source dimension had produced a raised in heat transfer.

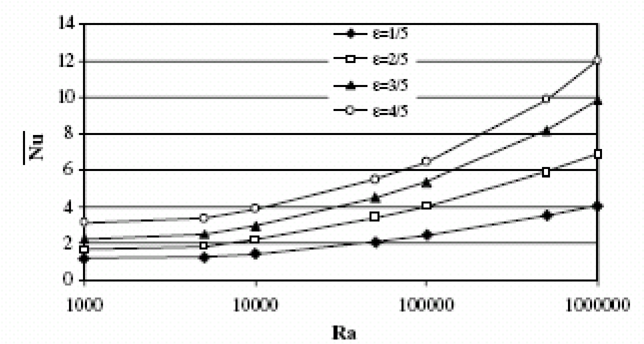


Figure 2.7: Numerical $\overline{Nu}$ -Ra for several dimensionless heat source lengths (Calcagni et al., 2005).

Dalal and Das (2007) carried-out research on natural convection in a complicated cavity heated from top with sinusoidal temperature and cooled from other sides. A steady, laminar flow in a two-dimensional enclosure with three flat and one wavy wall, was numerically investigated. Air was considered as the working fluid. This problem was numerically solved by SIMPLE algorithm with deferred QUICK scheme in non-orthogonal curvilinear coordinates. The mesh generation had been done by solving the partial differential equation with grid control functions. Tests were carried-out for Rayleigh number, Ra from 10^0 to 10^6 while the Prandtl number, Pr was kept constant. The effect of Ra on the flow pattern and heat transfer had been studied. The mode of heat transfer was conductive for the range of

Ra = 10^0 - 10^3. This had been observed by the deep penetration of the isotherms inside the domain. When the Ra was increased from 10^4 to 10^6, the mode of heat transfer changed transition and finally to convection. In these cases, the isotherms gradually came closer to the heated wall and they formed a boundary layer type of pattern.

Rayleigh number, Ra in the range of 10^3 to 10^6 had been considered by Ouertatani et al. (2008) in the numerical simulation of two-dimensional Rayleigh-Benard convection in an enclosure. A numerical approach based on the finite volume method and a full multigrid acceleration was used. Fine grids were used and Benchmark solutions were proposed for Ra ranging from 10^3 to 10^6. Some streamlines and isotherms were presented to analyze the natural convective flow patterns set up by the buoyancy force. The steady solution obtained for Ra = 10^3 was used as an initial one for the next Ra and so on. As the velocity distribution indicated, the boundary layer was more closely confined to the walls with increased in the Ra. It was also observed that the velocity norm increased with the Ra, meaning that convection dominated at high Ra. For Ra = 10^4, the flow was symmetrical and was dominated by a recirculating motion in the core region. By increasing Ra, two secondary eddies were then observed at the top left and bottom right corners.